Zero-Forcing-Based Two-Phase Relaying with Multiple Mobile Stations

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***Abstract*—It is well known that relay stations improve the link performance between the base and mobile stations and thereby improve the total system throughput. Nonetheless, the additional resource consumption to deploy the relay station reduces the system throughput significantly. The zero-forcing (ZF)-based two-phase relaying scheme that requires only two phases to communicate a frame was suggested for the system where a single mobile station is considered [1]. The performance bottle-necks of the conventional ZF-based two-phase relaying are channel asymmetry and an ill-conditioned channel that reduces the effective channel gain. We propose a novel two-phase relaying scheme to support multiple users simultaneously for a given channel resource. The proposed relaying scheme includes the multiuser scheduling at the relay station and the additional precoding at the base station to make the effective channels from the base and mobile stations equal to each other. We evaluate the proposed relaying scheme by numerically comparing its sum rate to those of the conventional relaying schemes.**

I. Introduction

Two-way relaying based on network coding (NC) utilizes resource more effectively compared to conventional half duplex relaying and thereby dramatically improves the achievable rate of relay systems. Basically, two-way relaying using NC, which is also called physical layer network coding (PNC), utilizes the bit-level exclusive OR (XOR) operation and performs joint decoding [2]–[4]. Recently, there have been several efforts to deploy PNC to MIMO fading channels [1], [5]. In particular, [1] suggests an efficient and simple three-node relaying scheme that utilizes precoding at the base and mobile stations in phase 1.

However, the performances of these PNC-based two-way relaying are bound by the channels with the lower gains. Furthermore, the performance of the conventional ZF-based two-phase relaying that uses the channel inversion as precoding is severely degraded if the channel with the lower gain is ill-conditioned. If the channel gains are equal to one another and the channels are all orthogonal, the conventional ZF-based two-phase relaying shows twice the performance of the half duplex relaying. Thus, the performance bottleneck of the conventional ZF-based two-phase relaying is the ill-conditioned channel with the lower gain.

In this paper, we propose an efficient scheme that exploits the multiuser diversity and thereby improves the achievable rate of the system. Furthermore, we propose the mobile station (MS) selection method to maximize the total achievable rate of the system. Considering that the multiple MSs cannot cooperate with one another, the base station (BS) performs additional precoding. This additional precoding lowers the achievable rate of the link between the BS and relay station (RS); however, it improves the achievable rate of the link between the MSs and RS. Thus, if the achievable rate of the link between the BS and RS is higher than that of the link between the MSs and RS, the total achievable rate of the system is improved. This improvement is due to the fact that the total achievable rate of the two-phase relaying is bound by that with the lower value.

The remainder of this paper is organized as follows. In Section II, the conventional ZF-based two-phase relaying is briefly reviewed. In Section III we propose the ZF-based two-phase relaying for multiple MSs along with the user selection method. The performance of the proposed scheme is analyzed numerically and the simulation results are presented in Section IV, and the conclusions are drawn in Section V.

II. Conventional ZF-Based Two-Phase Relaying

In the conventional ZF-based two-phase relaying [1], the BS and the MS transmit the $(N \times L_s)$ signal matrices $\mathbf{X}_{\text{BS}^{\text{conv}}}$ and $\mathbf{X}_{\text{MS}^{\text{conv}}}$, respectively, as

$$\mathbf{X}_{\text{BS}}^{\text{conv}} = m_{\text{conv}}\mathbf{H}_{\text{BR}}^{-1}\mathbf{S}_1,\ \mathbf{X}_{\text{MS}}^{\text{conv}} = n_{\text{conv}}\mathbf{H}_{\text{MR}}^{-1}\mathbf{S}_2 \tag{1}$$

where

$$m_{\text{conv}} = \sqrt{P_{\text{BS}}/\text{tr}[\mathbf{H}_{\text{BR}}^{-1}\mathbf{H}_{\text{BR}}^{-H}]}, \tag{2}$$

$$n_{\text{conv}} = \sqrt{P_{\text{MS}}/\text{tr}[\mathbf{H}_{\text{MR}}^{-1}\mathbf{H}_{\text{MR}}^{-H}]} \tag{3}$$

with the power constraints P_{BS} and P_{MS}. Here, $\mathbf{S}_1$ and $\mathbf{S}_2$ denote the $(N \times L_s)$-dimensional symbol matrices to be transmitted at the BS and MS, respectively. The decoding method and performance analysis were presented in [1] as well as numerical simulations.

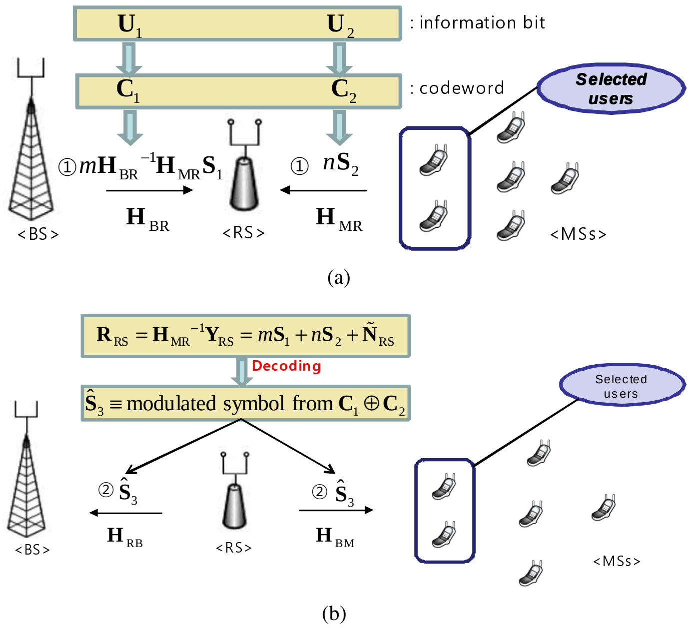


Fig. 1. Overall scheme in (a) phase 1 and (b) phase 2

III. ZF-Based Two-Phase Relaying with Multiple Mobile Stations

A. System Model

We consider the system involving a single relay station and multiple MSs as shown in Fig. 1. The BS, RS, and MS are assumed to have N antennas, respectively, so that each channel matrix is an $(N \times N)$-dimensional matrix. For simplicity, we assume that an independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variable for each element of the channel matrices. Each channel matrix is represented as $\mathbf{H}$, the superscript of which denotes the transmitter and receiver. For example, the channel between the BS and RS is represented as $\mathbf{H}_{\text{BR}}$. The number of MSs interested is assumed to be K and the number of time slots for a downlink or uplink is denoted as L. We also assume that the frequency flat and slowly varying channel so that the channels can be assumed to be constants during L_s.

Throughout the paper the following notations shall be used. $\mathbf{A}(i,j)$ shall denote the (i,j)th element of a matrix $\mathbf{A}$; $\mathbf{A}(:,i)$ represents the ith column of the matrix $\mathbf{A}$; $\mathbf{A}^H$ represents the Hermitian transpose of the matrix $\mathbf{A}$; $\text{tr}[\mathbf{A}]$ denotes the trace of the matrix $\mathbf{A}$; $P\{\cdot\}$ shall denote the probability.

B. Modified Precoding

We start to describe the algorithm by assuming that the augmented channel matrix $\mathbf{H}_{\text{MR}}$ is obtained as depicted in Fig. 1(a). If L users are chosen to form $\mathbf{H}_{\text{MR}}$, it can be represented as

$$\mathbf{H}_{\text{MS}} = \begin{bmatrix} \mathbf{h}_1^T \\ \mathbf{h}_2^T \\ \vdots \\ \mathbf{h}_L^T \end{bmatrix}, \tag{4}$$

where $\mathbf{h}_k$ is an $(N \times t_k)$-dimensional channel matrix or a vector between the kth MS and the BS. Here, t_k denotes the number of data streams that is allocated to the kth MS. $\mathbf{S}_1$ and $\mathbf{S}_2$ denote $(N \times L_s)$-dimensional symbol matrices to be transmitted at the BS and MS, respectively.

Considering that the MSs cannot cooperate with one another, the transmit signal and the power constant at the chosen MSs can be represented as

$$\mathbf{X}_{\text{MS}} = n\mathbf{S}_2, \; n = \sqrt{P_{\text{BS}}}. \tag{5}$$

To make the effective channels at the RS from the BS and MS equal to each other, the BS performs the additional precoding as in Fig. 1(a), and the transmit signal at the BS can be represented as

$$\mathbf{X}_{\text{BS}} = m\mathbf{H}_{\text{BR}}^{-1}\mathbf{H}_{\text{MR}}\mathbf{S}_1, \tag{6}$$

where

$$m = \sqrt{P_{\text{BS}}/\text{tr}[\mathbf{H}_{\text{BR}}^{-1}\mathbf{H}_{\text{MR}}\mathbf{H}_{\text{MR}}^{H}\mathbf{H}_{\text{BR}}^{-H}]}. \tag{7}$$

Then, the received signal at the RS can be represented as

$$\begin{aligned} \mathbf{Y}_{\text{RS}} &= \mathbf{H}_{\text{BR}}\mathbf{X}_{\text{BS}} + \mathbf{H}_{\text{MR}}\mathbf{X}_{\text{MS}} + \mathbf{N}_{\text{RS}} \\ &= m\mathbf{H}_{\text{MR}}\mathbf{S}_1 + n\mathbf{H}_{\text{MR}}\mathbf{S}_2 + \mathbf{N}_{\text{RS}}, \end{aligned} \tag{8}$$

where $\mathbf{N}_{\text{RS}}$ denotes an $(N \times L_s)$-dimensional i.i.d. complex Gaussian noise matrix with the zero-mean and the variance N_0.

In this paper, the linear ZF receiver is applied for simplicity, and the extension of the algorithm to the case of the minimum mean square error (MMSE) or nonlinear receivers such as the successive interference cancellation (SIC) receivers is straightforward, once the ZF case is described.

The received signal after the ZF equalization, $\mathbf{R}_{\text{RS}}$, can be represented as

$$\mathbf{R}_{\text{RS}} = \mathbf{H}_{\text{MR}}^{-1}\mathbf{Y}_{\text{RS}} = m\mathbf{S}_1 + n\mathbf{S}_2 + \tilde{\mathbf{N}}_{\text{RS}}, \tag{9}$$

where $\tilde{\mathbf{N}}_{\text{RS}} = \mathbf{H}_{\text{MS}}^{-1}\mathbf{N}_{\text{RS}}$.

We denote $\mathbf{S}_3$ as an $(N \times L_s)$-dimensional symbol matrix modulated from the codeword $\mathbf{C}_3$, where

$$\mathbf{C}_3 \equiv \mathbf{C}_1 \oplus \mathbf{C}_2. \tag{10}$$

Furthermore, if we define $\mathbf{U}_3 \equiv \mathbf{U}_1 \oplus \mathbf{U}_2$, $\mathbf{C}_3$ is a codeword that is generated from $\mathbf{U}_3$ by the property of linear codes [6].

Considering that the exclusively added codeword $\mathbf{C}_3$ is transmitted in phase 2, the RS does not separately decodes $\mathbf{C}_1$ and $\mathbf{C}_2$ rather decodes $\mathbf{C}_3$ itself. The log likelihood ratio (LLR) values for the (i,l)th element of $\mathbf{C}_3$ can be obtained from (11), where $\mathbf{S}_a(i,q)$, $a = 1,2$, and $\mathbf{Y}_{\text{RS}}(i,q)$, $q = 1,2,\ldots,L_s$, indicate the symbol and the received signal that corresponds to $\mathbf{C}_a^{i,l}$, respectively. Here, $\mathbb{G}_l^b$, $b = 0,1$, denotes the set of all symbol combinations associated with the code bit $\mathbf{C}_a(i,l) = b$. For example, the LLR value of the BPSK case is represented as (12), and the quadrature phase shift keying (QPSK) case can be directly extended from (12).

For M-PAM cases, the one-to-one mappings between the added symbol and the exclusively added codeword are derived

$$\begin{aligned} LLR_{i,l} &= \log \frac{P(\mathbf{C}_3(i,l)=1|\mathbf{Y}_{\text{RS}})}{P(\mathbf{C}_3(i,l)=0|\mathbf{Y}_{\text{RS}})} \\ &= \log \frac{\Sigma_{(\mathbf{S}_1(i,q),\mathbf{S}_2(i,q))\in\mathbb{G}_f^1} P(\mathbf{Y}_{\text{RS}}(i,q)|\mathbf{S}_1(i,q),\mathbf{S}_2(i,q))}{\Sigma_{(\mathbf{S}_1(i,q),\mathbf{S}_2(i,q))\in\mathbb{G}_f^0} P(\mathbf{Y}_{\text{RS}}(i,q)|\mathbf{S}_1(i,q),\mathbf{S}_2(i,q))}. \end{aligned} \tag{11}$$

$$\begin{aligned} LLR_{i,l}|_{\text{BPSK case}} &= \log \frac{P\{(\mathbf{Y}_{\text{RS}}(i,l)|\mathbf{S}_1(i,l)=1,\mathbf{S}_2(i,l)=-1) \text{ or } (\mathbf{Y}_{\text{RS}}(i,l)|\mathbf{S}_1(i,l)=-1,\mathbf{S}_2(i,l)=1)\}}{P\{(\mathbf{Y}_{\text{RS}}(i,l)|\mathbf{S}_1(i,l)=1,\mathbf{S}_2(i,l)=1) \text{ or } (\mathbf{Y}_{\text{RS}}(i,l)|\mathbf{S}_1(i,l)=-1,\mathbf{S}_2(i,l)=-1)\}} \\ &= \log \frac{\exp\{-\frac{(\mathbf{Y}_{\text{RS}}(i,l)-m+n)^2}{N_0}\}+\exp\{-\frac{(\mathbf{Y}_{\text{RS}}(i,l)+m-n)^2}{N_0}\}}{\exp\{-\frac{(\mathbf{Y}_{\text{RS}}(i,l)-m-n)^2}{N_0}\}+\exp\{-\frac{(\mathbf{Y}_{\text{RS}}(i,l)+m+n)^2}{N_0}\}}. \end{aligned} \tag{12}$$

in [1], which can be applied to extend the proposed relaying scheme. Once the one-to-one mapping is guaranteed, the LLR values can be calculated from (11), and finally the RS can decode the exclusively added codeword $\mathbf{C}_3$.

Assuming that the numbers of antennas at the RS and MSs are equal to each other, the kth MS among the selected MSs can obtain the information bits it needs by decoding $\mathbf{C}_3$ and taking the exclusively OR operation to $\mathbf{U}_3(k,:)$ with $\mathbf{U}_2(k,:)$ that it sent previously. The overall procedure in phase 2 is depicted in Fig. 1(b)

C. User Selection

As the sum-rate of the two-phase relaying [1] for MIMO channels is hard to find in a closed form, we choose to select the users who maximizes the effective received signal-to-interference-plus-noise-ratio (SINR). The SINRs between the BS and RS, and between the MS and RS can be easily obtained as [7]

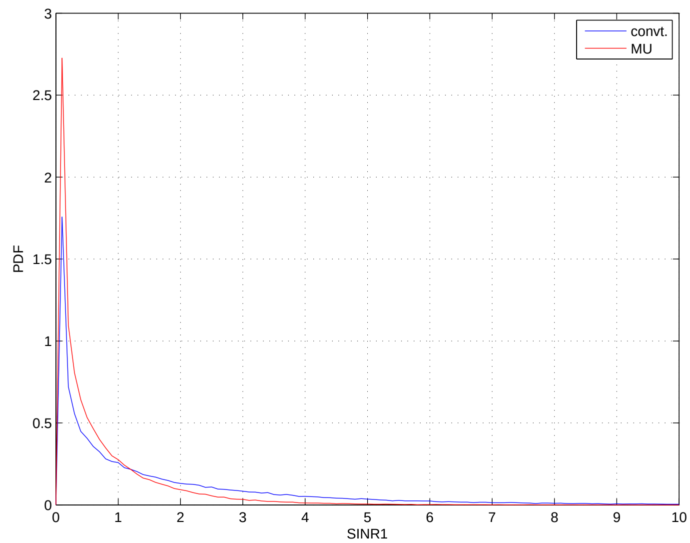
$$\begin{aligned} \gamma_{\text{BR}} &= m^2/\text{Var}(\tilde{\mathbf{N}}_{\text{RS}}(:,l)) \\ &= P_{\text{BS}}/(N_0 \text{tr}[\mathbf{H}_{\text{BR}}^{-1}\mathbf{H}_{\text{MR}}\mathbf{H}_{\text{MR}}^H\mathbf{H}_{\text{BR}}^{-H}] \\ &\quad \times 1/\text{tr}[\mathbf{H}_{\text{MR}}^{-1}\mathbf{H}_{\text{MR}}^{-H}]) \end{aligned} \tag{13}$$

$$\gamma_{\text{MR}} = n^2/\text{Var}(\tilde{\mathbf{N}}_{\text{RS}}(:,l)) = P_{\text{MS}}/(N_0\text{tr}[\mathbf{H}_{\text{MR}}^{-1}\mathbf{H}_{\text{MR}}^{-H}]). \tag{14}$$

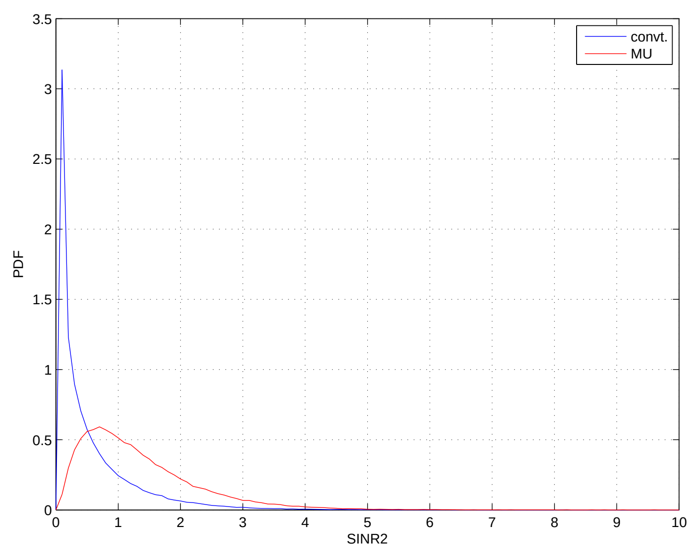
Note that the additional precoding of $\mathbf{H}_{\text{MR}}$ lowers γ_{MR} compared to the conventional ZF-based two-phase relaying as depicted in Fig. 2(a). However, $\mathbf{H}_{\text{MR}}$ can be improved by using the user selection in the proposed scheme; and γ_{MR} becomes higher than that of the conventional ZF-based two-phase relaying as depicted in Fig. 2(b). Denoting the values for the conventional and proposed schemes as $\cdot|_{\text{conv}}$ and $\cdot|_{\text{MR}}$, respectively, we assume the following assumptions:

$$\gamma_{\text{BR}}|_{\text{conv}} > \gamma_{\text{MR}}|_{\text{conv}}. \tag{15}$$

That is, the link between the BS and RS is assumed to have a line of sight and be higher than the link between the MSs and RS. Although the proposed scheme lowers γ_{BR}, the total achievable rate can be improved if the condition of $\gamma_{\text{BR}}|_{\text{prop}} > \gamma_{\text{MR}}|_{\text{prop}}$ is valid. If this condition is valid, the minimum SNR



(a)



(b)

Fig. 2. Effective SNR in phase 1 (a) at BS and (b) at MSs

that determines the total achievable rate can be represented as

$$\begin{aligned} &\min\{\gamma_{\text{BR}}|_{\text{prop}}, \gamma_{\text{MR}}|_{\text{prop}}\} = \gamma_{\text{MR}}|_{\text{prop}} \\ \geq &\min\{\gamma_{\text{BR}}|_{\text{conv}}, \gamma_{\text{MR}}|_{\text{conv}}\} = \gamma_{\text{MR}}|_{\text{conv}}. \end{aligned} \tag{16}$$

Given that the sum rate of the two-way relaying channels is bounded by the minimum value of the SINRs [8], we can represent the user selection problem as finding the optimal augmented matrix $\mathbf{H}_{\text{MR}}$ that satisfies the following problem:

$$\begin{aligned} \max_{\mathbf{H}_{\text{MR}}} \quad & \min(\gamma_{\text{BR}}, \gamma_{\text{MR}}) \\ \text{s.t.} \quad & \gamma_{\text{BR}} = P_{\text{BS}}/(N_0 \text{tr}[\mathbf{H}_{\text{BR}}^{-1}\mathbf{H}_{\text{MR}}\mathbf{H}_{\text{MR}}^{H}\mathbf{H}_{\text{BR}}^{-H}] \\ & \times 1/\text{tr}[\mathbf{H}_{\text{MR}}^{-1}\mathbf{H}_{\text{MR}}^{-H}]), \\ & \gamma_{\text{MR}} = P_{\text{MS}}/(N_0 \text{tr}[\mathbf{H}_{\text{MR}}^{-1}\mathbf{H}_{\text{MR}}^{-H}]). \end{aligned} \tag{17}$$

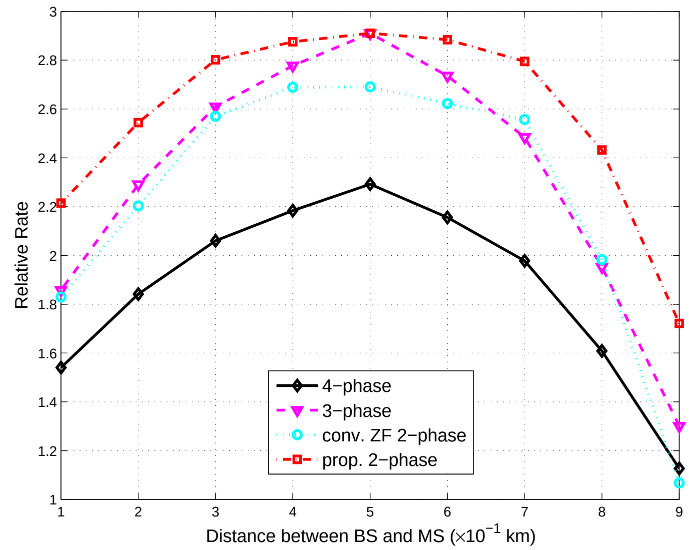
Solving (17) is equivalent to find the optimal $\mathbf{H}_{\text{MR}}$ that maximizes the minimum SNR and thereby maximizes the total achievable rate.

IV. Simulation Results

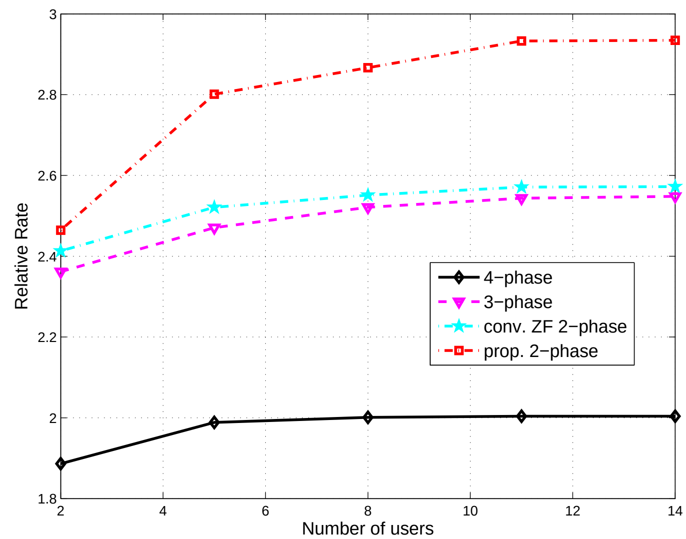
Considering multi-cell interferences for two-way relaying systems, we calculated the typical SINR values as in [1]. The SINRs were decided by the distances between the BS and RS, d_{BR}, and between the MS and RS, d_{MR}. It is assumed that $d_{\text{BR}} = 0.7$ km and $d_{\text{MR}} = |1 - d_{\text{BR}}| = 0.3$ km. For each of conventional schemes, the rate maximizing scheduling was assumed. For simplicity, the modulation for the ZF-based two-phase relaying schemes is fixed to QPSK; however, other modulations can be used for the proposed scheme as described in [1].

Fig. 3(a) denotes the achievable rates where $K = 5$. The curves of '3-phase' and '4-phase' in Fig. 3(a) denote the performances of the coded bi-directional relaying [9] and the conventional half duplex relaying [10], respectively. The coded bidirectional relaying using NC requires three phases to communicate a frame and shows up to 33 % gain compared to the half duplex relaying that requires four phases to communicate a frame. The curve 'conv. ZF 2-phase' denotes the achievable rate of the conventional ZF-based two-phase relaying with QPSK signalling. Because the conventional three-node ZF-based two-phase relaying cannot exploit the multiuser diversity gain and suffers from ill-conditioned matrices, it shows even worse performance than the coded bidirectional relaying using NC. However, the proposed scheme, which is depicted as 'prop. 2-phase' in Fig. 3(a), effectively exploits the multiuser diversity and obtains the maximum achievable rate for QPSK signalling.

Fig. 3(b) shows the achievable rates with respect to the number of MSs where $d_{\text{BR}} = 0.7$ km. The proposed scheme shows almost the same performance as the conventional schemes if the number of MSs is small. However, as the number of MSs increases, the diversity gain increases; and the total achievable rate increases after all. Furthermore, Fig. 3(b) shows that there are saturation points for all schemes where almost full multiuser diversity can be obtained.



(a)



(b)

Fig. 3. Relative rates with respect to (a) the distance and (b) the number of mobile stations

V. Conclusion

We have proposed an efficient ZF-based two-phase relaying for multiuser MIMO systems along with the scheduling method. The conventional ZF-based two-phase relaying can be extended to the multiuser scenario by applying the additional preocoding at the BS because the MSs cannot cooperate with one another. Due to the increased degree of freedom when making the augmented matrix $\mathbf{H}_{\text{MR}}$, the proposed scheme outperforms the conventional ones. As the number of MSs increases, the multiuser diversity gain increases.

Acknowledgment

This work was supported by the IT R&D program of MKE/IITA. [2005-S-609-04, Development of Core Technolo-

gies for Next Generation Mobile Multimedia Platform]

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