Effect of Other-Cell Interference on Multiuser Diversity in Cellular Networks

Seong Hwan Kim,Bang Chul Jung, and Dan Keun Sung
School of EECS., KAIST, 373-1, Guseong-dong, Yuseong-gu, Daejeon, 305-701, Korea
Email: shkim@cnr.kaist.ac.kr; bcjung@cnr.kaist.ac.kr; dksung@ee.kaist.ac.kr

***Abstract*— We present an analytical framework to evaluate the system capacity of two cellular systems in an interference-limited environment: a single-user system with and without other-cell interference and a multiuser system with and without other-cell interference. In a multiuser system, we adopt the multiuser diversity scheme. We derive closed-form expressions of the system capacity for the above four cases. The analytical results agree well with simulation result. Other-cell interference with large variances increases the achievable throughput, while the averaged other-cell interference reduces the system capacity. In addition, we show that a fairness problem arising between inner cells and outer cells in the cellular network can be improved by using other-cell interference in the multiuser system. Therefore, the multiuser diversity scheme can be a good candidate for future cellular networks.**

I. Introduction

One of major challenges in current cellular networks is other-cell interference induced from adjacent cells. A. J. Viterbi and A. M. Viterbi [1], [2] analyzed the user capacity in cellular power-controlled CDMA and showed that the reduction of other-cell interference increases the user capacity. However, the capacity was limited only in CDMA systems with voice traffic. However, the data traffic has increased in cellular network recently. Moreover, the user capacity which indicates the maximum number of users supported in a cell for a given blocking probability is not a good measure for wireless communication systems any more. Shamai [3] presented the theoretical link-capacity of several schemes in multicell environment including time division multiple access(TDMA) and code division multiple access(CDMA). They used a simplified cellular model in order to analyze the link-capacity of the cellular system in a multicell interference-limited environment. However, the link-capacity analyzed in [3] did not provide the system capacity of the cellular network with an opportunistic scheduler and it was derived only for the uplink.

Knopp and Humblet [4] introduced multi-user diversity as a means to provide diversity against channel fading in multi-user communication systems. The performance gain of multiuser diversity increases as the number of active users in the system becomes large [5]. However, most studies on the multiuser diversity have not considered the other-cell interference even if the wireless link generally suffers from other-cell interference as well as fading. In addition, a fairness problem occurs between users close to the base station and those far from the base station(BS) when a scheduler is used to maximize the multiuser diversity at the BS [6]. To solve the fairness problem, some modifications in opportunistic schedulers have been made [7], [8]. However, the fairness problem also should be controlled by considering the effect of other-cell interference. In this paper, we analyze the system capacity with multiuser diversity in an interference-limited environment. We derive a closed form of the achievable rate of the system. Furthermore, we investigate the effect of the other-cell interference on the fairness in cellular networks.

The rest of this paper is organized as follows: In Scetion II, we introduce the system model for mathematical analysis. In Section III, we derive the capacity of a single user system and a multiuser system with and without other-cell interference. In Section IV, we show numerical examples. In Sectioin V, we investigate the effect of other-cell interference on fairness in cellular networks. Finally, the conclusion is shown in Section VI.

II. System Model

Fig. 1 shows the system model of a downlink cellular network which is symmetric. We assume that each BS serves an MS among U candidates at a certain time. This model can be also applied to the orthogonal resource multiple access systems, such as TDMA and OFDMA. It is assumed that the received signal at each cell is interefered by the BSs of adjacent cells. The received symbol at an MS in the zero-th cell is expressed as

$$y_0 = a_0 x_0 + \alpha(b_1 x_1 + b_2 x_2 + \cdots + b_M x_M) + n_0, \qquad (1)$$

where x_m represents the symbol transmitted with fixed power P from the m-th BS. The coefficients a_0 and $b_m(m = 1, \cdots, M)$ denote the channel coefficient from a home -cell BS, i.e., the zero-th cell, and those of other-cells, respectively. We assume that all BSs and MSs have one antenna each and all wireless channels are assumed to be Rayleigh distributed, i.e., $a_0, b_m \sim \mathcal{CN}(0, \sigma^2)$ and $n_0 \sim \mathcal{CN}(0, N_0)$. Thus, the term σ^2 denotes the channel gain between the home-cell and the MS. The other-cell interference is modeled using a single parameter $\alpha \geq 0$ which represents the attenuation of the adjacent cell signals received at the home-cell. In general, α is smaller than 1, since each MS decides its home-cell with the strongest channel gain. We assume that $b_m (1 \leq m \leq M)$ is i.i.d with the same variance, σ^2. Thus, all the interference channel gains are equal to $\alpha^2\sigma^2$. γ denotes the output signal-to-interference plus noise ratio(SINR) at an MS in the zero-th cell, which is expressed as:

$$\gamma = \frac{{a_0}^2 \rho}{1 + \alpha^2 \rho \sum_{m=1}^{M} b_m^2}, \qquad (2)$$

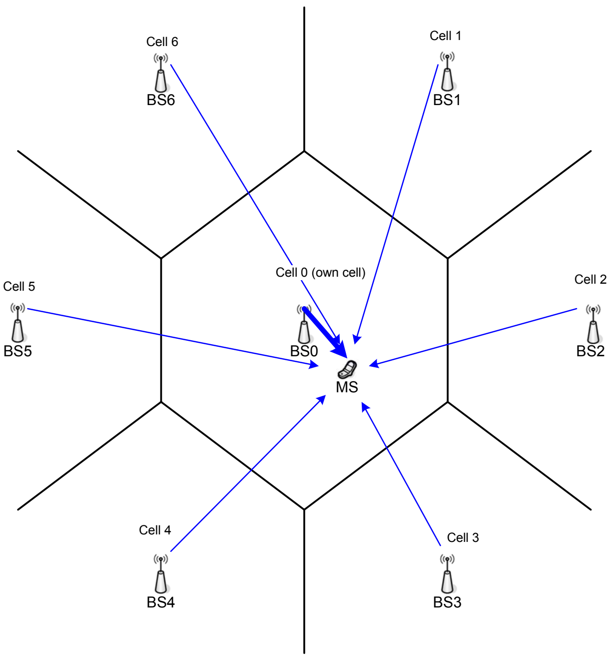

Fig. 1. System model of a downlink cellular network.

where $\rho = P/N_o$ is the input signal-to-noise ratio(SNR). We now assume that the system is in the interference-limited environment($\rho \to \infty$). Then, Eq. (2) can be rewritten as:

$$\gamma \approx \frac{{a_0}^2}{\alpha^2 \sum_{m=1}^{M} b_m^2}. \quad (3)$$

Let the random variable ξ_M be defined as

$$\xi_M = \frac{{a_0}^2}{\sum_{m=1}^{M} b_m^2}, \quad (4)$$

i.e, $\gamma = \xi_M/\alpha^2$. The probability density function(PDF) of ξ_M is obtained as [3]:

$$f_\xi(\nu) = \frac{M}{(\nu+1)^{M+1}}, \nu \geq 0. \quad (5)$$

Eq. (5) is the PDF of the SINR of the MS served in the home-cell with M other-cell interference(OCI) signals.

III. Capacity Anaysis of the Interference-Limited Cellular Networks.

A. Capacity analysis of a single-user system without other-cell interference

At first, we derive the ergodic capacity of a single-user in an isolated cell, i.e., there is no other-cell interference. The system is now in the noise limited environment. γ denotes the output SNR of a user, i.e., $\gamma = {a_0}^2 P/N_0$. Therefore, the PDF of γ is given by

$$f_\gamma(\gamma) = \frac{1}{\rho}\exp\left(-\frac{\gamma}{\rho}\right), \quad (6)$$

where $\rho = \sigma^2 P/N_0$ denotes the average output SNR. The ergodic capacity of a single-user without other-cell interference is expressed by

$$\begin{aligned} C_1 &= \mathbb{E}[\log_2(1+\gamma)] = \int_0^\infty \log_2(1+\gamma) f_\gamma(\gamma) d\gamma \\ &= \int_0^\infty \log_2(1+\gamma)\frac{1}{\rho}\exp\left(-\frac{\gamma}{\rho}\right) d\gamma \\ &= \frac{e^{1/\rho}}{\ln 2} E_i\left(\frac{1}{\rho}\right), \end{aligned} \quad (7)$$

where we use the integral equality expressed as [9]:

$$\int_0^\infty e^{-\mu x}\ln(1+dx)dx = \frac{1}{\mu}e^{\mu/d}E_i\left(\frac{\mu}{d}\right). \quad (8)$$

B. Capacity of the multiuser system without other-cell interference

We extend the single-user system to the multiuser without other-cell interference. In the multiuser system, the BS is assumed to select the user with the largest SNR among U candidate MSs. Let γ_Γ be the output SINR of the selected user among U candidates. According to the order statistics [10], the PDF of γ_Γ can be obtained as

$$\begin{aligned} f_{\gamma_{\Gamma:U}}(\gamma) &= \frac{dF(\gamma)^U}{d\gamma} \\ &= U F_\gamma(\gamma)^{U-1} f_\gamma(\gamma) \\ &= U(1-e^{-\frac{\gamma}{\rho}})^{U-1}\frac{1}{\rho}e^{-\frac{\gamma}{\rho}}. \end{aligned} \quad (9)$$

Substituting Eq. (6) into the ergodic capacity formula, the ergodic capacity of the multiuser system scheme with U users in a cell is given as:

$$\begin{aligned} C_U &= \int_0^\infty \log_2(1+\gamma)U(1-e^{-\frac{\gamma}{\rho}})^{U-1}\frac{1}{\rho}e^{-\frac{\gamma}{\rho}}d\gamma \\ &= \frac{U}{\rho\ln 2}\int_0^\infty \ln(1+\gamma)(1-e^{-\frac{\gamma}{\rho}})^{U-1}e^{-\frac{\gamma}{\rho}}d\gamma \\ &= \frac{U}{\rho\ln 2}\int_0^\infty \ln(1+\gamma)\sum_{k=0}^{U-1}\binom{U-1}{k} \\ &\quad \times(-1)^k e^{-\frac{(k+1)\gamma}{\rho}}d\gamma \\ &= \frac{U}{\ln 2}\sum_{k=0}^{U-1}\binom{U-1}{k}\frac{(-1)^k e^{\frac{k+1}{\rho}}}{k+1}E_1\left(\frac{k+1}{\rho}\right). \end{aligned} \quad (10)$$

In Eq.(10), we use the binomial expansion and the integral equality noted in Eq.(8). As the number of users in a cell increases, the capacity also increases.

C. Capacity of the single-user with M other-cell interferers

Hereafter, we consider a cellular network in the interference-limited environment. γ_M denotes the output SINR at an MS with M interferers, i.e., $\gamma_M = \xi_M/\alpha^2$, where ξ_M is defined as Eq. (4). By using Eq. (5), the PDF of γ_M can be expressed as

$$f_{\gamma_M}(\gamma) = \frac{\alpha^2 M}{(\alpha^2\gamma+1)^{M+1}}, \gamma \geq 0. \quad (11)$$

The ergodic capacity of the single-user system with M interferers is expressed as:

$$\begin{aligned} C_1(M) &= \int_0^\infty \log_2(1+\gamma) f_{\gamma_u}(\gamma) d\gamma \\ &= \int_0^\infty \log_2(1+\gamma) \frac{\alpha^2 M}{(\alpha^2\gamma+1)^{M+1}} d\gamma \\ &= \frac{1}{\ln 2} \int_0^\infty \frac{1}{(1+\gamma)(\alpha^2\gamma+1)^M} d\gamma. \end{aligned} \tag{12}$$

Eq.(12) can be solved by integration by parts and some mathematical manupulation [3].

$$C_1(M) = \frac{-\log_2 \alpha^2}{(1-\alpha^2)^M} - \frac{1}{\ln 2}\sum_{m=1}^{M-1} \frac{1}{m(1-\alpha^2)^{M-m}}. \tag{13}$$

We assume that the average SINR of the MS, β, is a constant value regardless of the number of interferers, M. This assumption is useful to observe the capacity variation of the MS according to the number of interferences. When the number of interferers is set to M, the SINR of the MS can be expressed as :

$$\gamma_u = \frac{{a_0}^2}{\frac{1}{M\beta}\sum_{m=1}^M b_m^2} \tag{14}$$

($\because \beta = \mathbb{E}[\gamma_u] = \frac{1}{\alpha^2}\mathbb{E}[\xi_M] = \frac{1}{\alpha^2 M}$). The ergodic capacity of the MS whose average SINR is equal to β is given by substituting $1/M\beta$ for α^2 in Eq. (13). In general, the signal composed of a few but strong interference terms has a larger variance than the signal composed of many weak interference terms. The small variance of the other-cell interference in SINR reduces the ergodic capacity. The variance of other-cell interference decreases the performance of the cellular network.

We now show that other-cell interference approaches to a constant value as M increases. The other-cell interference is expressed as

$$\frac{1}{M\beta}\sum_{m=1}^{M} b_m^2. \tag{15}$$

We assume that the random variables $\frac{b_m^2}{M}(m = 1, \cdots, M)$ have the same mean and variance, i.e., A and B^2, respectively. According to the central limit theorem [11], the mean and variance of the random variable $\sum_{m=1}^M \frac{b_m^2}{M}$ are given by A and $\frac{B^2}{M}$, respectively. As M increases, the variance of the other-cell interference Eq. (15) tends to 0, and the SINR value becomes β. Thus, as M increases, the PDF of the SINR in the interference-limited system becomes close to the PDF of the SNR in the noise-limited system.

D. Capacity of the multiuser system with M other-cell interferers($\mathbb{E}[SINR] = \beta$)

We extend the interference-limited cellular network to the multiuser case. In the multiuser system, the BS select the MS with the largest SINR among U MSs. We assume that all MSs experience the same SNR characteristics, i.e., PDF of the SINR. Let $\gamma_{\Gamma:U}$ be the output SINR of of the user selected among U candidates. According to the order statistics [10], the PDF of $\gamma_{\Gamma:U}$ can be obtained as

$$\begin{aligned} f_{\gamma_{\Gamma:U,M}}(\gamma) &= \frac{dF_{\gamma_M}(\gamma)^U}{d\gamma} \\ &= UF_{\gamma_M}(\gamma)^{U-1} f_{\gamma_M}(\gamma) \\ &= U[1-\frac{1}{(\alpha^2\gamma+1)^M}]^{U-1}\frac{\alpha^2 M}{(\alpha^2\gamma+1)^{M+1}}. \end{aligned} \tag{16}$$

Substituting Eq(16) into the ergodic capacity formula, the ergodic capacity of the multiuser system scheme with U users is derived as

$$\begin{aligned} C_U(M) &= \int_0^\infty \log_2(1+\gamma) f_{\gamma_{\Gamma:U,M}(\gamma)} d\gamma \\ &= \int_0^\infty \log_2(1+\gamma) \\ &\quad \times U[1-\frac{1}{(\alpha^2\gamma+1)^M}]^{U-1}\frac{\alpha^2 M}{(\alpha^2\gamma+1)^{M+1}} d\gamma \\ &= \frac{\alpha^2 MU}{\ln 2}\int_0^\infty \ln(1+\gamma)\sum_{k=0}^{N-1}\begin{pmatrix} U-1 \\ k \end{pmatrix} \\ &\quad \times (-1)^k \left(\alpha^2\gamma+1\right)^{-kM}\left(\alpha^2\gamma+1\right)^{-M-1} d\gamma \\ &= \frac{\alpha^2 MU}{\ln 2}\sum_{k=0}^{N-1}\begin{pmatrix} U-1 \\ k \end{pmatrix}(-1)^k \\ &\quad \times \int_0^\infty \ln(1+\gamma)\left(\alpha^2\gamma+1\right)^{-kM-M-1} d\gamma. \end{aligned} \tag{17}$$

We can solve Eq.(17) by using the integration by part and it can be expressed as:

$$\begin{aligned} C_U(M) &= \frac{\alpha^2 MU}{\ln 2}\sum_{k=0}^{N-1}\begin{pmatrix} U-1 \\ k \end{pmatrix}(-1)^k\frac{1}{M(k+1)} \\ &\quad \times \int_0^\infty \frac{1}{(1+\gamma)(\alpha^2\gamma+1)^{M(k+1)}} d\gamma \\ &= \frac{\alpha^2 U}{\ln 2}\sum_{k=0}^{N-1}\begin{pmatrix} U-1 \\ k \end{pmatrix}\frac{(-1)^k}{(k+1)} \\ &\quad \times \left(\frac{-\ln\alpha^2}{(1-\alpha^2)^(M(k+1))} - \sum_{m=1}^{M-1}\frac{1}{m(1-s)^{M(k+1)-m}}\right) \end{aligned} \tag{18}$$

When M goes to infinity, the PDF expressed in Eq. (16) of the SINR in the interference-limited system is equal to the PDF expressed in Eq. (9) of the SNR in the noise-limited system. This means two capacity formulars are equal when M goes to infinity.

IV. Numerical Examples

A. Capacity of the single-user system for varing the number of interferers

We assume that the average value of SINR is set to 0dB. Fig. 2 shows the capacity for varing the number of interferers. The analytical results agree very well with computer simulation results. The system capacity decreases as the number of interferers increases. Note that the system with a small number of other-cell interferers yields better performance than the system with a large number of other-cell interferers.

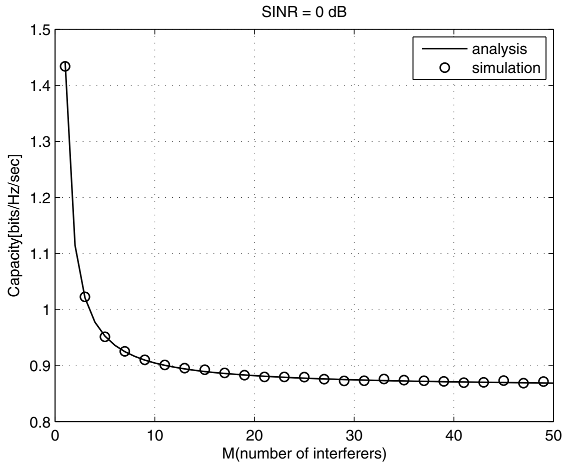

Fig. 2. Capacity of a single user system for varying the number of other-cell interferers.

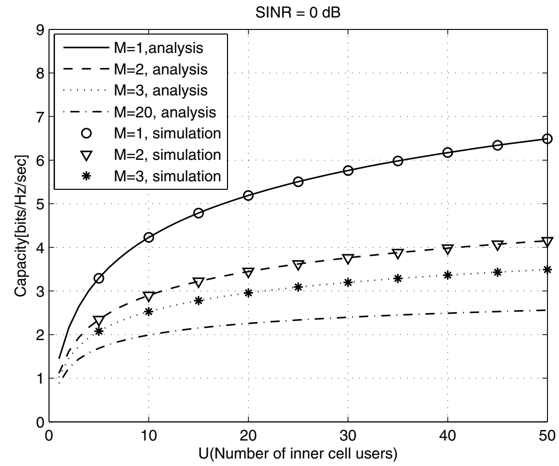

Fig. 3. Capacity versus the number of inner cell users for varying the number of other-cell interferers(SINR=0dB).

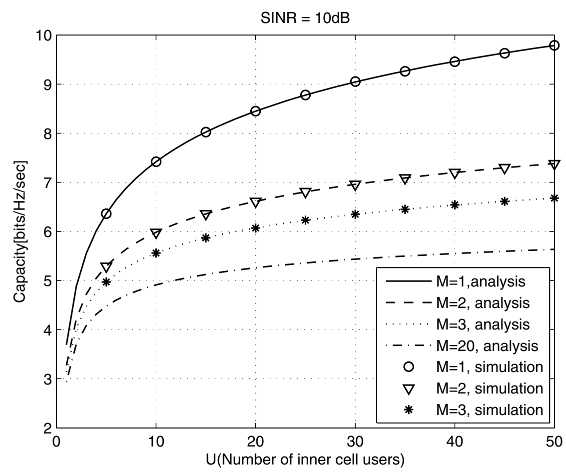

Fig. 4. Capacity versus the number of inner cell users for varying the number of other-cell interferers(SINR=10dB).

B. Capacity change of the multiuser system with the various number of interferers

We now investigate the capacity of the multiuser system when the number of the home-cell users and the number of other-cell interferers vary. We consider two cases where the SINR values are set to 0dB and 10dB. Fig. 3 shows the capacity versus the number of cell users in the multiuser system for varying the number of other-cell interferes when the SINR value is set to 0dB. The analytical result matches well with Monte Carlo simulation results. As the number of inner cell user increases, the system capacity also increases especially for $M = 1$. As the number of other-cell interferers increases, the multiuser system gain is reduced due to the low variances in the SINR. Fig. 4 show the capacity versus the number of inner cell users in the multiuser system for varying the number of other-cell interferers when the SINR value is set to 10dB. We observe a decrease in the capacity due to the reduced variance of the SINR values as the number of other-cell interferers, M increases. In summary, a small number of other-cell interferers improves the ergodic capacity of the multiuser cellular network for a given average SINR(β).

V. Effect of Other-Cell Interference on Fairness

In a cellular network, the SINR of a user is much dependent on the distance from the BS. In other words, users near the BS have relatively high SINR values, while users far from the BS usually have poor SINR values. In this aspect, we can divide the cell area as two parts. One is a high SINR region including a BS and the other is a low SINR region close to the cell boundary. Since the variance of other-cell interference is averaged in the CDMA case, the variance of other-cell interference is reduced to a very small value. If we adopt multiuser diversity in this system, users in the high SINR region are selected with higher probabilities and users in the low SINR region are hard to be selected. The system suffers from a fairness problem. This fairness problem can be improved by using the other-cell interference in the multiuser system. We consider the number of other-cell interfers in the each region. Fig. 5 shows the effect of the other cell interference on the users in the high SINR region in the proposed model. There are 6 other-cell interferers. Each average power is nearly identical and relatively small, compared to the power of the home-cell signal. Fig. 6 shows the effect of other-cell interference on the users in the low SINR region. The received signal in the low SINR region has two strong other-cell interference sources. This strong other-cell interference causes low output SINR values. However, this other-cell interference with a few strong interference sources yields large variances. As considered in Section III, this other-cell interference provides a large diversity gain in the multiuser system. To verify this analysis, we perform Monte Carlo simulations. We assume an interference-limited environment. We also assume that the number of users in each region is 10, the number of other-cell interferers to the MS in the high SINR region and the low SINR region is 6 and 2, respectively. The average output

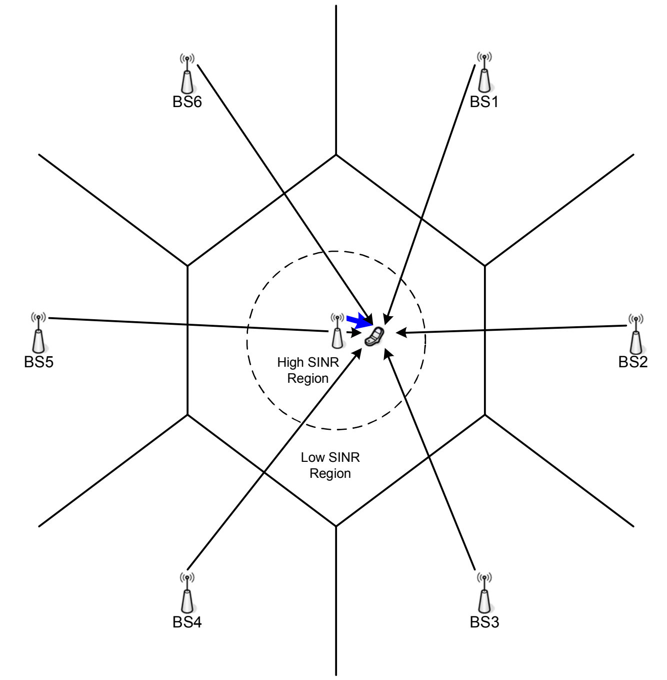

Fig. 5. Effect of other-cell interference on the users in the high SINR region.

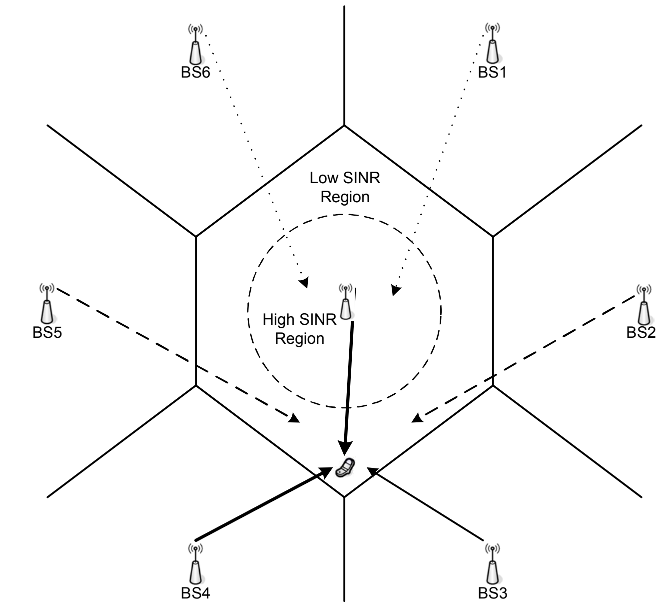

Fig. 6. Effect of other-cell interference on the users in the high SINR region.

TABLE I

THE RATIO OF THE CAPACITY OF USERS IN EACH REGION TO THE TOTAL SYSTEM CAPACITY. U=10, THE NUMBER OF OTHER-CELL INTERFERENCE SOURCES IS 6 AND 2 IN THE HIGH SINR REGION AND THE LOW SINR REGION, RESPECTIVELY

	High SINR region	*Low SINR region*	*High SINR region*	*Low SINR region*
E[SIR]	10 dB	4 dB	10 dB	7 dB
Averaged OCI	99.3%	0.7%	88.9%	11.1%
Non Averaged OCI	80.4%	19.6%	55.5%	44.5%

SIR of the MS in the high SINR region is 10 dB, and that in the low SINR region is set to 4dB and 7dB in the first and the second simulations, respectively. We estimate the ratio of the capacity of users in the high and low SINR regions to the total system capacity. Table I shows that the system with averaged other-cell interference(Averaged OCI) or in the noise-limited environment causes a severe unbalanced fairness problem between the high SINR region and the low SINR region. However, we can observe that the proportion of the capacity in the low SINR region increases in non-averaged OCI. In conclusion, the other- cell interference improves the fairness of the cellular network adopting a multiuser diversity scheme.

VI. CONCLUSIONS

We analyzed the effect of other-cell interference on the performance of cellular system, especially in the multiuser system. In contrast to a negative viewpoint on the other-cell interference, the other-cell interference with large variances can improve the system capacity. We showed that a few strong interferers help to increase the system capacity in the limited SINR condition. As a result, the capacity of the users in the low SINR region increases. Therefore, we can mitigate the fairness problem in multiuser systems. As further work, we will investigate multiuser diversity systems with MIMO antennas in multi-cell environments.

ACKNOWLEDGMENTS

This study has been supported in part by a grant from the Institute of Information Technology Assessment (IITA) and the BrOMA IT Research Center.

REFERENCES

[1] A. J. Viterbi and A. M. Viterbi, "Other-Cell Interference in Cellular Power-Controlled CDMA," *IEEE Trans. Commun.*, Vol. 42, No. 2, pp. 1501-1504, 1994.

[2] A.M. Viterbi and A. J. Viterbi, "Erlang Capacity of a Power Controlled CDMA System," *IEEE J. Select. Areas Commun.*, Vol. 11, No. 6, pp. 892-899, Aug. 1993.

[3] S Shamai (Shitz), "Information-Theoretic Considerations for Symmetric, Cellular, Multiple-Access Fading Channels," *IEEE Trans. Inform. Theory*, Vol. 43, No. 6, Nov. 1997.

[4] R. Knopp and P. Humblet, "Information capacity and power control in single cell multiuser communications," *in Proc. of IEEE ICC*, Vol. 1, pp. 331 - 335, Jun. 1995.

[5] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inform. Theory*, Vol. 48, No. 6, pp. 1277 - 1294, Jun. 2002.

[6] P. Viswanath and D. N. C. Tse, *Fundamentals of Wireless Communications*, Cambridge.

[7] R. Knopp, "Achieving Multiuser Diversity under Hard Fairness Constraints," *in Proc. IEEE ISIT*, Lausanne, Switzerland, July 2002.

[8] Gyasi-Agyei, A, "Multiuser Diversity based Opportunistic Scheduling for Wireless Data Networks," *IEEE Commun. Lett.*, Vol. 9, pp. 670 - 672, July 2005.

[9] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, Academic press, 6th edition, 2000.

[10] H. David and H. Nagaraja, *Order Statistics*, Wiley, 2003.

[11] A. Papoulis and S. U.Pillai, *Probability, Random Variables and Stochastic Processes*, McGrawHil.