A Cognitive p-Persistent CSMA Scheme for Spectrum Sharing Based Cognitive Radio Networks

Min Suk Kang and Bang Chul Jung

KAIST Institute for Information Technology Convergence,
373-1, Guseong-dong, Yuseong-gu, Daejeon, 305-701, KOREA
Email: minsuk.kang@kaist.ac.kr; bcjung@kaist.ac.kr

Abstract— **In this paper, we propose a cognitive p-persistent carrier sense multiple access (CpCSMA) scheme for spectrum sharing based cognitive radio (CR) networks. In order to guarantee the quality of service of the primary (licensed) network, secondary users are allowed to transmit their data as long as the interference received at a primary receiver is limited by the predetermined level. In the proposed CpCSMA scheme, secondary users adaptively control the transmit power with ON/OFF fashion according to whether the interference constraint at primary receivers is violated or not. We show that the proposed CpCSMA scheme can be mathematically analyzed by adopting the scaling factor to the conventional p-persistent CSMA analysis framework. The numerical examples illustrate that the analytical results match well to the simulation results and the throughput of the proposed CpCSMA scheme approaches to that of the conventional p-persistent CSMA scheme for high input load. Moreover, the proposed CpCSMA scheme is backward compatible and, thus, can easily be implemented with little modification of the conventional CSMA scheme.**

I. INTRODUCTION

Recently, cognitive radio (CR) technology [1] [2] has been considered as an attractive solution for efficiently utilizing scarce radio spectrum. In a spectrum sharing based CR system, the secondary (unlicensed) users, coexisting with single primary (licensed) users, are allowed to utilize licensed frequency spectrum as long as the interference power from the secondary transmitters to a primary receiver is below a threshold. This interference constraint guarantees that the primary receiver accepts only predetermined degradation of its quality of service (QoS). In order to meet the interference constraint, the secondary transmitters need to monitor the radio environment cognitively.

The study of a spectrum sharing based CR system was initially inspired by an interference temperature model, proposed by Federal communication commission (FCC) in 2003 as a means to regulating the received interference temperature at a primary receiver [3]. Although the study item of the interference temperature model was terminated by FCC in May 2007 [4], many recent academic researches still provide significant improvements in performance of spectrum sharing based CR systems [5] [6] [7] [8]. Moreover, it is stated that the termination of the model does not foreclose the further research the interference temperature model in FCC [9].

The fundamental studies of spectrum sharing based CR systems have been widely performed in various system environments [10] [11] [12] [5] [6] [7] [8]. Fundamental capacity limits of a spectrum sharing based CR system were studied in [10] and [11] under AWGN and fading channel environments, respectively. Later, a research in [5] studied the capacity limit of a spectrum sharing based CR system with multiple secondary users. The authors in [5] proposed a centralized spectrum sharing technique to perform efficient resource management for the secondary user network.

Many communication systems, such as IEEE 802.11a/b/g (commercial WLAN), IEEE 802.15.4 (ZigBee), or IEEE 802.11p (vehicular communications), are operated with decentralized scheduling protocols and most of them are based on carrier sense multiple access (CSMA) schemes. Cognitive random access schemes are needed for the decentralized networks to operate under a spectrum sharing based CR system and a few decentralized medium access control (MAC) schemes have been proposed for interweave CR systems [13] [15] [16]; however, there have been no study on cognitive random access schemes for a spectrum sharing based CR system.

In this paper, we propose a *cognitive* p-persistent CSMA (CpCSMA) scheme for a spectrum sharing based CR system. The proposed CpCSMA scheme operates with fixed transmit power control, which simply turns on/off the transmit power. The proposed CpCSMA is faster-to-market because it is backward compatible to the conventional pCSMA scheme. We provide rigorous mathematical analysis of the proposed CpCSMA scheme. Interestingly, it is found that the proposed CpCSMA scheme can achieve about 85% of the peak throughput of the system without the primary network even under a strict interference constraint at a primary receiver. Furthermore, we have found the condition that the CpCSMA can achieve the throughput of the conventional pCSMA without the primary network.

The rest of this paper is organized as follows. In Section II, the system model and the conventional p-persistent CSMA scheme are described. In Section III, we propose a cognitive p-persistent CSMA scheme. In Section IV, numerical and Monte-Carlo simulation results are shown and the accuracy of the numerical results is verified. We provide some observations about the throughput at low and high system offered load and peak throughput analysis. Finally, conclusions are drawn in Section V.

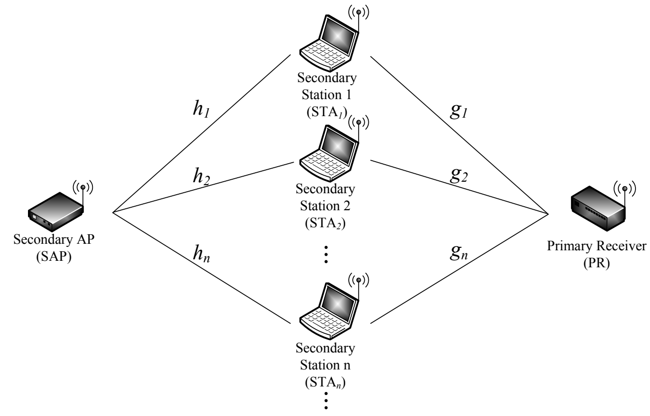


Fig. 1. System Model

II. SYSTEM MODEL AND THE CONVENTIONAL p-PERSISTENT CSMA

A. System model

Fig. 1 shows the secondary network with a single secondary access point (SAP), time-varying number of secondary stations (STAs), and a single primary receiver (PR) coexisting in the same spectrum. The secondary network is assumed to be a random access system; each STA accesses the channel and transmits its data in a decentralized and random manner. Since the primary network has the priority for the shared spectrum resource, the secondary network is required to guarantee the quality of service (QoS) constraints on the PR.

In our system model, STAs communicate with the SAP through quasi-static fading channels, which imply that the fading channel gain remains static until the transmission of a packet is finished and independently changes at the beginning of a new random access procedure. Assuming only one STA accesses the channel at time t, the received signal of the SAP at time t is given by

$$y(t) = h_i x_i(t) + n_i, \tag{1}$$

where h_i is a Rayleigh distributed data channel between the STA i and the SAP with the mean of μ_h, $x_i(t)$ represents the transmitted signal from the STA i, and n_i denotes an additive white Gaussian noise (AWGN) with the power of N_0. The power of the transmitted signal $x(t)$ is P_i and can be dynamically controlled by the STA i. For simpler mathematical analysis, the noise power N_0 is assumed to be 1 without loss of generality.

When the STA i transmits $x_i(t)$, the PR receives an unwanted interference of power $|g_i|^2 P_i$, where g_i is a Rayleigh distributed interference channel between the STA i and the PR with the mean of μ_g. To guarantee the QoS of the primary network, we assume the secondary network performs transmit power control according to the interference constraint, $|g_i|^2 P_i \le Q$, at the PR. The PR predetermines an allowable interference power, Q, and the STA i cognitively controls its parameters to meet the constraint

Each STA i is assumed to be able to obtain its own signal channel gain $|h_i|^2$ by measuring the signal strength of the received packet form the SAP. Moreover, since the time division duplex (TDD) is presumed in the primary network, the STA i is able to obtain the interference channel g_i by overhearing the preamble transmitted by the PR when it is operating as a transmitter.

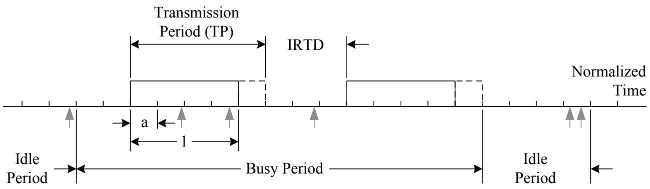


Fig. 2. Operation of the conventional p-Persistent CSMA

B. Conventional p-Persistent CSMA

The carrier sense multiple access (CSMA) protocols, invented in the 1970s, have been widely used in modern networking such as WLAN and ZigBee due to its flexibility for random access systems. One of the popular CSMA algorithms is p-persistent CSMA (pCSMA) protocol. When the shared medium is sensed idle, a node contending for the medium transmits with probability p and defers with the probability $(1-p)$ [17]. In [18], it is shown that the IEEE 802.11 WLAN MAC standard protocol can be well approximated by the pCSMA. Therefore, we adopt the pCSMA protocol as the baseline protocol for the secondary network.

Fig. 2 shows how the conventional pCSMA works and some of the definitions of the related terms. In pCSMA protocol, the system is slotted (the slot size is a, which is the normalized propagation delay) and all transmissions start at the beginning of a slot. We assume that the length of a transmission period (TP) is 1 for every transmission. If an STA transmits a packet at time $t = 0$, all the other STAs should wait until $t = 1 + a$ since a represents the propagation delay. All packets from all STAs are modelled as a single Poisson process with offered load G for mathematical simplicity. The term G is closely related to the average number of active STAs in the system. Since one or more STAs contend for the shared medium at every slot, there exists a random delay before a TP starts, called the initial random transmission delay (IRTD).

In this paper, we assume that each STA can achieve its channel capacity by using capacity achieving codes for each transmission. Thus, the STA i with the channel h_i can achieve $\log_2\left(1 + P_i|h_i|^2\right)$.

The performance of the conventional pCSMA is measured as the system throughput. The throughput of pCSMA can be defined as the number of successfully transmitted bits per unit time and given in a function of G, p, and a as follows [17]:

$$S_{\mathrm{C}}(G,p,a) = \frac{(1-e^{-aG})R_{\mathrm{C}}}{(1-e^{-aG})[a\overline{t'}\pi_0 + a\overline{t}(1-\pi_0) + 1 + a] + a\pi_0}, \tag{2}$$

where $\overline{t}$ and $\overline{t'}$ are the average length of the IRTD before the first and the other TPs, respectively, and π_n is defined as

$$\pi_n = e^{[-(1+a)G]} \frac{[(1+a)G]^n}{n!}. \tag{3}$$

The term R_{C} represents the average throughput over the channel fading statistics and is given in a closed-form as

$$R_{\text{C}} = R_0 \sum_{n=0}^{\infty} \pi_n P_s(n), \tag{4}$$

where $P_s(n)$ represents the probability that a packet is successfully sent without any collision in the presence of n STAs and is given as $\frac{npq^{n-1}}{1-q^n}$ for $n > 0$ where $q = 1-p$. The ergodic capacity R_0 of a single transmission with randomly selected h is given as

$$\begin{aligned} R_0 &= \int_0^{\infty} \frac{1}{\mu_h} e^{-h/\mu_h} \log_2(1 + P_i h) dh \\ &= \frac{e^{1/P_i \mu_h}}{\ln 2} E_1\left(\frac{1}{P_i \mu_h}\right), \end{aligned} \tag{5}$$

where $E_1(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt$ represents an exponential integral.

III. PROPOSED p-PERSISTENT CSMA SCHEME FOR COGNITIVE RADIO NETWORKS

A. Cognitive p-persistent CSMA (CpCSMA)

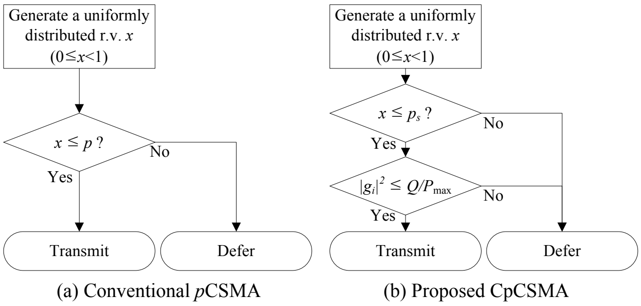
Unfortunately, the conventional pCSMA is not suitable for a spectrum sharing based CR system because randomly selected user by the conventional pCSMA could violate the interference constraint. Therefore, we propose a cognitive p-persistent CSMA (CpCSMA) considering the interference constraint as well as backward compatibility to the conventional pCSMA.

For designing the CpCSMA scheme for a spectrum sharing based CR system, we assume that a STA i operates with a *fixed transmit power control*. The power allocation rule for the fixed transmit power control at the STA i is given as

$$P_i = \begin{cases} P_{\max}, & |g_i|^2 \le Q/P_{\max} \\ 0, & |g_i|^2 > Q/P_{\max} \end{cases}. \tag{6}$$

The rule guarantees the QoS of the PR by turning off the transmit power of the STA when its maximum transmit power could violate the interference constraint. Although a *variable transmit power control*, which allows the STA to use continuous transmit power between 0 and $P_{\max}$, can be the better solution in terms of performance, in this paper, we only consider the fixed transmit power control due to its simplicity and backward compatibility to conventional random access networks. (Typical random access networks such as WiFi and ZigBee control data rates according to wireless channel quality and fix their transmit power to their maximum value.)

Moreover, in order to satisfy the backward compatibility to the pCSMA scheme, we devise a *p-scaling technique* so that an STA of the CpCSMA accesses the channel with the same probability p. By maintaining the same probability to transmit a packet, the STA of the CpCSMA can coexist with the other STAs of the pCSMA without any fairness problem. With the p-scaling, we scale-up the probability of channel access p to



(a) Conventional pCSMA (b) Proposed CpCSMA

Fig. 3. Flowcharts of the random channel access process of the conventional pCSMA and the proposed CpCSMA

$$p_s = \frac{p}{\gamma}, \tag{7}$$

where $\gamma \triangleq F_G\left(\frac{Q}{P_{\max}}\right) = 1 - e^{-\frac{Q}{\mu_g P_{\max}}}$ and $F_G(\cdot)$ represents the cumulative distribution function of the interference channel gain $|g_i|^2$. Fig. 3 shows how the random channel access processes of the two different CSMA schemes operate. In Fig. 3-(a), an STA of the pCSMA only goes through the *random channel access phase* (diamond), which determines the channel access with probability of p. However, an STA of the CpCSMA additionally undergoes one more phase, *interference constraint phase*, as in the second diamond in Fig. 3-(b). Considering the interference constraint phase of the CpCSMA, we scale-up the probability to p_s so that each STA finally transmits its packet with the same probability of p. When the PR does not exists (i.e., $Q \to \infty$), the proposed CpCSMA works the same as the conventional pCSMA, due to the fact that $\gamma = F_G\left(\frac{Q}{P_{\max}}\right) \to 1$.

B. Performance of the CpCSMA and the pCSMA

The performance of the proposed CpCSMA is closely related to the conventional pCSMA, since, as in the pCSMA, each STA in the CpCSMA transmits its packet with probability of p at each backoff slot. Although the two schemes have the same probability at each backoff slot, differences exist due to the fact that the random channel access process of the CpCSMA at each backoff slot is not independent because of the quasi-static fading assumption. The correlation between random channel access processes in successive backoff slots limits the performance of the proposed CpCSMA.

The relationship between the throughput of the CpCSMA and the conventional pCSMA is found in Proposition 1.

Proposition 1: The throughput of the proposed CpCSMA, $S(G, p, a)$, is equal to the throughput of the conventional pCSMA with the scaled-down G, $G_s = G\gamma$, and the scaled-up p, $p_s = p/\gamma$. That is,

$$S(G, p, a) = S_{\text{C}}\left(G\gamma, \frac{p}{\gamma}, a\right). \tag{8}$$

Proof: Due to the correlation between random channel access processes in successive backoff slots in the CpCSMA, in average, $1-\gamma$ of arrived packets cannot be transmitted during an IRTD because they violate the interference constraint. Since some portion of arrived packets do not participate in the contention, we can model the effective arrival process of the CpCSMA as the random selection of a Poisson process with rate G. Let $X(t)$ be the Poisson process for the arrival process with rate G. Let $Y(t)$ be the selected process. Then,

$$\begin{aligned} P\{Y(\Delta t) = 1\} =& \\ & P\{\text{selected}|X(\Delta t)=1\}P\{X(\Delta t)=1\} \\ &+ P\{\text{only one selected}|X(\Delta t)=2\}P\{X(\Delta t)\geq 2\} \quad (9) \\ =& \quad G\gamma\Delta t + o(\Delta t), \quad (10) \end{aligned}$$

where $o(h)$ is a little-o function, which satisfies the asymptotic relation $\lim_{h\to 0} o(h)/h = 0$. Therefore, from the Poisson process $X(t)$ if we select each arrival with probability γ, then the selected arrival process $Y(t)$ forms a new Poisson process with parameter $G\gamma$.

Moreover, the selected (with probability γ) STAs access the channel with probability of p/γ. Therefore, the throughput of the CpCSMA is equivalent to the throughput of the pCSMA with $G\gamma$, p/γ, and a. ■

IV. NUMERICAL ANALYSIS

In this Section, we perform simulations for evaluating the throughput performance of the CpCSMA scheme. Active STAs are modelled as a single Poisson process with offered load G and higher G implies that there are more average number of active STAs in the system. Simultaneous transmissions from two or more STAs are regarded as collision. The simulation results are compared to the analytical model obtained from (8) when $a = 0.01$, $p = 0.03$ and $\mu_h = \mu_g$ = 0 dB. The maximum transmit power $P_{\max}$ is set to 1. Lines and symbols represent analysis and simulation, respectively.

A. Throughput Analysis

Fig. 4 shows the throughput of the conventional pCSMA and the proposed CpCSMA schemes for varying offered load G and varying interference level Q. The throughput of the pCSMA is defined in (2) and that of the CpCSMA is analyzed in (8) in Proposition 1. It shows that the analysis well matches to the simulation results. For all schemes, it is observed that the throughput has its peak value at a certain point and decreases and finally converges to zero for higher G. It is a main characteristic of the throughput of CSMA based MAC protocols owing to the fact that the collision probability drastically increases as the offered load increases.

We have found two interesting characteristics on the throughput at low and high G. Firstly, at high G, it is observed that the throughput of the proposed CpCSMA converges to that of the pCSMA, no matter what interference level Q is set at the PR. The throughput convergence in high G region can be analyzed and quantified from the following Proposition.

Proposition 2: At high offered load, the throughput of the CpCSMA scheme approaches to the throughput of the pCSMA with no primary network. That is, for high G, $S(G,p,a) \to S_{\mathrm{C}}(G,p,a)$.

Proof: See Appendix I. ■

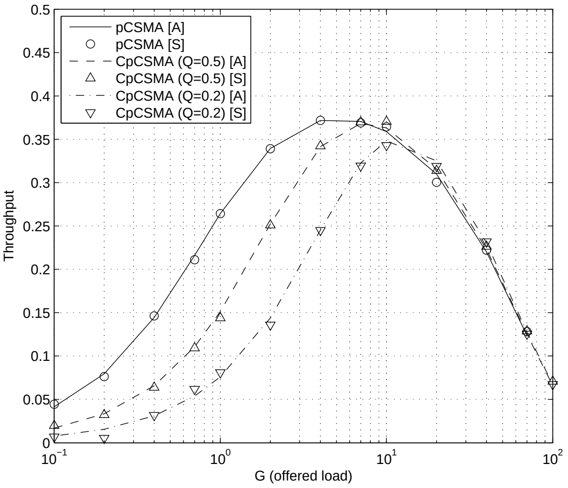


Fig. 4. Comparison of analyzed and simulated throughput of the pCSMA and the CpCSMA for varying G when $a = 0.01$, $p = 0.03$ and $\mu_h = \mu_g$ = 0 dB. [A] and [S] represent analysis and simulation, respectively.

The convergence of the throughput of the CpCSMA to that of the pCSMA at high offered load can also be intuitively explained as follows. When the offered load to the system is high, it is highly probable that number of STAs at each backoff slot 0 (the first backoff slot) is much greater than one. When active STAs are '*very crowded,*' the expected number of backoff slots before at least one STA transmits would be close to 1. Then the backoff process of the CpCSMA scheme finished at the first backoff slot with high probability and the operation of the CpCSMA becomes exactly the same as the pCSMA. In other words, with high load, it is highly probable that there are always going to be STAs that satisfy the interference constraint with packet to transmit. Therefore the CpCSMA would look as the pCSMA.

Secondly, at low G, it is observed that as Q increases, i.e., as it gets harder to satisfy the interference constraint, the throughput of the system degrades. The degradation in low G region can be analyzed and quantified from the following Proposition.

Proposition 3: At low offered load, the throughput of the CpCSMA scheme is the same as the throughput of the conventional pCSMA with the scaled-down $G_s = G\gamma$. That is, as $G \to 0$, $S(G,p,a) \to S_{\mathrm{C}}(G_s,p,a)$.

Proof: See Appendix II. ■

B. Peak Throughput Analysis

As shown in Fig. 4, the throughput of the pCSMA and the CpCSMA does not always grow as the offered load G increases, but decreases to zero after a peak point. Peak throughput is achieved at different G for different system parameters. Fig. 5 shows the peak throughput of the CpCSMA

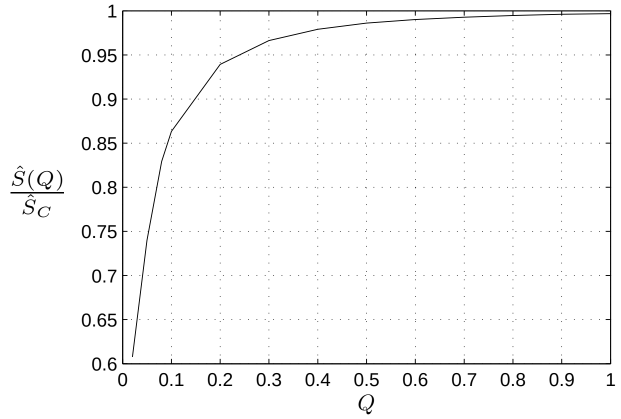


Fig. 5. Comparison of the normalized peak throughput of the pCSMA and the CpCSMA for varying Q when $a = 0.01$, $p = 0.03$ and $\mu_h = \mu_g = 0$ dB.

normalized by that of the pCSMA. As the Q at the PR increases, the normalized peak throughput reaches to 1. It is noteworthy that the secondary network can achieve about 85% of the peak throughput at $Q = 0.1$, compared to the case when there is no interference constraint. That the interference level Q equals to 0.1 means that the allowable received interference power is 10% of the thermal noise at the PR, which is a extremely strict interference constraint at the PR.

V. CONCLUSIONS

We proposed a p-persistent carrier sense multiple access (CSMA) scheme for spectrum sharing based cognitive radio (CR) systems as a decentralized medium access control protocol for spectrum sharing based CR systems. In this paper, we assume that secondary users make a decision on transmitting data according to the interference channel between the secondary users and the primary receiver. It was found that the proposed CpCSMA scheme can achieve about 90% of the peak throughput of the conventional p-persistent CSMA scheme without a primary network even under a strict interference constraint. Furthermore, we have found the condition that the CpCSMA can achieve the throughput of the conventional p-persistent CSMA without the primary network. The CpCSMA is backward compatible to the conventional p-persistent CSMA because it maintains the same probability p to access the medium. The proposed CpCSMA scheme can be extended to consider hidden node problems, collisions at the primary receiver, and practical protocols for secondary transmitters to monitor interference channels, which remain as our further study.

Appendix I

Proof: From Proposition 1, $S(G, p, a) = S_{\mathrm{C}}(G_s, p_s, a)$, where $G_s = G\gamma$ and $p_s = \frac{p}{\gamma}$.

Due to the fact that e^{-aG} and π_0 go to zero for high G and $a \ll 1$, the throughput of the CpCSMA is simplified for high G as follows:

$$
\begin{aligned}
&S_{\mathrm{C}}(G_s, p_s, a) = \\
&\quad \frac{(1-e^{-aG_s})R_{\mathrm{C}}}{(1-e^{-aG_s})[a\overline{t'}\pi_0 + a\overline{t}(1-\pi_0) + 1 + a] + a\pi_0} \quad (11)\\
&\rightarrow \quad \frac{R_{\mathrm{C}}}{a\overline{t}+1+a} \rightarrow \frac{R_{\mathrm{C}}}{a\overline{t}+1} \quad (12)
\end{aligned}
$$

For further analysis, we analyze the two terms in (12): R_{C} and $\overline{t}$. At first, let us investigate the average throughput R_{C} in (12). Because R_{C} defined in (4) is a function of the offered load, transmission probability, and slot size, we represent it with the parameters. The average throughput for the CpCSMA is given as

$$
\begin{aligned}
&R_{\mathrm{C}}(G_s, p_s, a)\\
&= R_0 \sum_{n=0}^{\infty} \pi_n P_s(n) \rightarrow R_0 \sum_{n=1}^{\infty} \pi_n P_s(n) \quad (13)\\
&= R_0 \sum_{n=1}^{\infty} \pi_n P_s(n) \sum_{k=0}^{\infty} P_K(k) \quad (14)\\
&= R_0 \left[(1+a)G_s p_s \sum_{k=0}^{\infty} e^{-(1+a)G_s(1-q_s^{k+1})} q_s^k\right] \quad (15)\\
&= R_0 \left[(1+a)G_s p_s \sum_{k=0}^{\infty} T(k)\right] \quad (16)
\end{aligned}
$$

where $q_s = 1-p_s$ and $T(k) = e^{-(1+a)G_s(1-q_s^{k+1})}q_s^k$. The term $P_K(k)$ in (14) represents the probability that at least one STA begins transmitting a packet in backoff slot k and is defined as $P_K(k) = q_s^{kn}(1-q_s^n)$ and satisfies $\sum_{k=0}^{\infty} P_K(k) = 1$. From (14) to (15), $e^x = \sum_{n=0}^{\infty} x^n/n!$ is used. Since $T(j)$ for $j \geq 2$ is negligible compared to $T(0)$, for high G, the equation (16) is further analyzed as

$$
\begin{aligned}
&R_{\mathrm{C}}(G_s, p_s, a) \rightarrow\\
&\qquad \left[(1+a)G_s p_s e^{-(1+a)G_s(1-q_s)} q_s^0\right] R_0 \quad (17)\\
&= \left[(1+a)G_s p_s e^{-(1+a)G_s p_s}\right] R_0. \quad (18)
\end{aligned}
$$

Second, the term $\overline{t}$ in (12) represents the average number of backoff slots elapsed until some packet is transmitted and is defined in [17] as follows:

$$
\begin{aligned}
&\overline{t} = \\
&\sum_{n=1}^{\infty}\sum_{k=0}^{\infty} q_s^{(k+1)n} \exp\left\{aG_s\left(\frac{q_s(1-q_s^k)}{p_s} - k\right)\right\}\left(\frac{\pi_n}{1-\pi_0}\right) \quad (19)\\
&\rightarrow \quad e^{-(1+a)G_s} \sum_{k=0}^{\infty} \exp\left(aG_s\left(\frac{q_s(1-q_s^k)}{p_s} - k\right)\right) \times\\
&\qquad \left[\exp\left((1+a)G_s q_s^{(k+1)}\right) - 1\right] \quad (20)\\
&= \quad \sum_{k=0}^{\infty} \left[\exp(G_s A(k)) - \exp(G_s B(k))\right] \quad (21)
\end{aligned}
$$

where $A(k)$ and $B(k)$ are defined as

$$A(k) = a\left(\frac{q_s(1-q_s^k)}{p_s} - k\right) - (1+a)(1-q_s^{k+1}) \quad (22)$$

$$B(k) = a\left(\frac{q_s(1-q_s^k)}{p_s} - k\right) - (1+a) \quad (23)$$

Thus, both $\exp(G_sA(k))$ and $\exp(G_sB(k))$ go to zero for high G. Since $B(k) < A(k)$, $\exp(G_sA(k)) - \exp(G_sB(k))$ also goes to zero. Therefore, for high G, $\bar{t}$ converges to a small value and it is finally trivial that

$$a\bar{t} + 1 \rightarrow 1. \quad (24)$$

From (24), we continue the equation (12) as follows:

$$S_{\mathrm{C}}(G_s, p_s, a) \rightarrow R_{\mathrm{C}} \quad (25)$$

Finally, from (18) and (25), the ratio of the throughput of the pCSMA and the CpCSMA is given as

$$\frac{S(G,p,a)}{S_{\mathrm{C}}(G,p,a)} = \frac{S_{\mathrm{C}}(G_s,p_s,a)}{S_{\mathrm{C}}(G,p,a)} \quad (26)$$

$$\rightarrow \frac{R_{\mathrm{C}}(G_s,p_s,a)}{R_{\mathrm{C}}(G,p,a)} \quad (27)$$

$$\rightarrow \frac{\left[(1+a)G_sp_se^{-(1+a)G_s(1-q_s)}q_s^0\right]R_{0,1}}{\left[(1+a)Gpe^{-(1+a)G(1-q)}q^0\right]R_{0,1}} \quad (28)$$

$$= \frac{(1+a)G_sp_se^{-(1+a)G_s(1-q_s)}}{(1+a)Gpe^{-(1+a)G(1-q)}} \quad (29)$$

$$= \frac{(1+a)G_sp_se^{-(1+a)(G\gamma)(p/\gamma)}}{(1+a)Gpe^{-(1+a)Gp}} = 1 \quad (30)$$

Therefore, $S(G,p,a) \rightarrow S_{\mathrm{C}}(G,p,a)$ is proved for high G. ■

Appendix II

Proof: The throughput of the pCSMA can be simplified at low G as follows.

$$S_{\mathrm{C}}(G,p,a)$$

$$= \frac{(1-e^{aG})R_{\mathrm{C}}}{(1-e^{aG})[a\overline{t'}\pi_0 + a\bar{t}(1-\pi_0) + 1 + a] + a\pi_0} \quad (31)$$

$$\rightarrow \frac{(1-e^{aG})R_{\mathrm{C}}}{a} \rightarrow \frac{(1-(1-aG))R_{\mathrm{C}}}{a} = G \times R_{\mathrm{C}} \quad (32)$$

$$= R_0G\sum_{n=0}^{\infty}\pi_nP_s(n) \quad (33)$$

$$\rightarrow R_0G\pi_0P_s(0) = R_0Ge^{-(1+a)G}. \quad (34)$$

Equation (32) is obtained from (4). Equation (32) is simplified to (33) because π_j ($j \geq 1$) is negligible compared to π_0. Therefore, as $G \rightarrow 0$, the throughput of the pCSMA scheme only depends on the offered load G, not on p or a.

From Proposition 1 and (34), we obtain $S(G,p,a) = S_{\mathrm{C}}(G_s,p_s,a) \rightarrow S_{\mathrm{C}}(G_s,p,a)$, as $G \rightarrow 0$. ■

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