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PERFORMANCE ANALYSIS OF TWO RELAY SELECTION SCHEMES FOR COOPERATIVE DIVERSITY

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ABSTRACT

We propose two relay selection in cooperative relay communications. In a fixed scheme, M multiple relays that have strong signal strength are selected out of K relays and forward their received data from a source node to a destination node. As an alternative approach, a threshold-based adaptive relay selection scheme is also proposed to minimize the number of forwarding relays while satisfying a given outage requirement because if the number of forwarding relays increases, then the number of interfering sources also increases. The minimum number of relays that can prevent an outage event are selected to forward data to a destination. The performance of both schemes are evaluated through numerical analysis and Monte-Carlo simulations in terms of end-to-end outage probability and the number of forwarding relays. The result presents a bound that the fixed and adaptive relay selection schemes can achieve information-theoretically. Furthermore, the outage performance of the adaptive relay selection scheme is identical to that of the fixed relay selection scheme with $M = K$, while the number of forwarding relays is much less than that of the fixed relay selection scheme with $M = K$.

I. INTRODUCTION

In wireless communications, diversity techniques using antenna arrays or RAKE receivers can effectively combat against fading [1], [2]. In addition to those conventional diversity techniques, diversity gains can be also achieved through cooperation among geographically distributed nodes or terminals. This cooperative diversity was first studied in [3], [4] and low-complexity cooperative diversity protocols were proposed and analyzed in [5], where two user cooperation strategies that use a single relay node were studied. The cooperative diversity techniques using multiple relays have been recently proposed and studied [6], [7]. Specifically, the opportunistic selection of a relay among multiple relays was proposed and analyzed in [7]. The opportunistic relay selection was shown to be theoretically optimal when only a small amount of feedback information such as channel quality indicator (CQI) is allowed from a destination node to the relays. From an analytical point of view, the opportunistic relay selection corresponds to a conventional transmit selection diversity technique.

However, if each node knows a channel gain from each relay to its destination node, the theoretically optimal diversity performance is achieved when transmit maximal ratio combining (MRC) is performed across all the relays. For the same total power consumption by relays, the transmit MRC based cooperative diversity techniques will certainly show better performance than the opportunistic relay selection scheme with the help of channel information. However, the opportunistic relay selection can be more beneficial in terms of the number of interference sources affecting other neighboring receiving nodes except the destination node despite the same power consumption by the relays. On the contrary, the opportunistic relay selection is not always able to satisfy the required outage probability at a destination node if the number of relays is not sufficient. In this context, we propose and analyze two relay selection schemes in which a subset of the relays is selected based on the channel gains from multiple relays to a destination node for transmission in the second hop. The proposed relay selection schemes correspond to a generalized selection combining (GSC) from an analytical point of view in the second hop. The analysis of this paper has a unique contribution, compared to the previous studies on GSC because our analysis incorporates multi-hop effects in addition to the nature of GSC in the relay selection.

The rest of this paper is organized as follows. In Section II, a system model is described and two relay selection schemes are proposed. In Section III, the performance of two proposed relay selection schemes is evaluated in terms of end-to-end outage probability and the average number of forwarding relays in the second hop. In Section IV, results obtained from numerical analysis and Monte-Carlo simulation are compared. Finally, conclusions are presented in Section V.

II. SYSTEM MODEL AND PROPOSED RELAY SELECTION SCHEMES

A. System Model

We consider a system model where there exist one source, one destination, and K relays. Fig. 1 shows a half-duplex scheme. We consider a decode-and-forward (DF)-based dual-hop relaying system. During the first hop, a source transmits its data to relays and, then, a destination listens to the source. In our system model, we assume that relays do not forward their data if a direct transmission from a source to a destination is successful. If the direct transmission fails, relays transmit data received from the source to a destination during the second hop and, finally, the destination receives the data. Since this cooperative communication takes place during two hops, the required spectral efficiency per hop should be equal to $2R$ if the spectral efficiency is equal to that of direct communication, R. If we assume that the corresponding channels remain constant during more than two hops, node B's received signal from node A can be defined as

$$y_B = h_{A,B} x_A + n_B \tag{1}$$

where x_A is the signal transmitted at the node A, $h_{A,B} \sim CN(0, \sigma_{A,B})$ is the channel gain of a link from A to B, and $n_B \sim CN(0, N_0)$ represents the additive white Gaussian noise (AWGN) at the node B. The terms A and B can be either source (s) and relay (r) or relay and destination (d), respectively. $E\{|x_A|^2\}$ is the transmission power of source or relay (P_s or P_r) and $\gamma_{A,B} \triangleq |h_{A,B}|^2$ is an exponential random vari-

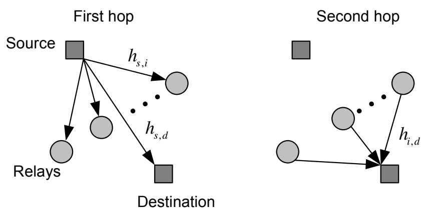

Figure 1: A half-duplex dual-hop system

able with mean $\sigma_{A,B}$. If we assume source and relays use the same transmission power(P), the channel can be modeled as the $SNR \times \gamma_{A,B}$, where $SNR \triangleq P/N_0$. We also assume that the channels between the first and the second hops are uncorrelated, and the channels are independent and identically-distributed (i.i.d.), and the destination has full knowledge of these channel parameters of both of the first and the second hops in order to obtain a theoretical bound of performance.

B. Relaying Protocols

1) Fixed Relay Selection Scheme

In a DF-based cooperative diversity scheme, a relay forwards data received from a source to a destination only when it successfully decoded the received data in order to prevent performance degradation [5]. We define a set of relays that successfully decoded the signal transmitted from the source for a given required spectral efficiency R as a decode set ($\mathcal{D}$), which is a subset of whole relay set ($\mathcal{S}$) and can be denoted by

$$\mathcal{D} = \{k \in \mathcal{S} : \gamma_{s,k} \geq R'\} \tag{2}$$

where R' is defined as

$$R' \triangleq \frac{2^{2R}-1}{SNR}.$$

In a fixed relaying scheme, a destination orders all relays in a decoding set in terms of the channel gain of the second hop and selects M ($\leq |\mathcal{D}|$) relays with best channels. The selected relays forward their decoded data to the destination.

2) Threshold-Based Adaptive Relay Selection Scheme

In a threshold-based adaptive relaying scheme, a destination computes an optimum threshold and broacasts it to relays in the reverse link. Only relays whose channel gain toward the destination is better than the threshold will forward their received data to the destination. The threshold is determined so that the number of relays that forward data to the destination becomes a minimum without causing an outage event. In order to perform this, the destination orders the second hop channel gains of all relays in a decode set and compute the optimum threshold as follows:

$$\gamma_{th} = \begin{cases} \infty, & \gamma_{s,d} \geq R' \\ \infty, & (\gamma_{s,d} < R') \cap \left(\gamma_{s,d} + \sum_{i=1}^{|\mathcal{D}|} \gamma_{i,d}\right) < R' \\ \gamma_{T,d}, & (\gamma_{s,d} < R') \cap \left(\gamma_{s,d} + \sum_{i=1}^{|\mathcal{D}|} \gamma_{i,d}\right) \geq R' \end{cases} \tag{3}$$

where $\gamma_{1,d} \geq \gamma_{2,d} \geq \cdots \geq \gamma_{|\mathcal{D}|,d}$ and T is defined as

$$T \triangleq \arg \min_{t \in \mathcal{D}} \text{ s.t. } \left(\gamma_{s,d} + \sum_{i=1}^{t} \gamma_{i,d}\right) \geq R'. \tag{4}$$

Under a given threshold, γ_{th}, the forwarding set consisting of relays that will forward data to the destination is defined as

$$\mathcal{F} = \{k \in \mathcal{D} : \gamma_{k,d} \geq \gamma_{th}\}, \tag{5}$$

and all relays in the forward set will forward their received data to the destination. If $\gamma_{th} = \infty$, then $|\mathcal{F}|$ is equal to 0. This event occurs when a direct transmission in the first hop from a source to a destination succeeds or the required spectral efficiency can not be satisfied even if all relays forward their data.

III. Performance Analysis

A. Outage Behavior

1) Fixed Relay Selection Scheme

An outage event for the fixed relay selection scheme is equivalent to the event

$$\left[\left(\gamma_{s,d} < R'\right) \cap \left(|\mathcal{D}| = 0\right)\right] \cup \left[\left(\gamma_{s,d} < R'\right) \cap \left(\gamma_{s,d} + \sum_{j=1}^{L} \gamma_{j,d} < R' \,\middle|\, \gamma_{s,d} < R'\right) \cap \left(|\mathcal{D}| \neq 0\right)\right] \tag{6}$$

where L is the number of relays that actually forward to a destination, which can not be greater than $|\mathcal{D}|$. Thus, L is defined as $\min(M, |\mathcal{D}|)$. Using the outage event defined in Eq. (6), the definition of a conditional probability [8], and the following fact

$$\left(\gamma_{s,d} + \sum_{j=1}^{L} \gamma_{j,d} < R'\right) \subset \left(\gamma_{s,d} < R'\right), \tag{7}$$

the outage probability for a given M can be described as

$$p_f^{out}(M) = \Pr\left[\gamma_{s,d} < R'\right] \Pr\left[|\mathcal{D}| = 0\right] + \sum_{i=1}^{K} \Pr\left[|\mathcal{D}| = i\right] \times \Pr\left[\gamma_{s,d} + \sum_{j=1}^{L} \gamma_{j,d} < R'\right], \tag{8}$$

where $\Pr[\gamma_{s,d} < R']$ and $\Pr[|\mathcal{D}| = i]$ can be computed as

$$\Pr\left[\gamma_{s,d} < R'\right] = 1 - e^{-\frac{R'}{\sigma_{sd}}}.$$

$$\Pr\left[|\mathcal{D}| = i,\ 0 \leq i \leq K\right] = \binom{K}{i} \left(e^{-\frac{R'}{\sigma_{sr}}}\right)^i \left(1 - e^{-\frac{R'}{\sigma_{sr}}}\right)^{K-i}.$$

A new random variable, Z, is introduced as

$$Z = X\left(\triangleq \gamma_{s,d}\right) + Y\left(\triangleq \sum_{j=1}^{M} \gamma_{j,d}\right), \tag{9}$$

where X is an exponential random variable and Y is the sum of M largest ordered statistics out of K exponential random variables. For a given M and K, the CDF of Z can be computed as [8]

$$\begin{aligned} F_{Z,M,K}(z) &\triangleq \Pr[Z<z] = \int_0^z \int_0^{z-x} f_{x,y}(X,Y)\,dydx \\ &= \int_0^z \frac{1}{\sigma_{s,d}} e^{-\frac{x}{\sigma_{s,d}}} \int_0^{z-x} f_Y(y)\,dydx \\ &= \int_0^z \frac{1}{\sigma_{s,d}} e^{-\frac{x}{\sigma_{s,d}}} F_Y(z-x)\,dx, \end{aligned} \quad (10)$$

where the second equality is due to the fact that X and Y are independent. The CDF of Y can be computed as [9]

$$\begin{aligned} F_Y(y) \cong\, & 2^{1-C} e^{A/2} \sum_{c=0}^{C} \binom{C}{c} \sum_{b=0}^{c+B} (-1)^b \alpha_0^{\alpha_b} \times \\ & \Re\left[\phi_Y\left(\frac{A+j2\pi b}{2y}\right) \Big/ (A+j2\pi b) \right], \end{aligned} \quad (11)$$

where $\alpha_0 = 0.5$, $\alpha_b = 1$ for any $b \geq 1$, and the constants A, B, and C are arbitrarily chosen to be 30, 18, and 24, respectively, which yield a good numerical accuracy for our purpose and $\phi_Y(s)$ is the moment generating function (MGF), which is given by the following closed form if $\gamma_{1,d}, \cdots, \gamma_{M,d}$ are i.i.d exponential variables

$$\phi_Y(s) = \frac{1}{(1+s\sigma_{r,d})^{M-1}} \prod_{r=M}^{K} \frac{1}{(1+s\sigma_{r,d}M/r)}. \quad (12)$$

Using Eq. (10), the outage probability for the fixed relaying scheme in Eq. (8) can be rewritten as

$$\begin{aligned} p_f^{out}(M) =& \left(1-e^{-\frac{R'}{\sigma_{sd}}}\right)\left(1-e^{-\frac{R'}{\sigma_{sr}}}\right)^K + \sum_{i=1}^{K}\binom{K}{i} \times \\ & \left(e^{-\frac{R'}{\sigma_{sr}}}\right)^i \left(1-e^{-\frac{R'}{\sigma_{sr}}}\right)^{K-i} F_{Z,L,K}(R'), \end{aligned} \quad (13)$$

which can be numerically computed using Eqs. (11) and (12).

2) Adaptive Relay Selection Scheme

The maximum number of relays that the adaptive relay selection scheme can use is the cardinality of $\mathcal{D}$, thus outage event for the threshold based adaptive scheme is equivalent to the event

$$\begin{aligned} &\left[\left(\gamma_{s,d}<R'\right)\cap\left(|\mathcal{D}|=0\right)\right]\cup\left[\left(\gamma_{s,d}<R'\right)\cap\right. \\ &\left.\underbrace{\left(|\mathcal{F}|=0 \,\middle|\, \gamma_{s,d}<R'\right)}_{\left(\gamma_{s,d}+\sum_{j=1}^{|\mathcal{D}|}\gamma_{j,d}<R' \,\middle|\, \gamma_{s,d}<R'\right)} \cap \left(|\mathcal{D}|\neq 0\right)\right]. \end{aligned} \quad (14)$$

Using Eqs. (7) and (14), the outage probability for the adaptive relaying scheme can be written as

$$\begin{aligned} p_a^{out} =& \Pr[\gamma_{s,d}<R']\Pr[|\mathcal{D}|=0] + \sum_{i=1}^{K}\Pr[|\mathcal{D}|=i] \times \\ & \Pr\left[\gamma_{s,d}+\sum_{j=1}^{i}\gamma_{j,d}<R'\right]. \end{aligned} \quad (15)$$

The outage probability for the threshold-based adaptive relaying scheme in Eq. (15) can be rewritten as

$$\begin{aligned} p_a^{out}(M) =& \left(1-e^{-\frac{R'}{\sigma_{sd}}}\right)\left(1-e^{-\frac{R'}{\sigma_{sr}}}\right)^K + \sum_{i=1}^{K}\binom{K}{i} \times \\ & \left(e^{-\frac{R'}{\sigma_{sr}}}\right)^i \left(1-e^{-\frac{R'}{\sigma_{sr}}}\right)^{K-i} F_{Z,i,K}(R'), \end{aligned} \quad (16)$$

which can be numerically computed using Eqs. (11) and (12).

B. Average Number of Forwarding Relays

In this section, the average number of forwarding relays, $E[N]$ is analyzed.

1) Fixed Relay Selection Scheme

In the fixed relay selection scheme, if a direct transmission from a source to a destination succeeds, then no relays forward data to the destination. On the contrary, if the direct transmission fails, then the number of relays that forward data to the destination will be determined as L, which is limited by $|\mathcal{D}|$. Thus, $E[N]$ can be expressed as

$$E[N] = \Pr[\gamma_{s,d}<R'] \times \sum_{i=1}^{K}\Pr[|\mathcal{D}|=i] \times \min(M,i) \quad (17)$$

2) Adaptive Relay Selection Scheme

In the adaptive relay selection scheme, the number of relays that forward data to the destination is given by $|\mathcal{F}|$, which is dependent on $|\mathcal{D}|$. Thus, $E[N]$ for the adaptive relay selection scheme can be described as

$$\begin{aligned} E[N] =& \Pr[\gamma_{s,d}<R'] \times \sum_{i=1}^{K}\Pr[|\mathcal{D}|=i] \times \\ & \sum_{j=1}^{i}\Pr\left[|\mathcal{F}|=j \,\middle|\, |\mathcal{D}|=i\right] \times j, \end{aligned} \quad (18)$$

where the last conditional probability is given by

$$\Pr[|\mathcal{F}|=j \,|\, |\mathcal{D}|=i] = \begin{cases} 1-F_{Z,j,i}(R'), & j=1 \\ F_{Z,j-1,i}(R')-F_{Z,j,i}(R'), & j\neq 1. \end{cases} \quad (19)$$

IV. NUMERICAL RESULTS

We evaluate the performance of both schemes in terms of end-to-end outage probability and average number of forwarding relays. We assume that a destination has full knowledge of channel states including the first hop for both of the fixed and adaptive relay selection schemes. This assumption is used to find an information-theoretical bound. Fig. 2 shows the outage probability and the average number of forwarding relays in an asymmetric network topology where a link between a relay node and a destination node is better than two other links. Monte Carlo simulations were performed over a range of SNR values of $0 \sim 16$ [dB]. Fig. 2-(a) shows the outage performance of both schemes described by Eqs. (13) and (16). The outage performance of the fixed relay selection scheme is improved as the parameter values of M increase because the combining gain increases according to the parameter values of M, where the performance of the fixed relay selection scheme with $M = 1$ corresponds to that of an opportunistic relay selection scheme. It is shown that the outage performance is saturated at $M \geq 3$, while the average number of forwarding relays shown in Fig. 2-(b) continuously increases as the parameter values of M increase. The outage probability of the adaptive relay selection scheme that uses the optimum threshold in Eqs. (3) and (4) is theoretically identical to that of the fixed relay selection scheme with $M = K$, where all relays in a decode set forward data to a destination. The reason is that if $M = K$, then $L = |\mathcal{D}|$, then the outage probability of the adaptive relay selection scheme in Eq. (16) becomes numerically identical to that of the fixed relay selection scheme in Eq. (13). From Fig. 2-(b), it is shown that at low and high SNR values, the number of forwarding relays decreases. At low SNR values, it decreases because of a decreased value of $|\mathcal{D}|$, while at high SNR values it decreases because the probability that direct transmission from a source to a destination is successful becomes high as the SNR values increase. The average number of forwarding relays in the adaptive relay selection scheme is less than that of fixed relay selection scheme regardless of M values at low SNR values because the probability that no relays forward data to the destination ($|\mathcal{F}| = 0$) by the second case of Eq. (3) increases. At high SNR values, the average number of forwarding relays in the adaptive relay selection scheme approaches to that of the fixed relay selection scheme with $M = 1$ at high SNR values because a given outage requirement can be satisfied with a single relay that has the strongest channel. Fig. 3 shows the outage probability and the normalized transmission power consumption in the second hop in a symmetric network topology where a direct link quality between a source and a destination is the same as the link quality of the other two links. Contrary to the asymmetric topology, the outage performance enhancement of the adaptive and fixed relay selection scheme with $M \geq 2$ is not significant compared to the opportunistic relay selection scheme, which corresponds to the fixed relay selection scheme with $M = 1$ because the symmetric topology yields a good direct channel between a source and a destination, and, thus, the effect of relay selection schemes decreases.

V. Conclusions

In this paper, we evaluate the performance of two relay selection schemes for cooperative relaying when there are multiple relays between a source and a destination. Opportunistic relay selection schemes are optimal when a limited amount of channel feedback information is available. However, the proposed schemes can achieve better performance than the opportunistic schemes by using the transmit MRC. The outage performance of the fixed relay selection scheme is improved as the parameter values of M increases. However, the performance enhancement is saturated at a certain M values, which is 2 or 3 in our models. The adaptive relay selection scheme can minimize the number of forwarding relays required to satisfy a given outage requirement. In the adaptive relay selection scheme, relays that forward data to a destination are determined by the optimum threshold computed and updated by the destination. This indicates that the number of forwarding relays in the adaptive relay selection scheme is variable and determined as a minimum level where outage can be prevented. The results obtained through numerical analysis and simulation show that the outage performance of the adaptive relay selection scheme is better than for the opportunistic relay selection schemes, specially in the asymmetric network topology and is identical to that of the fixed relay selection scheme where all relays transmit($M = K$), while the average number of forwarding relays of the adaptive relay selection scheme is always less than that of the fixed relay selection scheme regardless of M values at low SNR values and approaches to that of the opportunistic schemes at high SNR values.

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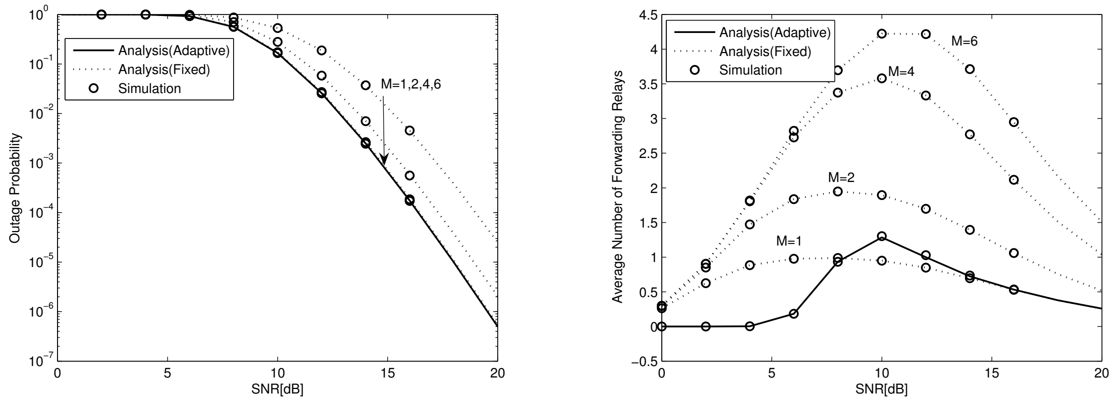
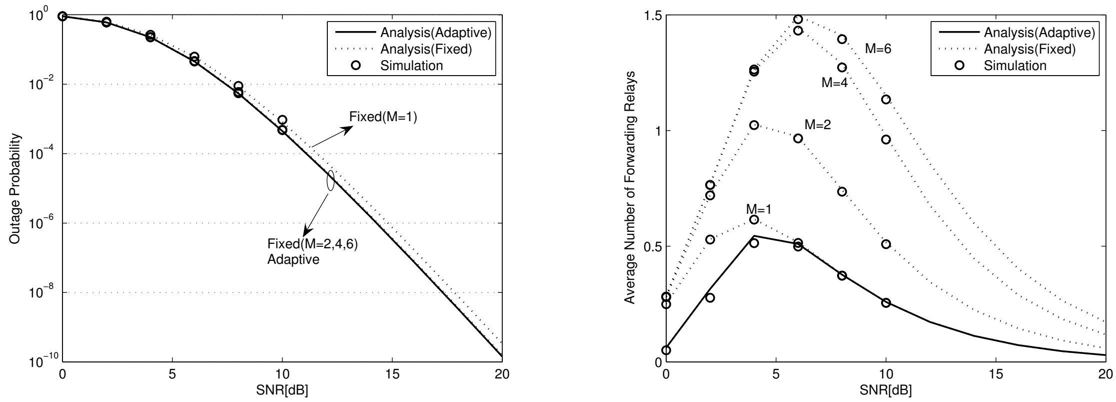


(a) Outage probability

(b) Average Number of Forwarding Relays

Figure 2: Performance for $R = 1$, $K = 6, \sigma_{s,d} = -10$ [dB], $\sigma_{s,r} = -10$ [dB], and $\sigma_{r,d} = 0$ [dB]



(a) Outage probability

(b) Normalized transmission power consumed in the second hop

Figure 3: Performance for $R = 1$, $K = 6$ and $\sigma_{s,d} = \sigma_{s,r} = \sigma_{r,d} = 0$ [dB]