

CAPACITY ANALYSIS OF A TH-PPM UWB SYSTEM USING A NEAR-INTERFERENCE ERASURE SCHEME IN MULTI-USER ENVIRONMENTS

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ABSTRACT

In this paper, the capacity of a TH-PPM UWB system with an outage constraint in multi-user environments is mathematically analyzed and evaluated. To mitigate a near-far problem, a near-interference erasure scheme is adopted and the capacity of the TH-PPM UWB system using the near-interference erasure scheme is evaluated through numerical analysis and Gaussian approximation. The numerical results show that the capacity of the TH-PPM UWB system using the near-interference erasure scheme is significantly improved and there exists an optimum threshold for the near-interference erasure scheme which maximizes the capacity. The capacity which is mathematically derived is very similar to that of numerical results in the region near the optimum threshold. Furthermore, the results are more accurate as the number of bit repetitions increases.

I INTRODUCTION

An ultra-wideband (UWB) impulse-based radio (IR) technology is one of promising candidates for next generation wireless personal area network communications. As a modulation and multiple access scheme for UWB systems, a time hopping pulse position modulation (TH-PPM) was proposed [1] and widely studied [2] [3] [4]. The TH-PPM UWB systems in multi-user environments suffer from a near-far problem which degrades the performance of a receiver due to transmission of other transmitters which are much closer to the receiver than a target transmitter. In cellular systems, the near-far problem can be resolved by a power control method. However, in the TH-PPM UWB system where users are distributed in an ad-hoc manner, a near-far problem still exists even if it uses the power control method in its own communication link due to simultaneous transmissions of transmitters in other communication links.

To solve the near-far problem in ad-hoc networks, several schemes have been proposed. First, a spatial exclusion scheme was proposed to set up an exclusive region so that transmissions are inhibited in order to reduce the effect of the near-interference [5]. This spatial exclusion scheme can significantly improve the capacity when an optimum size of the exclusion region is set. However, to implement the scheme, an appropriate scheduling mechanism in medium access control (MAC) layer is needed and additional control messages are required. Near-interference erasure schemes were proposed to eliminate a received pulse affected by near-interference among repeated pulses and to decide a bit using only remaining pulses [3] [6]. The BER performance of the TH-PPM UWB system using the near-interference erasure scheme was analyzed [4].

This scheme is limited in the system with bit repetitions. However, it does not need additional overheads in MAC layer and it can be simply implemented in physical layer.

In this paper, the capacity of a general TH-PPM UWB system and the capacity of a TH-PPM UWB system using a near-interference erasure scheme are mathematically analyzed with given outage constraints. The mathematical result is evaluated and compared with numerical results by simulation. Moreover, an optimum threshold for the near-interference erasure scheme is shown through mathematical analysis and numerical results.

This paper is organized as follows. In Section II, a system model is introduced to analyze system capacity mathematically. The system capacity of both a general TH-PPM UWB system and a TH-PPM UWB system using a near-interference erasure scheme is mathematically analyzed with given outage constraints in Section III. Both analytical results are compared with numerical results by simulation in Section IV. Finally, conclusions are presented in Section V.

II SYSTEM MODEL

Fig. 1 shows a multi-user environment of a TH-PPM UWB system. A target receiver is located at the center and multiple communication links can be simultaneously made in a given area \mathbf{R} . For each communication link, every transmitter may transmit data to its own receiver and it may interfere with the target receiver. In this paper, transmitters which interfere with the target receiver are distributed according to a Poisson Point Process (PPP) expressed as Eq. (1).

$$\Pi = \{X_i\} \text{ on a plane } \mathbb{R}^2 \text{ with parameter } \lambda_{\Pi}, \quad (1)$$

where X_i is a random variable representing the location of the transmitter of communication link i and λ_{Π} is a parameter which represents the density of transmitters.

An outage event occurs when the received signal-to-interference plus noise ratio (SINR) of a receiver in each communication link is less than a target SINR value. An outage constraint is a condition where the outage probability is less than or equal to a required target threshold to guarantee the system performance. If the outage constraint is satisfied, the error rate of received signal is less than the target threshold and the system performance can be guaranteed. We have the following three assumptions: 1) The transmission power in all communication links is set to ρ . 2) The distance between the transmitter and receiver in all communication links is set to r . 3) The received power for each receiver is expressed as $\rho r^{-\alpha}$ where α is the path loss exponent.

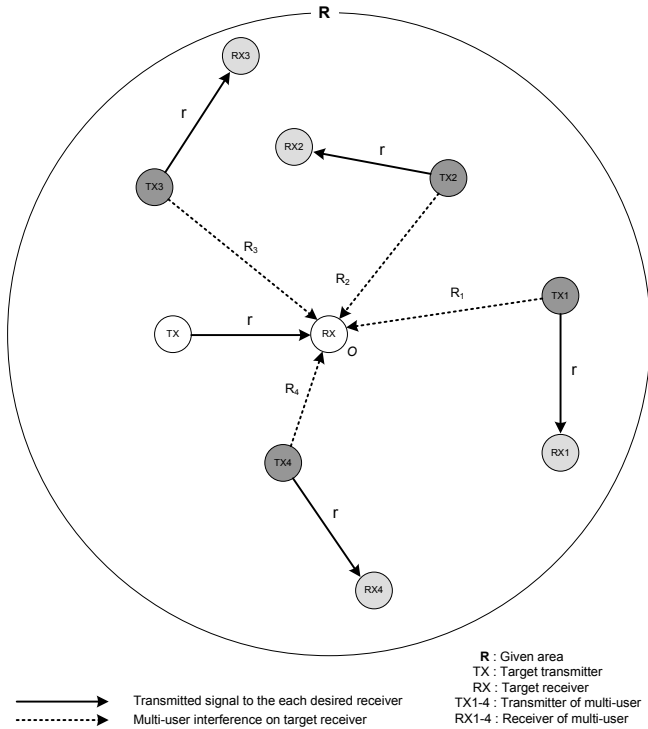


Figure 1: Multi-User Environment of a TH-PPM UWB System

Weber et al. [7] derived the system capacity of DS-CDMA and FH-CDMA with outage constraints. We first derive the outage constraint of a TH-CDMA system before deriving the outage constraint of a TH-PPM UWB system. The outage constraint of the TH-CDMA system is written as follows:

$$\mathbb{P}^o \left(\frac{\rho r^{-\alpha}}{\eta + \sum_{i \in \Pi_m} \rho R_i^{-\alpha}} \leq \beta \right) \leq \epsilon, \quad (2)$$

where $\rho r^{-\alpha}$ is the received power, η is the noise power within one time slot, R_i is the distance between the target receiver and the transmitter of communication link i which interferes with the target receiver, Π_m is a set of transmitters, which interfere with the target receiver within the time slot m , and the transmitters are distributed according to a PPP with density parameter λ . β denotes the target SINR and ϵ is the target threshold of the outage constraint.

In Eq. (2), $\sum_{i \in \Pi_m} \rho R_i^{-\alpha}$ represents the multi-user interference. Sousa and Silvester [8] derived a distribution of this multi-user interference for transmission power $\rho = 1$ and path loss exponent $\alpha > 2$ and showed that there exists a closed-form solution for only $\alpha = 4$. The probability density function (pdf) and the cumulative density function (cdf) of multi-user interference are derived as follows for $\alpha = 4$.

$$f_Y(y) = \frac{\pi}{2} \lambda y^{-\frac{3}{2}} e^{-\frac{\pi^3 \lambda^2}{4y}}. \quad (3)$$

$$F_Y(y) = \text{erfc} \left(\frac{\pi^{\frac{3}{2}} \lambda}{2\sqrt{y}} \right) = 2Q \left(\frac{\pi^{\frac{3}{2}} \lambda}{\sqrt{2y}} \right), \quad (4)$$

where Y is a random variable representing the multi-user interference and λ is the density of multi-user distributed according to a PPP.

III MATHEMATICAL ANALYSIS OF SYSTEM CAPACITY

A Capacity Analysis of a General TH-PPM UWB System

The system capacity obtained from a given outage constraint is defined as the maximum permissible user density per unit area so that maximum simultaneous transmissions are allowable in the unit area.

The TH-PPM UWB system primarily has bit repetitions due to limited transmission power. The outage constraint of the TH-PPM UWB system with bit repetitions is expressed as follows:

$$\mathbb{P}^o \left(\frac{N_s^2 \rho r^{-\alpha}}{N_s \eta + \sum_{n=1}^{N_s} \sum_{i \in \Pi_m} \rho R_{i,n}^{-\alpha}} \leq \beta \right) \leq \epsilon, \quad (5)$$

where N_s is the number of bit repetitions, $R_{i,n}$ is the distance between the target receiver and the transmitter of communication link i when the target transmitter transmits the n -th symbol. The received power at the target receiver has an N_s square term since it is coherently correlated. However, the noise power has an N_s term and the multi-user interference term has the sum of N_s random variables each of which represents the multi-user interference of the TH-PPM UWB system without bit repetitions since they are non-coherently correlated. Multi-user interference in Eq. (5) is the sum of interference induced by transmitters included in Π_m during N_s time slots. Multi-user interference in each time slot follows a Poisson distribution. The Poisson distribution is identical and independent distribution. Therefore, the multi-user interference superposed during N_s time slots is identical to that with N_s times larger density in one time slot. If Π' is a PPP with the density parameter $N_s \lambda$, Eq. (5) is expressed as follows:

$$\mathbb{P}^o \left(\frac{N_s^2 \rho r^{-\alpha}}{N_s \eta + \sum_{i \in \Pi'} \rho R_i^{-\alpha}} \leq \beta \right) \leq \epsilon, \quad (6)$$

where $\kappa = N_s \frac{r^{-\alpha}}{\beta} - \frac{\eta}{\rho}$.

Eq. (6) is expressed for the multi-user interference term as follows:

$$\mathbb{P}^o \left(\sum_{i \in \Pi'} R_i^{-\alpha} \geq N_s \kappa \right) \leq \epsilon. \quad (7)$$

Let the multi-user interference in Eq. (7) be $Y = \sum_{i \in \Pi'} R_i^{-\alpha}$. Eq. (7) is derived for the density of multi-user λ using Eq. (4) and the maximum value of λ indicates the system capacity. Therefore, the capacity of the TH-PPM UWB system with N_s bit repetitions is expressed as follows when the path loss exponent $\alpha = 4$.

$$C_o = \sqrt{\frac{2}{\pi^3} \frac{\kappa}{N_s}} Q^{-1} \left(\frac{1 - \epsilon}{2} \right). \quad (8)$$

B Capacity Analysis of a TH-PPM UWB System with a Near-Interference Erasure Scheme

Multi-user interference in a TH-PPM UWB system with a near-interference erasure scheme within one time slot is expressed as follows.

$$\bar{Y} = \begin{cases} \sum_{i \in \Pi_m} R_i^{-\alpha} & , y \leq \gamma_Y \\ 0 & , y > \gamma_Y. \end{cases} \quad (9)$$

where γ_Y is a threshold of the near-interference erasure scheme for interference intensity, \bar{Y} is the truncated multi-user interference which affects a symbol among N_s repeated symbols when we apply a near-interference erasure scheme. The survival probability δ that the multi-user interference is less than a given threshold γ_Y for interference intensity is written as Eq. (10).

$$\delta = F_Y(\gamma_Y) = \text{erfc} \left(\frac{\pi^{\frac{3}{2}} \lambda}{2\sqrt{\gamma_Y}} \right). \quad (10)$$

Therefore, if the number of bit repetitions N_s is sufficiently large, the outage constraint of the TH-PPM UWB system using the near-interference erasure scheme is expressed as Eq. (11) and Eq. (12).

$$\mathbb{P}^o \left(\frac{(\delta N_s)^2 \rho r^{-\alpha}}{\delta N_s \eta + \rho \sum_{n=1}^{N_s} \bar{Y}_n} \leq \beta \right) \leq \epsilon. \quad (11)$$

$$\mathbb{P}^o \left(\sum_{n=1}^{N_s} \bar{Y}_n \geq \delta N_s \kappa \right) \leq \epsilon, \quad (12)$$

where \bar{Y}_n is the truncated multi-user interference of the TH-PPM UWB system using the near-interference erasure scheme for the n -th symbol and $\kappa = N_s \frac{r^{-\alpha}}{\beta} - \frac{\eta}{\rho}$.

Let $\bar{Y}_{tot} = \sum_{n=1}^{N_s} \bar{Y}_n$ in Eq. (12). Since \bar{Y}_{tot} is accumulated for all repeated symbols after the heavy tail distribution of Y is removed by the near-interference erasure scheme, it can be approximated as a Gaussian distribution. To approximate it as a Gaussian distribution, we find mean and variance of multi-user interference for symbols. The mean $\mu_{\bar{Y}}$ and the variance $\sigma_{\bar{Y}}^2$ are expressed as follows:

$$\begin{aligned} \mu_{\bar{Y}} &= \int_0^{\infty} y dF_{\bar{Y}}(y) \\ &= \int_0^{\gamma_Y} y \cdot f_Y(y) dy \\ &= \frac{\pi \lambda}{2} \left[2\sqrt{\gamma_Y} \cdot \exp \left(\frac{-\pi^3 \lambda^2}{4\gamma_Y} \right) - \pi^2 \lambda \cdot \text{erfc} \left(\frac{\pi^{\frac{3}{2}} \lambda}{2\sqrt{\gamma_Y}} \right) \right]. \end{aligned} \quad (13)$$

$$\begin{aligned} \sigma_{\bar{Y}}^2 &= \int_0^{\infty} (y - \mu_{\bar{Y}})^2 dF_{\bar{Y}}(y) \\ &= \int_0^{\gamma_Y} (y - \mu_{\bar{Y}})^2 \cdot f_Y(y) dy \\ &= \int_0^{\gamma_Y} y^2 \cdot f_Y(y) dy - \mu_{\bar{Y}}^2 \\ &= \frac{\pi \lambda}{12} \left[\left(4\gamma_Y^{\frac{3}{2}} + 2\pi^3 \lambda^2 \sqrt{\gamma_Y} \right) \cdot \left(6\text{erfc} \left(\frac{\frac{3}{2}\lambda}{2\sqrt{\gamma_Y}} \right) - 1 \right) \right. \\ &\quad \cdot \left(\exp \left(\frac{-\pi^3 \lambda^2}{4\gamma_Y} \right) \right) - \pi \lambda \gamma_Y \cdot \exp \left(\frac{-\pi^3 \lambda^2}{2\gamma_Y} \right) \\ &\quad \left. - \pi^5 \lambda^3 \cdot \text{erfc} \left(\frac{\pi^{\frac{3}{2}} \lambda}{2\sqrt{\gamma_Y}} \right) \cdot \left(3\text{erfc} \left(\frac{\frac{3}{2}\lambda}{2\sqrt{\gamma_Y}} \right) - 1 \right) \right]. \end{aligned} \quad (14)$$

Since each symbol is independently transmitted in other time slot, the mean and variance of multi-user interference for a bit are the same as N_s times of $\mu_{\bar{Y}}$ and $\sigma_{\bar{Y}}^2$, respectively. Therefore, the mean and variance of \bar{Y}_{tot} are expressed as follows:

$$\mu_{\bar{Y}_{tot}} = N_s \cdot \mu_{\bar{Y}}. \quad (15)$$

$$\sigma_{\bar{Y}_{tot}}^2 = N_s \cdot \sigma_{\bar{Y}}^2. \quad (16)$$

The truncated multi-user interference of the TH-PPM UWB system using the near-interference erasure scheme is approximated as a Gaussian distribution using Eqs. (15) and (16). Its pdf and cdf can be written as Eqs. (17) and (18), respectively.

$$f_{\bar{Y}_{tot}}(y) = \frac{1}{\sqrt{2\pi\sigma_{\bar{Y}_{tot}}^2}} \exp \left(-\frac{(y - \mu_{\bar{Y}_{tot}})^2}{2\sigma_{\bar{Y}_{tot}}^2} \right). \quad (17)$$

$$\begin{aligned} F_{\bar{Y}_{tot}}(y) &= \int_{-\infty}^y \frac{1}{\sqrt{2\pi\sigma_{\bar{Y}_{tot}}^2}} \exp \left(-\frac{(y - \mu_{\bar{Y}_{tot}})^2}{2\sigma_{\bar{Y}_{tot}}^2} \right) dz \\ &= \int_{-\infty}^{\frac{y - \mu_{\bar{Y}_{tot}}}{\sigma_{\bar{Y}_{tot}}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= 1 - Q \left(\frac{y - \mu_{\bar{Y}_{tot}}}{\sigma_{\bar{Y}_{tot}}} \right). \end{aligned} \quad (18)$$

If we apply Eq. (18) to Eq. (12), then Eq. (12) is derived as follows:

$$\begin{aligned} \mathbb{P}^o (\bar{Y}_{tot} \geq \delta(\lambda, \gamma_Y) \cdot N_s \cdot \kappa) &= 1 - \mathbb{P}^o (\bar{Y}_{tot} \leq \delta(\lambda, \gamma_Y) \cdot N_s \cdot \kappa) \\ &= 1 - F_{\bar{Y}_{tot}} (\delta(\lambda, \gamma_Y) \cdot N_s \cdot \kappa) \\ &= Q \left(\frac{\delta(\lambda, \gamma_Y) \cdot N_s \cdot \kappa - \mu_{\bar{Y}_{tot}}(\lambda, \gamma_Y)}{\sigma_{\bar{Y}_{tot}}(\lambda, \gamma_Y)} \right) \leq \epsilon. \end{aligned} \quad (19)$$

Table 1: Parameters for Numerical Results

Notation	Description	Value
α	Path loss exponent	4
r	Transmission radius	10 m
ρ	Transmission power	0 dBm
η	Noise power spectral density (SNR)	-60 dB (20 dB)
β	Target SINR	10 dB
ϵ	Outage probability requirement	0.05

The maximum value of λ that satisfies a given outage constraint is the system capacity. Consequently, the capacity of the TH-PPM UWB system using the near-interference erasure scheme for a given threshold value γ_Y is represented as Eq. (20).

$$\tilde{C}_o = \frac{1}{N_s} \operatorname{argmax}_{\lambda} \left[\delta(\lambda) \cdot N_s \cdot \kappa - \mu_{\tilde{Y}_{tot}}(\lambda) - Q^{-1}(\epsilon) \cdot \sigma_{\tilde{Y}_{tot}}(\lambda) = 0 \right]. \quad (20)$$

Since additional time resources are consumed and the capacity is reduced as the number of bit repetitions N_s increases, the capacity is normalized by the number of bit repetitions N_s .

IV NUMERICAL RESULTS

Table 1 shows the parameters used to obtain numerical results. Fig. 2 represents the capacity of a general TH-PPM UWB system for varying the number of bit repetitions using the above parameters. As shown in the figure, the capacity of the general TH-PPM UWB system decreases as the number of bit repetitions N_s increases because the probability that at least one symbol among all the symbols is affected by the near interferers becomes larger as N_s increases. The near-interference significantly degrades the received SINR. If the TH-PPM UWB system adopts a soft decision method which decides a bit by adding all of the symbol values, then the error probability increases because a symbol affected by near interferers has a dominant value, compared with other symbol values. Therefore, the TH-PPM UWB system using the soft decision method needs a scheme to efficiently remove the effect of near-interference.

Fig. 3 shows the outage probability of a general TH-PPM UWB system and a TH-PPM UWB system using a near-interference erasure scheme for varying the density of multi-user λ when the number of bit repetitions $N_s=10$. γ_{SIR} represents the threshold value for signal-to-interference ratio (SIR) of the near-interference erasure scheme. This value has a relation with a threshold value for interference intensity γ_Y , $\gamma_{SIR} = \frac{\rho r^{-\alpha}}{\gamma_Y}$. We represent a threshold value to γ_{SIR} instead of γ_Y to give intuition for the threshold value. In Fig. 3, threshold values for SIR of the TH-PPM UWB system using the near-interference erasure scheme is set to -10 dB, 0 dB,

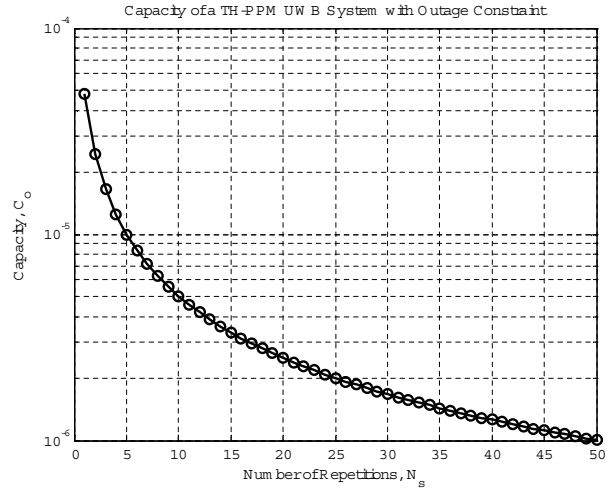


Figure 2: Capacity of a General TH-PPM UWB System

and 10 dB. The optimum threshold value for $N_s = 10$, which minimizes the outage probability, is 0 dB. Every outage probability increases up to 1 with increasing the density of multi-user λ because increase of λ means to increase the amount of multi-user interference. A solid line represents the outage probability of a general TH-PPM UWB system without a near-interference erasure scheme and a dashed line represents the outage probability of a TH-PPM UWB system using a near-interference erasure scheme with an optimum threshold value. The outage probability of the TH-PPM UWB system using the near-interference erasure scheme with an optimum threshold is significantly smaller than that of the general TH-PPM UWB system. In other aspect, if we consider the density of multi-user λ for a specific outage probability, then the λ value means the system capacity with an outage constraint. For example, if we choose a specific outage probability to 0.05, then λ values of the general TH-PPM UWB system and the TH-PPM UWB system using the near-interference erasure scheme have 1.5×10^{-5} and 1.25×10^{-3} , respectively. In this case, each λ value is the capacity of each system and the capacity of the TH-PPM UWB system using the near-interference erasure scheme is about 80 times larger than that of the general TH-PPM UWB system. Therefore, If we apply the near-interference erasure scheme to the TH-PPM UWB system with an optimum threshold, then the system capacity can be significantly improved.

Fig. 4 shows the capacity of a TH-PPM UWB system using a near-interference erasure scheme. We obtain numerical results by using a Monte Carlo simulation and mathematical results by using a Gaussian approximation. Marks represent the numerical results and lines indicate the mathematical results. As the γ_{SIR} value increases, most of received symbols are erased and the capacity is reduced because symbols are erased although the received power is sufficiently larger than the multi-user interference. In the contrast, as the γ_{SIR} value decreases, it is similar to the case that the TH-PPM UWB system dose not adopt the near-interference erasure scheme because symbols are only erased when multi-user interference is significantly larger than the received power. The capacity for a given N_s

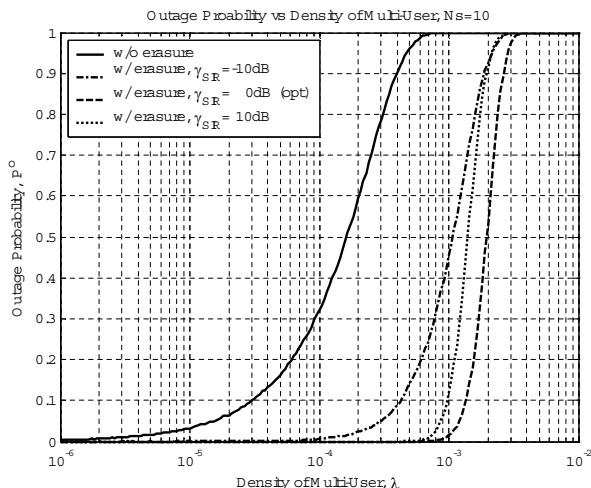


Figure 3: Outage Probability Comparison between a General TH-PPM UWB System and TH-PPM UWB Systems Using a Near-Interference Erasure Scheme

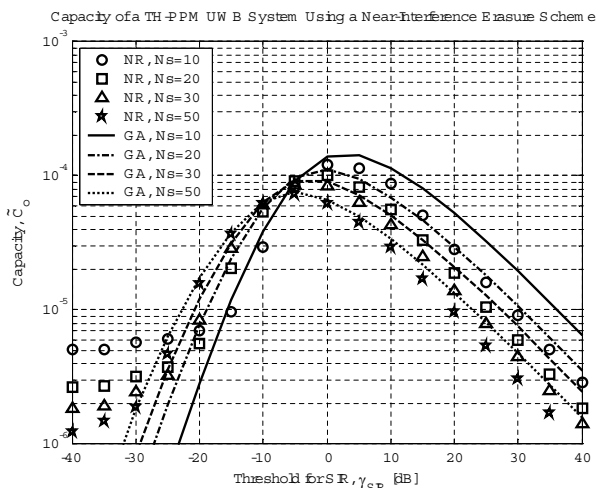


Figure 4: Capacity of a TH-PPM UWB System Using a Near-Interference Erasure Scheme

has a maximum value near $\gamma_{SIR} = 0$ dB. As the parameter N_s value increases, the maximum capacity is slightly reduced and the optimum threshold value which maximizes the capacity is also slightly reduced. This means that it is optimum to erase symbols for larger interference as the N_s value increases. As a threshold value γ_{SIR} is out of the optimum threshold range, the distribution of multi-user interference has a difference from a Gaussian distribution. However, the capacity mathematically derived by using a Gaussian approximation is very similar to that of numerical results in the region where the capacity is maximized by the near-interference erasure scheme with an optimum threshold. Hence, we can mathematically derive the maximum capacity of the TH-PPM UWB system with the near-interference erasure scheme. Furthermore, the capacity is more accurate as the number of bit repetitions N_s increases.

V CONCLUSION

In this paper, we mathematically analyzed the capacity of a general TH-PPM UWB system. Its performance is degraded by near-interference. To mitigate the near-far problem, we adopted a near-interference erasure scheme and mathematically derived the capacity of the TH-PPM UWB system using the near-interference erasure scheme through a Gaussian approximation. Moreover, we showed that there exists an optimum threshold for the near-interference erasure scheme which maximizes the capacity. Finally, we also showed the optimum threshold value through mathematical analysis and numerical results.

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