Opportunistic Interference Alignment for MIMO IMAC: Effect of User Scaling Over Degrees-of-Freedom

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***Abstract*—We consider a new opportunistic interference alignment (OIA) for the K-cell multiple-input multiple-output (MIMO) interfering multiple-access channel (IMAC) with time-invariant channel coefficients, where each cell consists of a base station (BS) with M antennas and N mobile stations (MSs) having L antennas each. In this paper, we propose three OIA techniques: antenna selection-based OIA, singular value decomposition (SVD)-based OIA, and vector-quantized (codebook-based) OIA. Then, their performance is analyzed in terms of user scaling law required to achieve KS degrees-of-freedom (DoF), where $S(\leq M)$ denotes the number of simultaneously transmitting MSs per cell. As our main result, it is shown that the antenna selection-based OIA does not fundamentally change the user scaling required to achieve KS DoF if L is fixed, compared with the single-input multiple-output (SIMO) IMAC case. In contrast, it is shown that the SVD-based OIA can greatly reduce the required user scaling to SNR$^{(K-1)S-L+1}$ through optimizing weight vectors at each MS. Furthermore, we show that the vector-quantized OIA can achieve the same user scaling as the SVD-based OIA case if the codebook size is beyond a certain value. For the vector-quantized OIA, we analyze a fundamental trade-off between the quantization level (i.e., codebook size) and the required user scaling.**

I. Introduction

Interference management is a crucial problem in wireless communications. Interference alignment (IA) [1]–[7] has emerged as a fundamental solution to achieve the optimal degrees-of-freedoms (DoF) in interference channels (ICs). It was shown that the DoF of IA schemes strictly surpass what is achievable on the interference, multiple-access, and broadcast components individually, for various scenarios: two-user X network [2], [3], multiuser IC [1], multiuser multiple-input multiple-output (MIMO) IC [7], and cellular IC [5], [6]. Recently, the concept of opportunistic interference alignment (OIA) was proposed in [8], [9] for the K-cell single-input multiple-output (SIMO) interfering multiple-access channel (IMAC) with time-invariant channel coefficients, where each base station (BS) has M antennas. The basic idea of OIA lies in the utilization of multiuser diversity via opportunistic user scheduling for implementing the IA principle. Unlike the case of the conventional IA schemes, the OIA scheme basically operates with local channel state information at transmitters (CSIT) and no time/frequency domain expansion. Furthermore, no iteration is needed prior to data transmission. The OIA scheme, thus, operates as a decentralized manner which does not involve joint processing among all communication links. It was shown in [9] that the OIA scheme achieves KS DoF if N scales faster than SNR$^{(K-1)S}$ in a high signal-to-noise ratio (SNR) regime, where $S(\leq M)$ is the number of selected mobile stations (MSs) in each cell.

In this paper, we introduce OIA for the MIMO IMAC, where each cell consists of a BS with M antennas and N MSs having L antennas each. In [10], the outer bound on the DoF in MIMO IMAC with time-invariant channel coefficients is characterized, and necessary conditions for M and L needed to achieve the optimal DoF are derived. However, the main goal of the proposed MIMO OIA is to study the required user scaling needed to achieve the target DoF of KS, which is optimal if $S = M$, for arbitrary M and L. More specifically, we propose the following three types of OIA: antenna selection-based OIA, singular value decomposition (SVD)-based OIA, and vector-quantized (codebook-based) OIA.

As our main result, we analyze the scaling condition between the required number of MSs per cell, N, and the received SNR under which KS DoF can be achieved. We show that for the antenna selection-based OIA, the required number of MSs, N, scales as L^{-1}SNR$^{(K-1)S}$. Thus, the required user scaling does not fundamentally change, compared with the SIMO IMAC case [9], if L is a constant independent of N. For the SVD-based OIA, each MS finds the weight vector that minimizes the *leakage of interference (LIF)* using the SVD method. We show that the SVD-based OIA can greatly reduce the required user scaling to SNR$^{(K-1)S-L+1}$. Note that since the local CSIT is assumed, information of weight vectors at each selected MS should be sent to the corresponding BS. Inspired by this fact, we further propose a vector-quantized OIA in which the weight vectors are chosen in a predefined codebook. It is shown that if the quantization level (i.e., codebook size) scales polynomially with respect to SNR, then the required user scaling can vary from SNR$^{(K-1)S}$ to SNR$^{(K-1)S-L+1}$ accordingly. This implies that there exists

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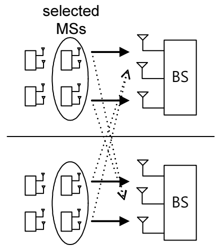

Fig. 1. MIMO IMAC with $K = 2$, $M = 3$, $S = 2$, and $L = 2$.

a fundamental trade-off between the quantization level and the user scaling while achieving KS DoF.

Notations: $\mathbb{C}$ indicates the field of complex numbers. The function $f(x)$ defined by $f(x) = \omega(g(x))$ implies that $\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the conjugate transpose, respectively.

II. System and Channel Models

Let us consider the K-cell MIMO IMAC, each cell of which consists of a BS with M antennas and N MSs, each with L antennas. The number of MSs selected to transmit uplink signals in each cell is denoted by $S \le M$. The case where $K = 2$, $M = 3$, $S = 2$, and $L = 2$ is depicted in Fig. 1.

It is assumed that each selected MS transmits a single spatial stream. To consider nontrivial cases, we assume that $L < (K-1)S+1$, because all the inter-cell interference can be canceled at the MSs otherwise. The channel matrix from the j-th MS in the i-th cell to the k-th BS (in the k-th cell) is denoted by $\mathbf{H}_k^{[i,j]} \in \mathbb{C}^{M\times L}$. Frequency-flat fading is assumed and channel reciprocity between uplink and downlink channels is assumed.

Without loss of generality, the indices of the selected MSs in every cell are assumed to be $(1, \ldots, S)$. The overall DoF is defined by

$$\text{DoF} = \lim_{\text{SNR}\to\infty} \frac{\sum_{i=1}^{K}\sum_{j=1}^{S} R^{[i,j]}}{\log \text{SNR}}, \tag{1}$$

where $R^{[i,j]}$ denotes the achievable rate for the j-th MS in the i-th cell.

III. Achievability Results

For completeness, we briefly review the achievability result of OIA for SIMO IMAC. Then, we propose three OIA techniques for MIMO IMAC and analyze their performance in terms of DoF and the associated user scaling. In addition, our schemes are compared with an ideal scenario where there are no inter-cell interferences, resulting in an upper bound on the performance.

A. Review on OIA in SIMO IMAC

In SIMO IMAC, the channel matrix $\mathbf{H}_k^{[i,j]}$ becomes an $M \times 1$ vector. For consistency with literature, let us denote this channel vector by $\mathbf{h}_k^{[i,j]} \in \mathbb{C}^{M\times 1}$. From the pilot signals from BSs (including home cell BS and neighboring cell BSs), each MS can estimate the channel vectors $\mathbf{h}_k^{[i,j]}$, $k = 1, \ldots, K$, utilizing the channel reciprocity. As in [8], [9], it is assumed that the interference space of the k-th BS, denoted by $\mathbf{Q}_k = [\mathbf{q}_{k,1}, \ldots, \mathbf{q}_{k,M-S}]$, is determined offline and is known to all the BSs and MSs, where $\mathbf{q}_{k,i} \in \mathbb{C}^{M\times 1}$ is an orthonormal basis vector.

In the OIA in SIMO IMAC, each BS opportunistically selects a set of MSs who generate the minimum interference to the other BSs. For computing its generating interference to other BSs, the j-th MS in the i-th cell calculates the *leakage of interference (LIF)* [8], [9], which is expressed as:

$$\eta_{\text{SIMO}}^{[i,j]} = \sum_{k=1,k\neq i}^{K} \left\| \text{Proj}_{\perp \mathbf{Q}_k}\left(\mathbf{h}_k^{[i,j]}\right)\right\|^2, \tag{2}$$

where $\text{Proj}_{\perp \mathbf{A}}$ denotes the orthogonal projection of the basis $\mathbf{A}$. Each MS reports this metric to the associated BS, and each BS selects S MSs with smallest LIF metrics among N MSs.

The following theorem is the main result of OIA for the required user scaling.

Theorem 1 (Theorem 1, [9]): The OIA scheme with the scheduling metric given in (2) and the zero-forcing (ZF) filter at each BS achieves

$$\text{DoF} \ge KS \tag{3}$$

with high probability (whp) if

$$N = \omega\left(\text{SNR}^{(K-1)S}\right). \tag{4}$$

B. OIA for MIMO IMAC

In this subsection, the overall procedure of the proposed OIA techniques for MIMO IMAC is presented.

The interference space of the k-th BS is denoted by $\mathbf{Q}_k$ as in Section III-A. Let us denote the null space of $\mathbf{Q}_k$ as

$$\mathbf{U}_k = [\mathbf{u}_{k,1}, \ldots, \mathbf{u}_{k,S}] \triangleq \text{null}(\mathbf{Q}_k), \tag{5}$$

where $\mathbf{u}_{k,i} \in \mathbb{C}^{M\times 1}$ is orthonormal. A simple way to determine $\mathbf{Q}_k$ and $\mathbf{U}_k$ would be choosing $M - S$ columns of a left or right singular matrix of any $M \times M$ matrix as $\mathbf{Q}_k$ and choosing the rest of the S columns as $\mathbf{U}_k$.

Let us define $\mathbf{w}^{[i,j]}$ as the weight vector at the j-th MS in the i-th cell. Then, the LIF metric is defined by

$$\eta_{\text{MIMO}}^{[i,j]} = \sum_{k=1,k\neq i}^{K} \left\| \text{Proj}_{\perp \mathbf{Q}_k}\left(\mathbf{H}_k^{[i,j]}\mathbf{w}^{[i,j]}\right)\right\|^2 \tag{6}$$

$$= \sum_{k=1,k\neq i}^{K} \left\| \mathbf{U}_k^H \mathbf{H}_k^{[i,j]}\mathbf{w}^{[i,j]}\right\|^2. \tag{7}$$

All the MSs report their LIF metrics to corresponding BSs. Then, each BS selects S MSs having smallest LIF metrics among N MSs in the cell. Subsequently, the j-th MS in the i-th cell forwards the information on $\mathbf{w}^{[i,j]}$ to the i-th BS for decoding.

The received signal at the i-th BS is expressed as:

$$\mathbf{y}_i = \underbrace{\sum_{j=1}^{S} \mathbf{H}_i^{[i,j]}\mathbf{w}^{[i,j]}x^{[i,j]}}_{\text{desired signal}} + \underbrace{\sum_{k=1,k\neq i}^{K}\sum_{m=1}^{S} \mathbf{H}_i^{[k,m]}\mathbf{w}^{[k,m]}x^{[k,m]}}_{\text{inter-cell interference}} + \mathbf{z}_i, \tag{8}$$

where $x^{[i,j]}$ is the transmit symbol with unit power, and $\mathbf{z}_i \in \mathbb{C}^{M\times 1}$ denotes additive noise, each element of which is independent and identically distributed complex Gaussian with zero mean and the variance of SNR^{-1}. As in SIMO IMAC [8], [9], the linear ZF detection is applied as

$$\mathbf{r}_i = [r_{i,1}, \ldots, r_{i,S}]^{\mathrm{T}} \triangleq \mathbf{F}_i{}^H \mathbf{U}_i^H \mathbf{y}_i, \tag{9}$$

$$\begin{aligned}\mathbf{F}_i &= [\mathbf{f}_{i,1}, \ldots, \mathbf{f}_{i,S}] \\ &\triangleq \left(\left[\mathbf{U}_i{}^H \mathbf{H}_i^{[i,1]} \mathbf{w}^{[i,1]}, \ldots, \mathbf{U}_i{}^H \mathbf{H}_i^{[i,S]} \mathbf{w}^{[i,S]} \right]^{-1} \right)^H.\end{aligned} \tag{10}$$

Consequently, the achievable rate of the j-th MS in the i-th cell, $R^{[i,j]}$, is given by (11) at the bottom of the next page.

In the following subsections, we consider three different strategies to determine the weight vector $\mathbf{w}^{[k,m]}$.

1) Antenna Selection-Based OIA: In the antenna selection-based OIA, we have $\mathbf{w}^{[i,j]} \in \{\mathbf{e}_1, \ldots, \mathbf{e}_L\}$, where $\mathbf{e}_l$ denotes the l-th column of the $(L \times L)$-dimensional identity matrix. Let us denote the l-th column of $\mathbf{H}_k^{[i,j]}$ by $\mathbf{h}_{k,l}^{[i,j]}$, $l = 1, \ldots, L$. Then, the j-th MS in the i-th cell chooses the optimal weight vector as $\mathbf{w}_{\text{AS}}^{[i,j]} = \mathbf{e}_{\hat{l}(i,j)}$, where the index $\hat{l}(i,j)$ is obtained from

$$\hat{l}(i,j) = \arg \min_{1\le l\le L} \sum_{k=1,k\neq i}^{K} \left\| \mathbf{U}_k{}^H \mathbf{h}_{k,l}^{[i,j]} \right\|^2. \tag{12}$$

Then, the corresponding LIF metric is given by

$$\eta_{\text{AS}}^{[i,j]} = \sum_{k=1,k\neq i}^{K} \left\| \mathbf{U}_k{}^H \mathbf{h}_{k,\hat{l}(i,j)}^{[i,j]} \right\|^2 \tag{13}$$

and is reported to the associated BS.

The following theorem establishes the DoF achievability of the antenna selection-based OIA.

Theorem 2: The antenna selection-based OIA with the scheduling metric (13) achieves

$$\text{DoF} \ge KS \tag{14}$$

whp if

$$N = \omega\left(L^{-1}\text{SNR}^{(K-1)S} \right). \tag{15}$$

Proof: For compactness, we only provide a brief sketch of the proof. The main difference in the proof of *Theorem 2* compared to the proof of [9, Theorem 1] is that the LIF metric of each selected MS, given by (13), can be represented as the minimum of L independent LIF metrics in SIMO IMAC. That is, $\eta_{\text{AS}}^{[i,j]}$ is the minimum of L independent Chi-square random variables with degrees of freedom $2(K-1)S$. Following the footsteps of the proof of [9, Theorem 1] with this change leads to the N scaling result given in (15). The detailed proof is included in [11]. ■

Thus, the antenna selection-based OIA does not fundamentally change the user scaling if L is fixed. Note that, however, the required user scaling is reduced if L scales polynomially with respect to SNR.

2) SVD-Based OIA: In the SVD-based OIA, each MS finds the optimal weight vector that minimizes its LIF metric. Let us rewrite the LIF metric for MIMO IMAC as

$$\eta_{\text{SVD}}^{[i,j]} = \sum_{k=1,k\neq i}^{K} \left\| \mathbf{U}_k{}^H \mathbf{H}_k^{[i,j]} \mathbf{w}^{[i,j]} \right\|^2 = \left\| \mathbf{G}^{[i,j]} \mathbf{w}^{[i,j]} \right\|^2, \tag{16}$$

$$\begin{aligned}\mathbf{G}^{[i,j]} \triangleq \Bigg[&\left(\mathbf{U}_1{}^H \mathbf{H}_1^{[i,j]} \right)^T, \ldots, \left(\mathbf{U}_{i-1}{}^H \mathbf{H}_{i-1}^{[i,j]} \right)^T, \\ &\left(\mathbf{U}_{i+1}{}^H \mathbf{H}_{i+1}^{[i,j]} \right)^T, \ldots, \left(\mathbf{U}_K{}^H \mathbf{H}_K^{[i,j]} \right)^T \Bigg]^T.\end{aligned} \tag{17}$$

Let us further denote the SVD of $\mathbf{G}^{[i,j]}$ as

$$\mathbf{G}^{[i,j]} = \mathbf{\Omega}^{[i,j]} \mathbf{\Sigma}^{[i,j]} \mathbf{V}^{[i,j]H}, \tag{18}$$

where $\mathbf{\Omega}^{[i,j]} \in \mathbb{C}^{(K-1)S\times L}$ and $\mathbf{V}^{[i,j]} \in \mathbb{C}^{L\times L}$ consist of orthonormal columns, and $\mathbf{\Sigma}^{[i,j]} = \text{diag}\left(\sigma_1^{[i,j]}, \ldots, \sigma_L^{[i,j]}\right)$, where $\sigma_1^{[i,j]} \ge \cdots \ge \sigma_L^{[i,j]}$. Then, it is apparent that the optimal $\mathbf{w}^{[i,j]}$ is determined as

$$\mathbf{w}_{\text{SVD}}^{[i,j]} = \mathbf{v}_L^{[i,j]}, \tag{19}$$

where $\mathbf{v}_L^{[i,j]}$ is the L-th column of $\mathbf{V}^{[i,j]}$. With this choice the LIF metric is simplified to

$$\eta_{\text{SVD}}^{[i,j]} = \sigma_L^{[i,j]2}. \tag{20}$$

Theorem 3: The proposed SVD-based OIA scheme with the scheduling metric (20) achieves

$$DoF \ge KS \tag{21}$$

whp if

$$N = \omega\left(\text{SNR}^{(K-1)S-L+1} \right). \tag{22}$$

Proof: The SINR for the j-th MS in the i-th cell, defined in (11), can be expressed as:

$$\text{SINR}^{[i,j]} \ge \frac{\text{SNR}/\left\|\mathbf{f}_{i,j}\right\|^2}{1 + \sum_{k=1,k\neq i}^{K} \sum_{m=1}^{S} \left\| \mathbf{U}_i^H \mathbf{H}_i^{[k,m]} \mathbf{w}_{\text{SVD}}^{[k,m]} \right\|^2 \text{SNR}}. \tag{23}$$

Thus, it is apparent that the DoF of KS is achieved if the interference term $\sum_{k=1,k\neq i}^{K} \sum_{m=1}^{S} \left\| \mathbf{U}_i{}^H \mathbf{H}_i^{[k,m]} \mathbf{w}_{\text{SVD}}^{[k,m]} \right\|^2 \text{SNR}$ remains smaller than ϵ for increasing SNR, where $\epsilon > 0$ is an arbitrary positive constant. Let us define $\mathcal{P}_{\text{SVD}}$ as

$$\begin{aligned}\mathcal{P}_{\text{SVD}} \triangleq \lim_{\text{SNR}\to\infty} \Pr\Bigg\{ &\sum_{k=1,k\neq i}^{K} \sum_{m=1}^{S} \left\| \mathbf{U}_i{}^H \mathbf{H}_i^{[k,m]} \mathbf{w}_{\text{SVD}}^{[k,m]} \right\|^2 \text{SNR} \le \epsilon, \\ &\forall \text{ MS } j, j = 1, \ldots, S, \forall \text{ BS } i, i = 1 \ldots, K \Bigg\}.\end{aligned} \tag{24}$$

Then, the DoF is bounded as

$$\text{DoF} \ge KS \cdot \mathcal{P}_{\text{SVD}}. \tag{25}$$

When calculating this lower bound, we assumed that the DoF of KS is achieved if the interference remains smaller than ϵ for increasing SNR, and that zero DoF is achieved otherwise. It is important to note that the sum of the received interference terms that appears in (24) is equivalent to the sum of the LIF metrics of the selected MSs. That is,

$$\sum_{i=1}^{K}\sum_{k=1,k\neq i}^{K}\sum_{m=1}^{S}\left\|\mathbf{U}_i{}^H\mathbf{H}_i^{[k,m]}\mathbf{w}_{\text{SVD}}^{[k,m]}\right\|^2=\sum_{i=1}^{K}\sum_{j=1}^{S}\eta_{\text{SVD}}^{[i,j]}. \tag{26}$$

Subsequently, we exploit the fact that the LIF metric given in (20) is the smallest eigen-value of the complex central Wishart matrix $\mathbf{G}^{[i,j]^H}\mathbf{G}^{[i,j]}$, i.e., $\mathbf{G}^{[i,j]^H}\mathbf{G}^{[i,j]}\sim\mathcal{CW}_{(K-1)S}(L,\mathbf{I})$ [12]. The result for the polynomial probability density function of this smallest eigenvalue derived in [13, Theorem 4] is also used. Here, we need to choose ϵ small enough such that $\frac{\epsilon\text{SNR}^{-1}}{KS^2}<1$ for arbitrarily large SNR. The detailed proof is shown in [11]. ■

3) Vector-Quantized OIA: Let us denote the codebook set consisting of N_f elements as

$$\mathcal{C}_f\triangleq\left\{\mathbf{c}_1,\ldots,\mathbf{c}_{N_f}\right\}, \tag{27}$$

where $\mathbf{c}_1,\ldots,\mathbf{c}_{N_f}\in\mathbb{C}^{L\times 1}$ are chosen from the L-dimensional unit sphere. Then, the number of bits to represent $\mathcal{C}_f$ is denoted as

$$n_f=\lceil\log_2 N_f\rceil. \tag{28}$$

Each MS finds the weight vector in the codebook from

$$\mathbf{w}_{\text{VQ}}^{[i,j]}=\arg\min_{1\le n\le N_f}\left\|\mathbf{G}^{[i,j]}\mathbf{c}_n\right\|^2. \tag{29}$$

Noting that $\mathbf{w}_{\text{VQ}}^{[i,j]}$ with an infinitely large codebook becomes $\mathbf{v}_L^{[i,j]}$ in (19), we employ the widely-used suboptimal rule [14]–[16], which is defined as

$$\hat{\mathbf{w}}_{\text{VQ}}^{[i,j]}=\arg\max\nolimits_{1\le n\le N_f}\left|\left(\mathbf{v}_L^{[i,j]}\right)^H\mathbf{c}_n\right|^2. \tag{30}$$

This rule is mathematically much more tractable than the optimal rule in (29), and rapidly approaches to the original rule in terms of the performance as N_f grows [14]–[16].

Theorem 4: The vector-quantized OIA with the weight vector given by (30) and the optimal Grassmannian codebook $\mathcal{C}_f$ [14] achieve whp

$$\text{DoF}\ge KS, \tag{31}$$

if

$$N=\omega\Bigg(\max\Big\{\text{SNR}^{(K-1)S-L+1+\varepsilon_\kappa}, N_f{}^{-(K-1)S/(L-1)}\cdot\text{SNR}^{(K-1)S+\varepsilon'_\kappa}\Big\}\Bigg), \tag{32}$$

where $\epsilon_\kappa,\epsilon'_\kappa>0$ are arbitrarily small constants.

Proof: Using the results on the vector quantization in [14], [16], [17], we show that the upper-bound of the LIF metric can be written by two independent terms as

$$\eta_{\text{VQ}}^{[i,j]}=\left\|\mathbf{G}^{[i,j]}\hat{\mathbf{w}}_{\text{VQ}}^{[i,j]}\right\|^2 \tag{33}$$

$$\le\tilde{\varepsilon}_\kappa^{-1}\sigma_L^{[i,j]^2}+(1+\mu)\nu_f\tilde{\varepsilon}_\kappa^{-1}\left\|\mathbf{G}^{[i,j]}\tilde{\mathbf{u}}\right\|^2, \tag{34}$$

where $\tilde{\varepsilon}_\kappa,\mu>0$ are arbitrarily small constants, and $\tilde{\mathbf{u}}$ becomes an independent random vector isotropically distributed over the L-dimensional unit sphere as $\tilde{\varepsilon}_\kappa\to 0$. Here, ν_f denotes the Gilbert-Vashamov bound on the distance of any two codebook vectors, which is given by [14], [18]

$$\nu_f\triangleq N_f{}^{-1/(L-1)}. \tag{35}$$

Note that the first term of (34) is a function of $\sigma_L^{[i,j]^2}$, and hence, similar derivations in Section III-B2 are used for this term. On the other hand, the second term becomes a Chi-square random variable with degrees of freedom $2(K-1)S$ for arbitrarily small $\tilde{\varepsilon}_\kappa$, and thus, the results in Section III-B1 are also used. The detailed proof is shown in [11]. ■

Thus, a trade-off between N_f and the required user scaling is clearly observed from (32). Specifically, if N_f scales polynomially with respect to SNR, then, the required user scaling varies accordingly.

Corollary 1: From Theorem 4, the loss due to a finite codebook becomes negligible and we only require $N=\omega\left(\text{SNR}^{(K-1)S-L+1}\right)$ to achieve the DoF of KS, if $N_f=\omega\left(\text{SNR}^{(L-1)^2/((K-1)S)}\right)$.

We complete Section III-B with the following remarks.

Remark 1: If $L=1$, then the channel becomes SIMO IMAC, and the user scaling results given in (15), (22), and (32) (for any $N_f>1$) become $N=\omega\left(\text{SNR}^{(K-1)S}\right)$, which is consistent with the result in Theorem 1.

Remark 2: Note that if $L\ge(K-1)S+1$, then $\mathbf{w}_{\text{SVD}}^{[i,j]}$ can be chosen such that $\eta_{\text{SVD}}^{[i,j]}=0$ as seen from (18) to (20).

C. Comparison

For comparison, we now consider an upper limit on the DoF in MIMO IMAC. From a Genie-aided removal of all the inter-cell interferences, we obtain K parallel MAC systems. Under the basic assumption that S MSs in a cell transmit simultaneously, the DoF for each MAC is thus upper-bounded by S, which is the same as the SIMO IMAC case [9]. Hence, this ideal upper bound on the total DoF, given by KS, matches our lower bound that is achieved using the OIA based only on local CSIT at each MS.

$$R^{[i,j]}=\log\left(1+\text{SINR}^{[i,j]}\right)=\log\left(1+\frac{\text{SNR}}{\|\mathbf{f}_{i,j}\|^2+\sum_{k=1,k\neq i}^{K}\sum_{m=1}^{S}\left|\mathbf{f}_{i,j}{}^H\mathbf{U}_i{}^H\mathbf{H}_i^{[k,m]}\mathbf{w}^{[k,m]}\right|^2\text{SNR}}\right). \tag{11}$$

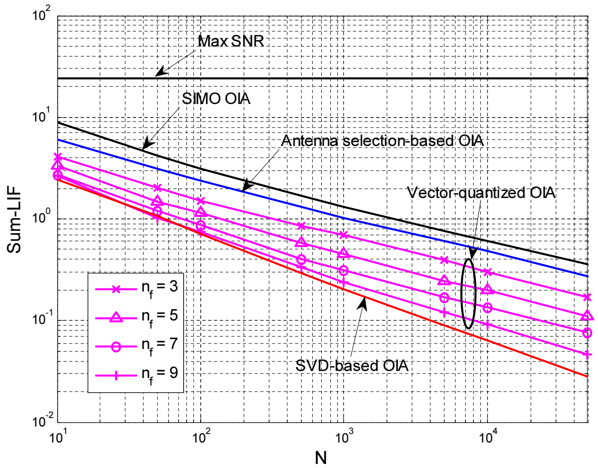

Fig. 2. Sum-LIF vs. N in MIMO IMAC where $K = 2$, $M = 4$, $S = 3$, $L = 2$, and SNR=10dB.

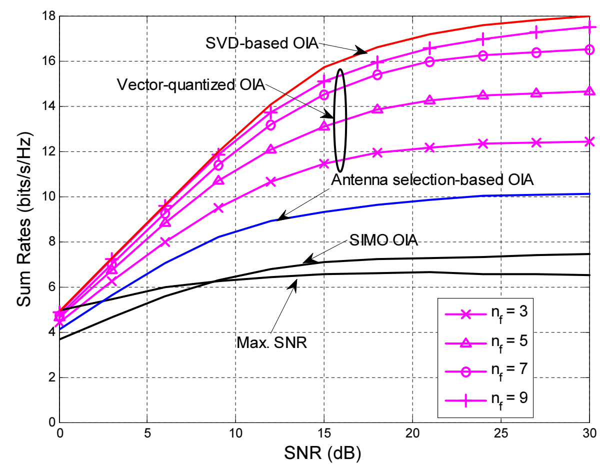

Fig. 3. Sum-rates vs. SNR in MIMO IMAC where $K = 2$, $M = 4$, $S = 3$, $L = 2$, and $N = 100$.

IV. Numerical Examples

For comparison, the max-SNR scheme was considered, in which each MS employs eigen-beamforming to maximize its SNR and the BS selects best S MSs that have higher effective SNRs than the rest. In addition, the OIA scheme employing a fixed weight vector, i.e., $\mathbf{w}^{[i,j]} = \mathbf{e}_1$ for all MSs, is considered, which is equivalent to the OIA scheme for SIMO IMAC. Thus, we refer this scheme as 'SIMO OIA'. For the proposed vector-quantized OIA, the random codebook was assumed, each element of which was independently and uniformly generated in a unit sphere.

Fig. 2 depicts the log-log plot of the sum-LIF, the sum of the LIF metrics of the selected MSs, vs. N when $K = 2$, $L = 2$, $M = 4$, $S = 3$, and SNR is 10dB. This performance measurement enables us to measure the quality of the proposed OIA schemes, as shown in [4]. The sum-LIF of the antenna selection-based OIA as well as that of the vector-quantized OIA with fixed n_f degrease with respect to N at the same rate of the SIMO OIA, because all these schemes are subject to the required user scaling of $\text{SNR}^{(K-1)S}$. On the other hand, the decreasing rate of the SVD-based OIA is higher, which is subject to the required user scaling of $\text{SNR}^{(K-1)S-L+1}$.

Fig. 3 illustrates the sum-rates of the considered schemes when $K = 2$, $M = 4$, $S = 3$, $N = 100$, and $L = 2$. A trade-off between the sum-rate for given N and the quantization level N_f is clearly observed. The sum-rate of the vector-quantized OIA with $n_f = 9$ nearly attains the sum-rate of the SVD-based OIA.

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