A Feasibility Study on Opportunistic Interference Alignment: Limited Feedback and Sum-Rate Enhancement

Hyun Jong Yang[1], Won-Yong Shin[2], Bang Chul Jung[3], and Arogyaswami Paulraj[1]
[1]Dept. of EE, Stanford Univ., Stanford, CA 94305, E-mail: {hjdbell, apaulraj}@stanford.edu
[2]Div. of MSE, Dankook Univ., Yongin, 448-701, Korea, E-mail: wyshin@dankook.ac.kr
[3]Dept. of ICE, Gyeongsang National Univ., Tongyeong 650-160, Korea, E-mail: bcjung@gnu.ac.kr

***Abstract*—In this paper, we illuminate opportunistic interference alignment (OIA) for multi-input multi-output (MIMO) interfering multiple-access channels (IMACs) by tackling the following two practical challenges: i) feedback overhead due to scheduling metrics sent by users and ii) sub-optimality in terms of sum-rate derived from the conventional OIA. To reduce the feedback overhead, we first propose an opportunistic feedback strategy based on interference leakage, which guarantees the total generating interference to other-cell base stations to be smaller than a predefined threshold. In addition, we also consider the channel gain of the desired link for improving the achievable sum-rate for uplink, while the conventional OIA only uses the generating interference to other cells as a scheduling metric. Simulation results show that the *effective* sum-rate of the proposed technique is significantly higher than that of conventional scheduling methods, including the conventional OIA, for all signal-to-noise ratio (SNR) regimes. The effective sum-rate is obtained by considering the effect of feedback overhead on the total throughput of the MIMO IMAC model.**

I. INTRODUCTION

Interference management is one of the most critical issues in wireless communications, and its importance has been increasingly stressed in promising future wireless networks such as femtocell or heterogeneous networks. Over the past decade, interference alignment (IA) [1]–[8] has emerged as a fundamental solution to achieve the optimal degrees-of-freedoms (DoF) in interference channels (ICs). Compared to the three traditional methods handling interference (i.e., decoding and subtracting, treating as noise, or orthogonalizing), IA greatly increases the achievable DoF by aligning the interference received at each receiver to predefined linear spaces. However, it is worth noting that DoF only characterizes the asymptotic slope of the capacity and may not be a suitable performance measure that can be taken into account in practical communication environments. In this paper, we aim to address how to further enhance the achievable sum-rate of the K-cell interfering multiple-access channel (IMAC) [5], i.e., multi-cell uplink network over the use of existing IAs, where multiple users transmit simultaneously to their corresponding base station (BS) in each cell.

The conventional IA schemes used in the IMAC or multiuser IC operate under several infeasible conditions: global channel state information (CSI) at all nodes and large dimension extension of the channel, the size which grows polynomially or exponentially with respect to K [1], [5], [7]. Recently, opportunistic interference alignment (OIA) was introduced in the K-cell single-input multi-output (SIMO) IMAC, where each cell has one BS with M antennas and N single-antenna users [9]–[11]. Under the OIA scheme, interference is aligned to predefined spaces by opportunistically selecting users causing the minimum leakage of interference (LIF). The OIA resolves most of the aforementioned feasibility issues, since it operates with local CSI, no time/frequency dimension extension, and no iterative transceiver optimization. In addition, the calculation of the scheduling metric (i.e., LIF) is carried out at each user in a distributed manner. The main analytical result is the user scaling law that characterizes the trade-off between the achievable DoF and the minimum required number of users. It was shown in [11] that the minimum number of users per-cell, N, needs to scale faster than $\text{SNR}^{(K-1)S}$ to achieve KS DoF, where $S \leq M$ denotes the number of selected users per cell and SNR denotes the received signal-to-noise ratio.

The OIA scheme and its achievability result were extended in [12], [13] for the multi-input multi-output (MIMO) IMAC case, where each user has L antennas. In [12], [13], it was shown that the use of transmit beamforming at each user can significantly reduce the user scaling condition required to achieve the target DoF of KS over the SIMO case.

However, the previous OIA framework has some practical challenges in realistic environments including finite SNR regime and/or finite block length. First, the OIA scheme focused only on the DoF optimality for given N, and thus may not guarantees a sufficiently high sum-rate, where the transmit beamforming and the user selection were performed only in terms of minimizing the generating interference to neighboring cells, denoted by LIF. Second, since the existing OIA framework operates based on the feedback of scheduling metrics for all the users, the feedback overhead greatly increases with increasing number of users, resulting in a reduced effective sum-rate performance for a finite block length.

In this paper, we first propose a novel opportunistic feedback strategy for the MIMO IMAC, in which a threshold-based scheduling is employed by allowing the limited number of users to transmit feedback signals, thereby reducing the feedback overhead. Since the eligibility of the feedback is determined by the level of LIF, the total sum-LIF can be guaranteed to be below the predefined threshold. In addition,

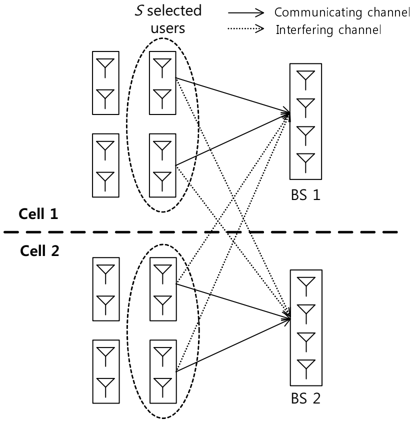

Fig. 1. MIMO IMAC with $K = 2$, $N = 4$, $M = 4$, $S = 2$, and $L = 2$

to enhance the achievable sum-rate, for each user eligible to feed back scheduling metrics, the channel gain of the desired link is also fed back to the corresponding BS. The threshold value for the opportunistic feedback strategy is optimized for given SNR through computer simulations so that the sum-rate is maximized. Moreover, the number of selected users per cell are optimized to further improve the sum-rate performance as in the conventional OIA case. Simulation results show that the effective sum-rate of the proposed scheme is higher than those of conventional distributed scheduling schemes under the MIMO IMAC model with finite block length for all SNR regimes.

The remainder of this paper is organized as follows. Section II defines the system and channel models. The proposed scheme is presented in Section III, and simulation results are provided in Section IV. Finally, Section V concludes the paper.

Notations: $\mathbb{C}$ indicates the field of complex numbers. The function $f(x)$ defined by $f(x) = \omega(g(x))$ implies that $\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the conjugate transpose, respectively. $\mathbf{I}_m$ denotes the $(m \times m)$-dimensional identity matrix.

II. SYSTEM AND CHANNEL MODELS

The K-cell MIMO IMAC with N users per cell is considered. Each BS has M antennas whereas each user is equipped with L antennas, constituting an $(L \times M)$-MIMO channel for each link. In each cell, S $(\leq M)$ users are assumed to be selected to transmit uplink signals. Fig. 1 illustrates the MIMO IMAC with $K = 2$, $N = 4$, $M = 4$, $S = 2$, and $L = 2$. Block fading with the block length T is assumed, namely, the channel coefficients are invariant during the period of T symbols.

The channel from the user j in the cell i to the BS k is denoted by $\mathbf{H}_k^{[i,j]} \in \mathbb{C}^{M\times L}$. Assuming the time division duplex reciprocity, it is assumed that each user accurately estimates the channels of its own links using the pilots transmitted from the BSs; that is, the user j in the cell i has the knowledge of $\mathbf{H}_k^{[i,j]}$, $k = 1, \ldots, K$. In the interference channel, this CSI assumption is referred to as *local CSI* [14].

We assume a rank one transmission at each user, i.e., beamforming is used with a single spatial stream, because it is sufficient to achieve the optimal performance in the MIMO IMAC [12], [13]. The transmit symbol at the user j in the cell i is denoted by $x^{[i,j]}$ and the unit-norm weight vector is denoted by $\mathbf{w}^{[i,j]} \in \mathbb{C}^{L\times 1}$, i.e., $\left\|\mathbf{w}^{[i,j]}\right\|^2 = 1$. The transmit signal vector is given by $\mathbf{w}^{[i,j]}x^{[i,j]}$ satisfying

$$E\left\|\mathbf{w}^{[i,j]}x^{[i,j]}\right\|^2 = \left|x^{[i,j]}\right|^2 \leq P, \tag{1}$$

where P is the power constraint.

At this point, we assume without loss of generality that the indices of the selected users are denoted by $(1, \ldots, S)$ in every cell for notational simplicity. The received signal $\mathbf{y}_i$ at the BS i is written by

$$\begin{aligned}\mathbf{y}_i = &\sum_{j=1}^{S} \mathbf{H}_i^{[i,j]}\mathbf{w}^{[i,j]}x^{[i,j]} \\ &+ \sum_{k=1,k\neq i}^{K}\sum_{m=1}^{S} \mathbf{H}_i^{[k,m]}\mathbf{w}^{[k,m]}x^{[k,m]} + \mathbf{z}_i,\end{aligned} \tag{2}$$

where $\mathbf{z}_i$ represents the additive white Gaussian noise (AWGN), each element of which is identically and independently distributed (i.i.d.) with zero mean and variance of σ_z^2.

III. PROPOSED OIA WITH OPPORTUNISTIC SCHEDULING METRIC FEEDBACK

A. *Initialization: Reference Interference Basis*

The reference interference basis is determined at all BSs independently prior to the communication. Since S $(\leq M)$ users will be selected to transmit uplink signals, $M - S$ dimensions are reserved for the reference interference basis to which the interference signals from neighbouring cells are aligned. Each BS independently and randomly constructs the reference interference basis $\mathbf{Q}_i \in \mathbb{C}^{M\times(M-S)}$. In addition, it also calculates the orthogonal (null) space of $\mathbf{Q}_i$, i.e., the basis for desired signals, as

$$\mathbf{U}_i = \text{null}(\mathbf{Q}_i) \in \mathbb{C}^{M\times S}. \tag{3}$$

An intuitive construction is to select the first $(M-S)$ columns of any $(M \times M)$-dimensional orthonormal random matrix for $\mathbf{Q}_i$ and to select the rest of the S columns for $\mathbf{U}_i$.

Then, all the K BSs broadcast $\mathbf{U}_i$ to users in the network such that every user acquires the information of $\mathbf{U}_i$, $i = 1, \ldots, K$. Note that this calculation and feedback effort are required only once, and are independent of channel variations.

B. *Stage 1: Scheduling Metric Feedback*

Since the the user j in the cell i has only the knowledge of $\mathbf{H}_k^{[i,j]}$, $k = 1, \ldots, K$, it calculates two different metrics as follows:

$$I^{[i,j]} = \sum_{k=1,k\neq i}^{K} \tilde{I}_k^{[i,j]} = \sum_{k=1,k\neq i}^{K} \left\|\mathbf{U}_k^H\mathbf{H}_k^{[i,j]}\mathbf{w}^{[i,j]}\right\|^2 \tag{4}$$

$$\text{SNR}^{[i,j]} = \left\|\mathbf{H}_i^{[i,j]}\mathbf{w}^{[i,j]}\right\|^2. \tag{5}$$

Here, $\tilde{I}_k^{[i,j]}$ accounts for the power of the residual received signal at BS k, which is remained in the signal basis $\mathbf{U}_k$, and $\text{SNR}^{[i,j]}$ represents the gain of the message link.

To minimize the LIF it generates, each user finds the weight vector that minimizes the LIF based on the SVD of the interference matrix, i.e., the weight vector design follows that of the SVD-based OIA [12], [13]. The justification of this approach shall be discussed later in this section. Define the horizontally stacked interference channel matrix $\mathbf{G}^{[i,j]} \in \mathbb{C}^{(M-1)S\times L}$ by

$$\mathbf{G}^{[i,j]} \triangleq \Bigg[\left(\mathbf{U}_1{}^H\mathbf{H}_1^{[i,j]}\right)^T, \ldots, \left(\mathbf{U}_{i-1}{}^H\mathbf{H}_{i-1}^{[i,j]}\right)^T, \left(\mathbf{U}_{i+1}{}^H\mathbf{H}_{i+1}^{[i,j]}\right)^T, \ldots, \left(\mathbf{U}_K{}^H\mathbf{H}_K^{[i,j]}\right)^T \Bigg]^T. \quad (6)$$

Let us further denote the SVD of $\mathbf{G}^{[i,j]}$ as

$$\mathbf{G}^{[i,j]} = \mathbf{\Omega}^{[i,j]}\mathbf{\Sigma}^{[i,j]}\mathbf{V}^{[i,j]H}, \quad (7)$$

where $\mathbf{\Omega}^{[i,j]} \in \mathbb{C}^{(K-1)S\times L}$ and $\mathbf{V}^{[i,j]} \in \mathbb{C}^{L\times L}$ consist of orthonormal columns, and $\mathbf{\Sigma}^{[i,j]} = \text{diag}\left(\sigma_1^{[i,j]}, \ldots, \sigma_L^{[i,j]}\right)$, where $\sigma_1^{[i,j]} \geq \cdots \geq \sigma_L^{[i,j]}$. Then, the solution that minimizes $I^{[i,j]}$ is given by

$$\mathbf{w}^{[i,j]} = \arg\min_{\mathbf{v}} I^{[i,j]} = \mathbf{v}_L^{[i,j]}, \quad (8)$$

where $\mathbf{v}_L^{[i,j]}$ is the L-th column of $\mathbf{V}^{[i,j]}$.

At this point, to control the overall sum-interference level, we define the threshold for LIF per user by δ, which is a design parameter for given average SNR and S. Subsequently, each user employs the following opportunistic scheduling metric feedback:

- if $I^{[i,j]} \leq \delta$, the user feeds back $\text{SNR}^{[i,j]}$ to the corresponding BS,
- otherwise, i.e., $I^{[i,j]} > \delta$, then the user remains silent without transmitting any feedback.

C. Stage 2: User Scheduling

Upon receiving the $\text{SNR}^{[i,j]}$ values, $i = 1, \ldots, N_i$, where N_i denotes the eligible (or active) users in the cell i, the BS i selects S users with smaller $\text{SNR}^{[i,j]}$'s among them. Subsequently, the BSs broadcast the user selection information to the users in the cell such that the selected users can transmit uplink signals.

In advance of the uplink transmission, each selected user feeds forward $\mathbf{w}^{[i,j]}$ as an effective pilot signal such that the corresponding BS can estimate the effective channel $\mathbf{H}^{[i,j]}\mathbf{w}^{[i,j]}$.

D. Stage 3: Uplink Communication

All the selected users transmit uplink signals simultaneously, and the received signal $\mathbf{y}_i$ at the BS i is given by (2). To null the interference aligned at the reference interference basis $\mathbf{Q}_i$, the BS i obtains the following:

$$\mathbf{r}_i = \mathbf{U}_i^H\mathbf{y}_i = \sum_{j=1}^{S}\mathbf{U}_i^H\mathbf{H}_i^{[i,j]}\mathbf{w}^{[i,j]}x^{[i,j]} + \underbrace{\sum_{k=1,k\neq i}^{K}\sum_{m=1}^{S}\mathbf{U}_i^H\mathbf{H}_i^{[k,m]}\mathbf{w}^{[k,m]}x^{[k,m]} + \mathbf{U}_i^H\mathbf{z}_i}_{\triangleq \mathbf{q}_i}. \quad (9)$$

Note that if the interference from the user m in the cell k is perfectly aligned at $\mathbf{Q}_i$, i.e., $\mathbf{H}_i^{[k,m]}\mathbf{w}^{[k,m]} \in \text{span}(\mathbf{Q}_i)$, then it is nulled from $\mathbf{y}_i$ by multiplying $\mathbf{U}_i^H$.

E. Effective Sum-Rate Calculation

We calculate the effective sum-rate of the proposed scheme considering the feedback overhead for the scheduling metric feedback. Defining $\mathbf{b}_i^{[k,m]} \triangleq \mathbf{U}_i^H\mathbf{H}_i^{[k,m]}\mathbf{w}^{[k,m]}$, the effective noise covariance matrix is given by

$$\mathbf{C}_i \triangleq E\left\{\mathbf{q}_i\mathbf{q}_i^H\right\} = P\sum_{k=1,k\neq i}^{K}\sum_{m=1}^{S}\mathbf{b}_i^{[k,m]}\left(\mathbf{b}^{[k,m]}\right)^H + \sigma_z^2\mathbf{I}_S. \quad (10)$$

Let us further denote the effective channel matrix for the desired signals by

$$\mathbf{F}_i \triangleq \left[\mathbf{U}_i^H\mathbf{H}_i^{[i,1]}\mathbf{w}^{[i,1]}, \ldots, \mathbf{U}_i^H\mathbf{H}_i^{[i,S]}\mathbf{w}^{[i,S]}\right]. \quad (11)$$

Now, to calculate the effective sum-rate, let us assume that one symbol duration is required for each user to feed back the scheduling metric as assumed in [15]. Note that we only concern the effort for the scheduling metric feedback of KN users in Stage 1, which dominates the other efforts as N increases, such as the acquisition of the local CSI through the channel reciprocity, broadcast of the user selection information, and feed forward of $\mathbf{w}^{[i,j]}$ for selected users. Recall that the block length is denoted by T and the number of eligible users at the cell i in Stage 1 is denoted by N_i. Then, the effective sum-rate is obtained by [15]

$$R = \sum_{i=1}^{K}\frac{T-N_i}{T}\cdot\log\det\left(P\mathbf{C}_i^{-1/2}\mathbf{F}_i\mathbf{F}_i^H\mathbf{C}_i^{-1/2} + \mathbf{I}_S\right). \quad (12)$$

We conclude the discussion on the proposed scheme by providing the following remark, which justifies the proposed weight vector design and user scheduling method in comparison to the previous OIA scheme.

Remark 1: In the DoF-maximizing OIA for the MIMO IMAC [12], [13], the weight vector as well as the user scheduling method was designed only to minimize the sum-LIF, or equivalently the sum-interference, not taking into consideration the message link. As a consequence, the DoF-maximizing OIA does not guarantee anything in terms of the achievable rate in the low to mid SNR regime. On the other hand, the proposed user selection is based on the SNR maximization for the eligible users, and thus we can expect higher achievable rates in the practical SNR regime. Furthermore, through the LIF-minimizing weight vector design and the threshold-based eligible user determination, the total sum-interference, which is equivalent to the sum of the LIFs of all selected users [13], can be guaranteed to be lower than the predefined threshold given by $KS\delta$. In addition, the feedback overhead can be greatly reduced by optimizing δ, enhancing the effective sum-rate significantly.

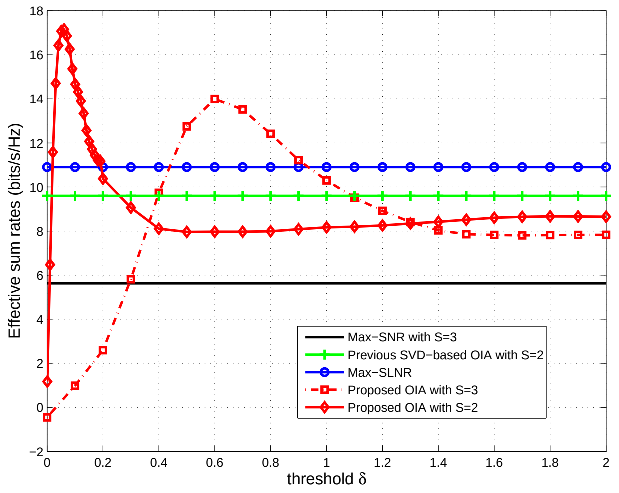

Fig. 2. Effective sum-rates versus δ for the MIMO IMAC with $K = M = 3$, $L = 4$, $N = 20$, and SNR=15dB.

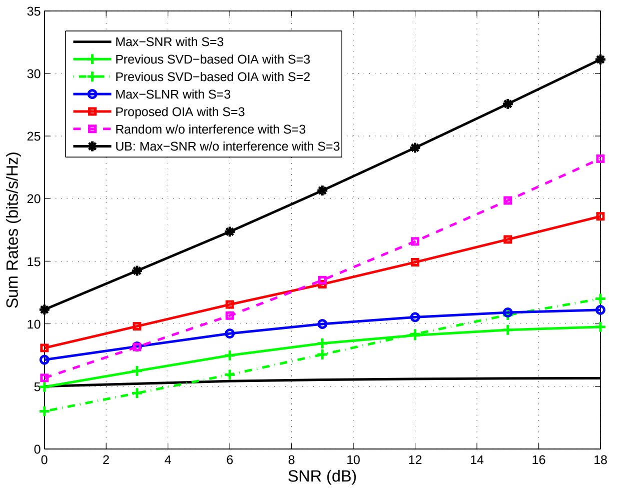

Fig. 3. Effective sum-rates versus SNR for the MIMO IMAC with $K = M = 3$, $L = 4$, and $N = 20$. For given SNR, the parameters S and/or δ was optimized for each of the schemes.

IV. SIMULATION RESULTS

The proposed scheme is evaluated for the i.i.d. Rayleigh fading scenario where $K = 3$, $M = 3$, $L = 4$, $N = 20$, and $T = 100$. For comparison, the naive SNR maximizing beamformming and user scheduling method is considered as a base line scheme. In addition, the signal-to-leakage-and-noise ratio (SLNR), or virtual signal-to-noise-and-interference ratio (SINR), maximizing scheme is compared, in which the weight vector design and the user scheduling are performed to maximize $\text{SLNR} = \text{SNR}/(\text{LIF}/\sigma_z^2 + 1)$ [16], [17]. The SLNR maximizing scheme is known to provide significant sum-rate gain over the naive SNR maximizing scheme in the interference channel. Moreover, the previous SVD-based OIA that maximizes the achievable DoF is also compared.

Figure 2 illustrates the effective sum-rates with respect to the threshold δ for the average SNR, P/σ_z^2, of 15dB. Note that the selection of S also changes the maximum achievable sum-rate, and thus should be optimized along with δ. It can be seen from the figure that the effective sum-rate of the proposed scheme is maximized at $S = 2$ and $\delta = 0.06$ and outperforms the previous schemes based on local CSI with this choice.

Figure 2 shows the effective sum-rates versus SNR, where the parameters S and/or δ were optimized for given SNR if necessary. For a further comparison, we considered two different ideal schemes: i) max-SNR weight design and user selection with genie-aided interference cancelation and ii) random user selection with the max-SNR weight design and with genie-aided interference cancelation. From this figure, it is seen that the effective sum-rate of the proposed scheme is higher than those of the previous schemes in all SNR regime. Interestingly, the proposed scheme even outperforms the second ideal scheme, denoted by 'Random w/o interference,' in the low SNR regime. In this noise-limited regime, the gain of the proposed scheme over the random user selection comes from the fact that the proposed scheme exploits the benefit of considering the SNRs of the message link for user scheduling. As the SNR increases, i.e., in the interference-limited regime, the second ideal scheme outperforms the proposed scheme as the interference in the proposed scheme cannot be perfectly cancelled with finite N.

V. CONCLUSIONS

A modified OIA framework has been proposed to reduce the signaling overhead due to the scheduling metric feedback from users and to further improve the sum-rate of the selected users in each cell. It turned out that through the LIF-based opportunistic feedback scheme, the feedback overhead for uplink is significantly reduced, which also guarantees the generating sum-interference level to other cells to be lower than a predefined threshold. It was also shown that the user scheduling based on both the channel gain of the desired link and the LIF-level greatly increases the resulting SINR at each BS, yielding the performance improvement on the sum-rate. Our achievability result indicated that by optimizing system parameters including the LIF threshold according to the number of users in a cell, the proposed scheme significantly outperforms the existing uplink scheduling schemes including the conventional SVD-based OIA in terms of effective sum-rate for all SNR regimes.

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REFERENCES

[1] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[2] S. A. Jafar and S. Shamai (Shitz), "Dgrees of freedom region of the MIMO X channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 151–170, Jan. 2008.

[3] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, “Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis,” *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3457–3470, Aug. 2008.

[4] K. Gomadam, V. R. Cadambe, and S. A. Jafar, “A distributed numerical approach to interference alignment and applications to wireless interference networks,” *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3309–3322, June 2011.

[5] C. Suh and D. Tse, “Interference alignment for cellular networks,” in *Proc. 46th Annual Allerton Conf. Communication, Control, and Computing*, Urbana-Champaign, IL, Sept. 2008, pp. 1037 – 1044.

[6] A. S. Motahari, O. Gharan, M.-A. Maddah-Ali, and A. K. Khandani, “Real interference alignment: exploiting the potential of single antenna systems,” *IEEE Trans. Inform. Theory*, submitted for publication, Preprint, [Online]. Available: http://arxiv.org/abs/0908.2282.

[7] T. Gou and S. A. Jafar, “Degrees of freedom of the K user M X N MIMO interference channel,” *IEEE Trans. Inf. Theory*, vol. 56, no. 12, pp. 6040–6057, Dec. 2010.

[8] T. Kim, D. J. Love, and B. Clerckx, “On the spatial degrees of freedom of multicell and multiuser MIMO channels,” *IEEE Trans. Inform. Theory*, submitted for publication, Preprint, [Online]. Available: http://arxiv.org/abs/1111.3160 2011.

[9] B. C. Jung, D. Park, and W.-Y. Shin, “A study on the optimal degree-of-freedoms of cellular networks: Opportunistic interference mitigation,” in *44th Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, CA, 2011, pp. 2067–2071.

[10] B. C. Jung and W.-Y. Shin, “Opportunistic interference alignment for interference-limited cellular TDD uplink,” *IEEE Commun. Lett.*, vol. 15, no. 2, pp. 148–150, Feb. 2011.

[11] B. C. Jung, D. Park, and W.-Y. Shin, “Opportunistic interference mitigation achieves optimal degrees-of-freedom in wireless multi-cell uplink networks,” *IEEE Trans. Commun.*, to appear, Preprint, [Online]. Available: http://arxiv.org/abs/1006.2380.

[12] H. J. Yang, W.-Y. Shin, B. C. Jung, and A. Paulraj, “Opportunistic interference alignment of MIMO IMAC : Effect of user scaling over degrees-of-freedom,” in *Proc. IEEE Int’l Symp. Inf. Theory (ISIT)*, Cambridge, MA, July 2012, pp. 2646–2650.

[13] ——, “Opportunistic interference alignment of MIMO interfering multiple-access channels,” *IEEE Trans. Wireless Commun.*, submitted for publication.

[14] K. Gomadam, V. R. Cadambe, and S. A. Jafar, “Approaching the capacity of wireless networks through distributed interference alignment,” in *IEEE GLOBECOM*, New Orleans, LO, Nov.-Dec. 2008.

[15] B. Hassibi and B. M. Hochwald, “How much training is needed in multiple-antenna wireless links?” *IEEE Trans. Inf. Theory*, vol. 49, no. 4, pp. 951–963, Apr. 2003.

[16] R. Zakhour and D. Gesbert, “Distributed multicell-MISO precoding using the layered virtual SINR framework,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 8, pp. 2444–2448, Aug. 2010.

[17] M. Sadek, A. Tarighat, and A. H. Sayed, “A leakage-based precoding scheme for downlink multi-user MIMO channels,” *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1711–1721, May 2007.