Opportunistic Noisy Network Coding for Fading Parallel Relay Networks

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***Abstract*—The recently developed noisy network coding naturally extends compress–forward coding for the relay channel by Cover and El Gamal to arbitrary relay networks. In particular, the noisy network coding scheme achieves the best known capacity lower bound for general Gaussian networks.**

Motivated by the recent development of noisy network coding, we propose a novel extension of noisy network coding specialized for the *fading* parallel relay network. In the new scheme, the relay observation is *opportunistically* compressed by adapting on the *local channel state information of the source–relay link*. More specifically, each relay node opportunistically compresses the collection of output symbols with channel gains above a certain threshold, and forwards the digital compression to the destination node using independent Gaussian codes. To present the potential of the new scheme, we focus on the symmetric setting in which the channel coefficients within each hop are identically and independently distributed. We show that in the large number of relays regime, our scheme achieves the capacity while outperforming other schemes such as amplify–forward and decode–forward. Our result demonstrates that adaptation using channel state information at the receiver side can be beneficial.

I. Introduction

In recent years, multihop and cooperative communication using relays has received a great deal of attention from both academia and industry due to its potential in wireless networks [1], [2], [3], [4], [5]. Both IEEE 802.16j and IEEE 802.16m systems have adopted multihop relays for coverage extension and higher throughput [6], [7]. Recently, 3GPP LTE-advanced system is also considering relays for the same purposes [8]. Due to the fact that multiple relays can significantly increase system performance, recent research has been focused on multiple relay configurations.

For relay networks, there are three core relaying schemes in the literature: decode–forward (DF), amplify–forward (AF), and compress–forward (CF). In the DF scheme [9], first developed by Cover and El Gamal for the three-node relay channel, the relay node recovers the message either fully or partially and forwards it to the destination node while coherently cooperating with the source node. In [10], [11], DF has been generalized to multiple relay networks. In the AF scheme [12], the relay simply sends an amplified version of its received signal and forwards it. In the CF scheme, also first developed in [9], the relay quantizes its received signal and forwards it. The CF scheme has been generalized to arbitrary noisy networks in [13], [14].

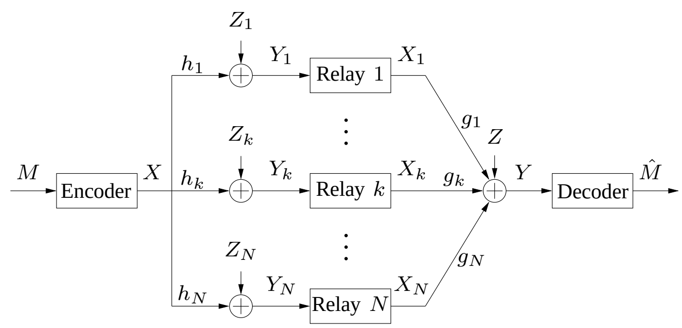

Fig. 1. Gaussian parallel relay networks.

The parallel relay network [12] depicted in Fig. 1 is a two hop network in which the source node communicates to the destination node by the help of a set of N relay nodes. The source node transmits to a set of relays through a broadcast channel, and the relay nodes transmits to the destination node through a multiple access channel. For the Gaussian parallel 2-relay network, the achievable rates of DF and AF have been analyzed in [15]. It was further shown that DF and AF achieve the capacity in some signal-to-noise ratio (SNR) regimes. The asymptotic characteristics of the parallel relay network was analyzed in [16] where it has been shown that in certain SNR regimes, AF can achieve the capacity as the number of relays N goes to infinity. The authors of [17] showed that the bursty AF scheme achieves the capacity of the symmetric Gaussian parallel relay network within a constant gap independent of SNR and the number of relays N. The noisy network coding scheme [14], originally developed under a general framework by considering networks with arbitrary topology and number of hops can be specialized to Gaussian networks which includes the Gaussian parallel relay network. It was shown that noisy network coding is universally within $1.26N$ bits/s/Hz of the capacity, where the capacity gap does not depend on the channel gains, power constraints, nor the topology of the network. However, the full potential of noisy network coding for *fading* relay networks is yet to be explored.

Motivated by the recent development of noisy network coding, we propose a novel extension of noisy network coding specialized for the fading parallel relay network. In the new scheme, the relay observation is opportunistically compressed by adapting on the source–relay channel state information (CSI) at the relay node. In particular, each relay node adaptively compresses a subset of observation symbols with channel gains above a certain threshold. By treating the digital compression of the "good" channel observations as an independent message, the relays send the digital compression to the destination node using independently generated Gaussian codes. Our proposed scheme does not require CSI at the transmitter side (CSIT), which is impractical for most wireless communications due to the time-varying nature of wireless channels and feedback overhead. The fast fading setup with CSI at the receiver side (CSIR) makes our work distinguishable from other models assuming block fading or global CSI [16], [18], [19].

For the general fading parallel relay network, we first show that our proposed scheme is within N bits/s/Hz of the capacity, which shows that noisy network coding can be effectively extended to fading relay networks. To further present the potential of the new scheme, we then focus on the symmetric setting in which the channel statistics within each hop is identically and independently distributed. We show that in the large number of relays regime, our scheme achieves the capacity while strictly outperforming other schemes such as AF and DF. Our result demonstrates that adaptation only using CSIR can be beneficial.

II. Problem Statement

Throughout the paper, we will use the following notation. Denote $[1:N] = \{1, 2, \cdots, N\}$, $x^n = \{x[1], \cdots, x[n]\}$, and $\mathsf{C}(x) = \log(1 + x)$, where the log operation is with respect to base 2. For $\mathcal{S} \subseteq [1:N]$, denote $\mathcal{S}^c = [1:N] \setminus \mathcal{S}$ and $X(\mathcal{S}) = (X_k : k \in \mathcal{S})$. We also use the notation $X^N = \{X_1, \ldots, X_N\}$, $H = \{h_1, \cdots, h_N\}$, and $G = \{g_1, \cdots, g_N\}$.

We consider the fading parallel relay network depicted in Fig. 1 in which the source node wishes to send a message to the destination node with the help of N relay nodes. The source node has a channel input X, relay node $k \in [1:N]$ has an input and output pair (X_k, Y_k), and the destination node has a channel output Y. Then the input–output relations at time t are given by

$$Y_k[t] = h_k[t]X[t] + Z_k[t]$$

and

$$Y[t] = \sum_{k=1}^{N} g_k[t]X_k[t] + Z[t]$$

where $Z_k[t]$ and $Z[t]$ are independent complex Gaussian noise with $\mathcal{N}_{\mathbb{C}}(0, 1)$. We assume average power constraint P for the source node and P_r/N for the relay nodes, i.e., $\mathsf{E}[|X[t]|^2] \le P$ and $\mathsf{E}[|X_k[t]|^2] \le P_r/N$ for all $k \in [1:N]$.

We assume time varying channels such that $h_k[t]$ and $g_k[t]$ are independently drawn from $\mathcal{N}_{\mathbb{C}}\left(0, \sigma^2_{h_k}\right)$ and $\mathcal{N}_{\mathbb{C}}\left(0, \sigma^2_{g_k}\right)$, respectively. We further assume that CSI is causally available only at receiver sides, i.e., relay node k knows $h_k[t]$ at time t and the destination knows $h_1[t]$ to $h_N[t]$ and $g_1[t]$ to $g_N[t]$ at time t. In the rest of the paper, we will omit the time index for notational convenience.

Let M be the message of the source, uniformly distributed over $[1:2^{nR}]$. A $(2^{nR}, n)$ code consists of an encoding function $x^n = \varphi(M)$, relaying functions at time t, $x_k[t] = \varphi_{k,t}(y_k^{t-1})$, for $k \in [1:N]$, and a decoding function $\psi(y^n) = \hat{M}$. A rate R is said to be *achievable* if there exist a sequence of $(2^{nR}, n)$ codes with $\mathsf{P}\{\hat{M} \ne M\} \to 0$ as $n \to \infty$, where $\mathsf{P}\{\hat{M} \ne M\}$ is the average probability of error. The capacity C_N of the fading parallel relay network with N relay nodes is the supremum of all achievable rates. When the context is clear, we will drop the subindex N throughout the paper.

III. Main Results

In this section, we propose an opportunistic noisy network coding scheme and show that it achieves the capacity of the fading symmetric parallel relay network in the limit of large N.

A. Cutset Upper Bound

The cutset upper bound [20], [21] on the capacity of the fading parallel relay network is given by

$$C \le \max \min_{\mathcal{S} \subseteq [1:N]} I(X, X(\mathcal{S}); Y(\mathcal{S}^c), Y | X(\mathcal{S}^c), H, G) \quad (1)$$

where the maximum is over all probability distributions $p(x, x_1, \ldots, x_N)$ such that the power constraints are satisfied. By using some Markov relations and the fact that an independent Gaussian distribution maximizes the multiple input single output channel with per antenna power constraint [22], (1) can be simplified to

$$C \le \min_{\mathcal{S} \subseteq [1:N]} \mathsf{E}\left[\mathsf{C}\left(\sum_{k \in \mathcal{S}^c} |h_k|^2 P\right) + \mathsf{C}\left(\sum_{k \in \mathcal{S}} |g_k|^2 \frac{P_r}{N}\right)\right]. \quad (2)$$

B. Opportunistic Noisy Network Coding

The noisy network coding lower bound [14] for discrete memoryless networks can be adapted for the fading parallel relay network with power constraint and state dependency (e.g., channel gains) which yields the following lower bound

$$\begin{aligned} C \ge \max \min_{\mathcal{S} \subseteq [1:N]} & I(X, X(\mathcal{S}); \hat{Y}(\mathcal{S}^c), Y | X(\mathcal{S}^c), H, G) \\ & - I(Y(\mathcal{S}); \hat{Y}(\mathcal{S}) | X, X^N, \hat{Y}(\mathcal{S}^c) Y, H, G) \end{aligned}$$

where the maximization is over all probability distributions $p(x) \prod_{k=1}^{N} p(x_k) p(\hat{y}_k | y_k, h_k)$ such that the power constraints are satisfied. We emphasize that the compressed output $\hat{Y}_k$ can depend on h_k at relay node k by considering $p(\hat{y}_k | y_k, h_k)$ instead of $p(\hat{y}_k | y_k)$. Thus, this adaptation can be done in a *distributed manner* based only on each relay's *local CSIR*.

Theorem 1: For the fading parallel relay network,

$$\begin{aligned} C \ge \max \min_{\mathcal{S} \subseteq [1:N]} \mathsf{E}\Bigg[& \mathsf{C}\left(\sum_{k \in \mathcal{S}^c} \frac{|h_k|^2 P}{1 + \eta_k(h_k)}\right) \\ & + \mathsf{C}\left(\sum_{k \in \mathcal{S}} |g_k|^2 \frac{P_r}{N}\right) - \sum_{k \in \mathcal{S}} \mathsf{C}\left(\frac{1}{\eta_k(h_k)}\right)\Bigg] \quad (3) \end{aligned}$$

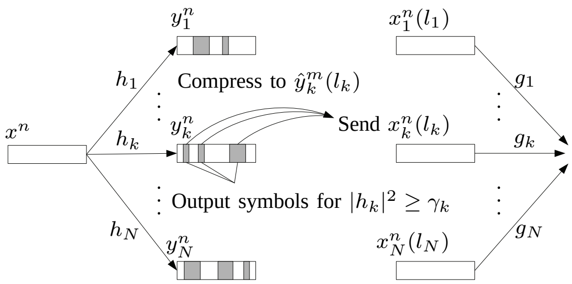

Fig. 2. Threshold-based adaptation for the opportunistic noisy network coding scheme.

where the maximization is taken over all functions of h_k, $\eta_k(h_k) > 0$, $k \in [1:N]$.

Proof: By using the Markov structure of the network, we can further simplify the opportunistic noisy network coding lower bound as

$$C \geq \max \min_{\mathcal{S} \subseteq [1:N]} I(X; \hat{Y}(\mathcal{S}^c)|H) + I(X(\mathcal{S}); Y|X(\mathcal{S}^c), G) - I(Y(\mathcal{S}); \hat{Y}(\mathcal{S})|X, X^N, H).$$

Then we choose the input distributions as $X \sim \mathcal{N}_{\mathbb{C}}(0, P)$, $X_k \sim \mathcal{N}_{\mathbb{C}}(0, P_r/N)$, and $\hat{Y}_k = Y_k + \hat{Z}_k$ where $\hat{Z}_k \sim \mathcal{N}_{\mathbb{C}}(0, \eta_k(h_k))$ for $k \in [1:N]$. ∎

Although Theorem 1 provides a general achievable rate which can be optimized over all possible adaptation functions $\eta_k(h_k)$, $k \in [1:N]$ on the compression accuracy, the optimization process itself is intractable for most cases. We propose the opportunistic noisy network coding scheme which makes use of a *threshold*-based adaptation function $\eta_k(h_k)$. As will be shown later, while having a simple structure, opportunistic noisy network has many desirable properties.

To define the threshold-based adaptation, let $\alpha_k \in (0,1]$ and $\gamma_k \geq 0$ such that $\mathsf{P}\{|h_k|^2 \geq \gamma_k\} = \alpha_k$, i.e., $\gamma_k = \sigma_{h_k}^2 \ln(1/\alpha_k)$. Then we define $\eta_k(h_k)$ as

$$\eta_k(h_k) = \begin{cases} Q_k & \text{if } |h_k|^2 \geq \gamma_k, \\ \infty & \text{otherwise} \end{cases}$$

where $Q_k > 0$. Figure 2 illustrates how the threshold-based adaptation operates in the opportunistic noisy network coding scheme. For relay node k, the collection of outputs with channel gains above γ_k is compressed to $\hat{y}_k^m(l_k)$, where $m \leq n$ is the number of symbols with $|h_k|^2 \geq \gamma_k$. The compression index l_k is then sent by independently generated Gaussian codes, i.e., $x_k^n(l_k)$. As a result, the outputs with high channel gains are opportunistically compressed and forwarded to the destination.

By fixing the compression noise level as the same as the channel noise variance, i.e., $Q_k = 1$ and $\alpha_k = 1$ (equivalently $\eta_k(h_k) = 1$), we provide the following performance guarantee for any channel parameters and power constraints.

Theorem 2: For the fading parallel relay network, opportunistic noisy network coding is within N bits/s/Hz of the capacity, independent of σ_{h_k}, σ_{g_k}, P, and P_r.

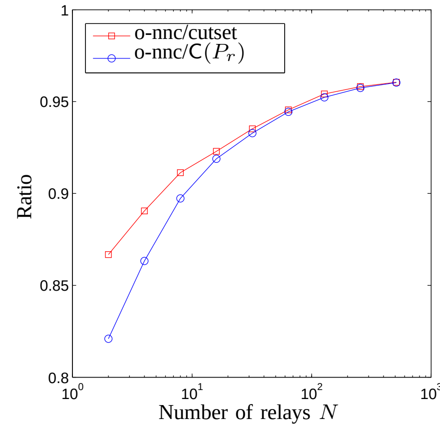

Fig. 3. Ratios between the achievable rate of opportunistic noisy network coding vs. $\mathsf{C}(P_r)$ and the cutset upper bound when $P = P_r = 20$ dB.

Proof: By substituting $\eta_k(h_k) = 1$ in (3),

$$C \geq \min_{\mathcal{S} \subseteq [1:N]} \mathsf{E}\left[\mathsf{C}\left(\sum_{k \in \mathcal{S}^c} |h_k|^2 \frac{P}{2} \right) + \mathsf{C}\left(\sum_{k \in \mathcal{S}} |g_k|^2 \frac{P_r}{N} \right) - |\mathcal{S}|\, \mathsf{C}(1) \right]$$

where $|\mathcal{S}|$ denotes the cardinality of $\mathcal{S}$. Then the rate gap from the cutset bound (2) can be shown within N bits/s/Hz. ∎

The above result extends the capacity gap result of [14] for Gaussian (non-fading) networks to fading parallel relay networks. This type of performance guarantee has many appealing features, for example, it implies that noisy network coding has optimal multiplexing gain. However, the capacity gap result does not say much when the number of relays is large.

The next theorem states that the proposed opportunistic noisy network coding scheme can achieve the capacity as the number of relays becomes large. The proof of Theorem 3 is provided in the next subsection.

Theorem 3: Consider the fading symmetric parallel relay network in which $\sigma_{h_k}^2 = \sigma_{g_k}^2 = 1$. Then $\lim_{N \to \infty} C_N = \mathsf{C}(P_r)$ for any P and P_r.

Figure 3 plots the opportunistic noisy network coding lower bound divided by $\mathsf{C}(P_r)$ and the cutset upper bound (2), respectively. As shown in the figure, the ratios converge to one as N increases.

C. Proof of Theorem 3

Consider the symmetric case where $\sigma_{h_k}^2 = \sigma_{g_k}^2 = 1$. From the cutset upper bound,

$$C_N \leq \mathsf{E}\left[\mathsf{C}\left(1 + \sum_{k=1}^{N} \frac{|g_k|^2 P_r}{N} \right) \right] \leq \mathsf{C}(P_r) \qquad (4)$$

where the second inequality holds from Jensen's inequality. Since (4) holds for any N, $\lim_{N \to \infty} C_N \leq \mathsf{C}(P_r)$.

Now consider the achievable rate of the opportunistic noisy network coding scheme. By symmetry, we set $\alpha_k = \alpha$

(equivalently $\gamma_k = \gamma$) and $Q_k = Q$ for all $k \in [1:N]$. Then the original 2^N rate constraints in Theorem 1 simplify to $N+1$ rate constraints by noticing the fact that the rate constraints corresponding to $\mathcal{S}$ are the same for all $\mathcal{S}$ having the same cardinality. Define $\tilde{h}_k$ whose probability density function is given by

$$f_{|\tilde{h}_k|^2}(x) = \begin{cases} f_{|h_k|^2}(x)/\alpha & \text{if } |h_k|^2 \ge \gamma, \\ 0 & \text{otherwise} \end{cases}$$

where $f_{|h_k|^2}(x) = e^{-x}$. Then, after some manipulation, we can show that

$$C_N \ge \max_{\alpha\in(0,1],Q>0} \min_{i\in[0:N]} R_{\text{o-nnc}}(i) \tag{5}$$

where

$$R_{\text{o-nnc}}(i) = \sum_{j=0}^{i-1} \binom{i}{j} \alpha^{i-j}(1-\alpha)^j \, \mathsf{E}\left[\mathsf{C}\left(\sum_{k=1}^{i-j} \frac{|\tilde{h}_k|^2 P}{1+Q}\right)\right] + \mathsf{E}\left[\mathsf{C}\left(\sum_{k=i+1}^{N} \frac{|g_k|^2 P_r}{N}\right)\right] - \alpha(N-i)\,\mathsf{C}\left(\frac{1}{Q}\right).$$

Next, we lower bound $R_{\text{o-nnc}}(i)$ as follows. For $i \in [1:N]$, $R_{\text{o-nnc}}(i)$ is lower bounded by

$$\begin{aligned} &\sum_{j=0}^{i-1} \binom{i}{j} \alpha^{i-j}(1-\alpha)^j \, \mathsf{C}\left(\frac{\gamma P}{1+Q}\right) - \alpha N\,\mathsf{C}\left(\frac{1}{Q}\right) \\ &= \left(1-(1-\alpha)^i\right) \mathsf{C}\left(\frac{\gamma P}{1+Q}\right) - \alpha N\,\mathsf{C}\left(\frac{1}{Q}\right) \end{aligned} \tag{6}$$

where we use $\mathsf{E}\left[\mathsf{C}\left(\sum_{k=1}^{i-j} \frac{|\tilde{h}_k|^2 P}{1+Q}\right)\right] \ge \mathsf{E}\left[\mathsf{C}\left(\frac{|\tilde{h}_1|^2 P}{1+Q}\right)\right]$ for $j \in [0:i-1]$ and $|\tilde{h}_1|^2 \ge \gamma$, and the equality holds since $\sum_{j=0}^{i-1}\binom{i}{j}\alpha^{i-j}(1-\alpha)^j = 1$. For $i \in [0:N]$, $R_{\text{o-nnc}}(i)$ is lower bounded by

$$\mathsf{E}\left[\mathsf{C}\left(\sum_{k=i+1}^{N} \frac{|g_k|^2 P_r}{N}\right)\right] - \alpha N\,\mathsf{C}\left(\frac{1}{Q}\right). \tag{7}$$

From (6) and (7), we show that $\lim_{N\to\infty} R_{\text{o-nnc}}(i) = \mathsf{C}(P_r)$ for all $i \in [0:N]$. To do this, we set $\alpha = \frac{\log\log(N)}{N}$ and $Q = \frac{P}{P_r}\ln(N)$. Then γ is given by $\ln(N/\log\log(N))$. First, consider the case where $i \in \left[\lceil N/\sqrt{\log\log(N)}\rceil, N\right]$. From (6), we have (8), where the first inequality holds since (6) is minimized when $i = \lceil N/\sqrt{\log\log(N)}\rceil$ and the equaility holds since $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x = \frac{1}{e}$. Next, consider the case where $i \in \left[0 : \lfloor N/\sqrt{\log\log(N)}\rfloor\right]$. From (7), we have (9), where we use

$$\lim_{N\to\infty} \frac{1}{N - \left\lfloor \frac{N}{\sqrt{\log\log(N)}} \right\rfloor} \sum_{k=\left\lfloor \frac{N}{\sqrt{\log\log(N)}} \right\rfloor+1}^{N} |g_k|^2 = 1. \tag{10}$$

Hence, from (8) and (9), $\lim_{N\to\infty} C_N \ge \mathsf{C}(P_r)$, which concludes the proof.

IV. Comparison

In this section, we compare the opportunistic noisy network coding scheme with AF and DF.

A. Amplify–Forward and Decode–Forward Relaying

Notice that a similar threshold-based adaptation used in Section III can also be applied to AF relaying. Specifically, relay node $k \in [1:N]$ sends $X_k = \zeta_k(h_k)Y_k$, where

$$\zeta_k(h_k) = \begin{cases} \sqrt{\frac{P_r/(\alpha_k N)}{|h_k|^2 P+1}} & \text{if } |h_k|^2 \ge \gamma_k, \\ 0 & \text{otherwise,} \end{cases}$$

which satisfies the power constraint. Then the opportunistic AF scheme results in the following lower bound

$$C \ge \max \mathsf{E}\left[\mathsf{C}\left(\frac{\left|\sum_{k=1}^N g_k h_k \zeta_k(h_k)\right|^2 P}{\sum_{k=1}^N |g_k|^2 \zeta_k^2(h_k) + 1}\right)\right] \tag{11}$$

where the maximization is taken over all $\alpha_k \in (0,1]$, $k \in [1:N]$. For the symmetric case, (11) is given by

$$C \ge \max_{\alpha\in(0,1]} \sum_{j=0}^{N-1} \binom{N}{j} \alpha^{N-j}(1-\alpha)^j \cdot \mathsf{E}\left[\mathsf{C}\left(\frac{\left|\sum_{k=1}^{N-j} g_k \tilde{h}_k \sqrt{\frac{P_r/(\alpha N)}{|\tilde{h}_k|^2 P+1}}\right|^2 P}{\sum_{k=1}^{N-j} \frac{|g_k|^2 P_r/(\alpha N)}{|\tilde{h}_k|^2 P+1} + 1}\right)\right]. \tag{12}$$

Note that the CSIR dependent adaptations used in noisy network coding and AF cannot be done for DF relaying due to the inherent difference between the schemes. Hence, the overall rate is limited by the minimum of the point-to-point

$$\begin{aligned} \lim_{N\to\infty} R_{\text{o-nnc}}(i) &\ge \lim_{N\to\infty}\left(1-\left(1-\frac{\log\log(N)}{N}\right)^{\left\lceil \frac{N}{\sqrt{\log\log(N)}} \right\rceil}\right) \mathsf{C}\left(\frac{\ln(N/\log\log(N))P}{1+\frac{P}{P_r}\ln(N)}\right) - \lim_{N\to\infty} \log(e)\frac{P_r}{P}\frac{\log\log(N)}{\ln(N)} \\ &\ge \lim_{N\to\infty}\left(1-\left(1-\frac{\log\log(N)}{N}\right)^{\frac{N}{\log\log(N)}\frac{\log\log(N)}{\sqrt{\log\log(N)}}}\right)\mathsf{C}(P_r) = \mathsf{C}(P_r). \end{aligned} \tag{8}$$

$$\lim_{N\to\infty} R_{\text{o-nnc}}(i) \ge \lim_{N\to\infty} \mathsf{E}\left[\mathsf{C}\left(\sum_{k=\left\lfloor N/\sqrt{\log\log(N)}\right\rfloor+1}^{N} \frac{|g_k|^2 P_r}{N}\right)\right] \ge \lim_{N\to\infty} \mathsf{C}\left(\frac{N - N/\sqrt{\log\log(N)}}{N} P_r\right) = \mathsf{C}(P_r). \tag{9}$$

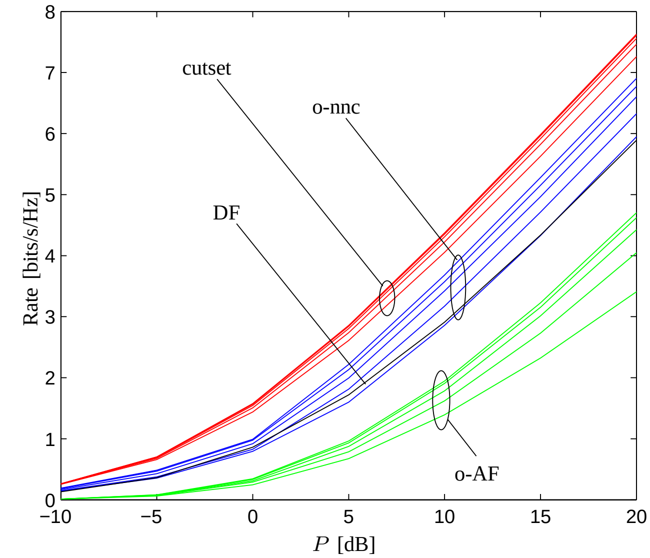

Fig. 4. Achievable rates for the symmetric case when $P_r = 2P$ for $N = 2, 4, 8, 16, 32$.

capacities between the source and each of the N relays which gives

$$C \geq \min\left\{\min_{k\in[1:N]} \mathsf{E}[\mathsf{C}(|h_k|^2 P)], \mathsf{E}\left[\mathsf{C}\left(\sum_{k=1}^{N}|g_k|^2\frac{P_r}{N}\right)\right]\right\}.$$

For the symmetric case, the above is simplified to

$$C \geq \min\left\{\mathsf{E}[\mathsf{C}(|h_1|^2 P)], \mathsf{E}\left[\mathsf{C}\left(\sum_{k=1}^{N}|g_k|^2\frac{P_r}{N}\right)\right]\right\}. \quad (13)$$

B. Rate Comparison

Figure 4 plots the achievable rates of the proposed scheme, AF, and DF for the fading symmetric parallel relay network which are given by (5), (11), and (13), respectively. As shown in the figure, opportunistic noisy network coding outperforms the other schemes in most cases, and the rate gap from the cutset upper bound converges to zero as the number of relays increases for any P and P_r. On the other hand, AF and DF cannot achieve the capacity even if $N \to \infty$. Due to the lack of CSIT at the relays, AF relaying cannot transmit coherently and, as a result, it can be shown that the right hand side of (11) is upper bounded by $\mathsf{E}[\mathsf{C}(|g_1|^2 P_r)]$ (approximately $\mathsf{C}(P_r) - 0.83$ at high SNR). Similarly, the right hand side of (13) is upper bounded by $\min\left\{\mathsf{E}[\mathsf{C}(|h_1|^2 P)], \mathsf{C}(P_r)\right\}$, which is again $\mathsf{E}[\mathsf{C}(|g_1|^2 P_r)]$ if $P = P_r$.

V. Conclusion

In this paper, we proposed the opportunistic noisy network coding scheme for fading parallel relay networks. We showed that the proposed scheme achieves the capacity within N bits/s/Hz for the general case. In the symmetric case, our scheme achieves the capacity in the limit of large number of relays. The optimal strategy is to compress fewer but better observations with higher channel gains as the number of relay increases. The framework presented in this paper can be widely adapted to current wireless network architectures since most systems basically measure CSI at the receiver sides.

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