A Novel Feedback Reduction Technique for Cellular Downlink with CDF-Based Scheduling

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***Abstract*—Cumulative distribution function (CDF)-based scheduling is known as an efficient scheduling method that can assign different time fractions for user access or, equivalently, satisfy different channel access ratio requirements of users in cellular downlink while exploiting multi-user diversity. In this paper, we propose CDF-FR, a feedback reduction technique for CDF-based scheduling that reduces feedback overhead from users in a cell. Although several threshold based feedback reduction schemes have been proposed for various scheduling algorithms, none of them considers users' different channel access ratio requirements for which CDF-based scheduling is designed. In the proposed technique, a single threshold is used for all users who have different channel access ratio requirements. We show that this simple setting is sufficient for CDF-FR to satisfy users' diverse channel access ratio requirements. It is proved that the average feedback overhead of CDF-FR is upper-bounded by $-\ln p$ for an arbitrary number of users in a cell, where p represents the probability that no user satisfies the threshold condition. Furthermore, the normalized throughput loss due to feedback reduction is upper-bounded by p in fading channels with arbitrary statistics.**

***Index Terms*—Cellular downlink, user scheduling, CDF-based scheduling, fairness, feedback overhead.**

I. Introduction

In wireless networks, independent fading of users can be exploited for multi-user diversity. In cellular networks with arbitrary fading channels, the optimal user scheduling to maximize the sum throughput is to select the user who has the largest channel gain at each time-slot. Although the above scheduling method can maximize the sum throughput, it may cause a fairness problem among users located at different distances from the base station (BS) because the BS tends to select users that are closer to it more frequently due to their higher average signal-to-noise ratios (SNRs). The fairness problem among users has been widely studied with various criteria, such as throughput requirements [1], proportional fairness [2], and fair resource sharing [3], [4].

Several scheduling algorithms [4]–[6] fairly assign channel resources to users based on the, cumulative distribution function (CDF) values of channel gains. This paper proposes feedback reduction in CDF-based scheduling [4]. In cellular systems, due to different service priorities or quality-of-service requirements, users may require different assignments of access time fractions, referred as channel access ratios in this paper, and CDF-based scheduling can exactly satisfy these requirements while exploiting multi-user diversity. As CDF-based scheduling can provide independent throughput performance for each user, it is robust to variations of system parameters such as traffic characteristics and number of users in a cell. Therefore, CDF-based scheduling has been studied under various network scenarios such as multi-cell coordination [7] and cheating of CDF values [8].

In order to exploit multi-user diversity, CDF-based scheduling requires all users to feedback their CDF values to BS in each time slot. For practical systems, feedback overhead is a challenging issue especially when a large number of users need to be scheduled in a cell. Therefore, it is of great interest to design a feedback reduction scheme for CDF-based scheduling to reduce the number of feedback users in each time slot. Several threshold-based feedback reduction schemes [9]–[11] have been proposed for various scheduling schemes such as proportional fair scheduling and normalized SNR-based scheduling. However, none of these schemes supports different channel access ratios among users, as CDF-based scheduling does. Consequently, these feedback reduction schemes cannot be applied to CDF-based scheduling. In this paper, we propose CDF-FR, a novel feedback reduction scheme for CDF-based scheduling, to reduce the feedback overhead. To the best of our knowledge, CDF-FR is the first feedback reduction scheme that considers diverse users who require different channel access ratios in scheduling.

It is notable that our design of CDF-FR employs a universal threshold for all users to decide whether to send feedback to BS. Despite the simplicity of this design, CDF-FR can maintain the different channel access ratio requirements of diverse users. CDF-FR also inherits the property of CDF-based scheduling in providing independent throughput performance for each user. Our analysis shows that the average feedback overhead of CDF-FR is upper-bounded by $-\ln p$ for an arbitrary number of users, where p represents the probability that no user satisfies the threshold condition. We also show that the throughput loss of CDF-FR relative to CDF-based scheduling with full feedback is upper-bounded by p in arbitrary channels.

The rest of this paper is organized as follows: Section II introduces the system model and reviews CDF-based scheduling. Section III presents our proposed CDF-FR and an analysis of its performance. Section IV discusses the numerical results. Finally, conclusions are drawn in Section V.

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II. System Model

We consider the downlink of a cell with a BS and n users. At each time slot, the BS selects one user to receive its transmission. The transmit power of the BS is assumed to be constant in each time slot. The BS and all users are assumed to have a single antenna. In time slot t, the received signal at the i-th user is given as

$$y_i(t) = h_i(t)x(t) + z_i(t), \qquad i = 1, 2, ..., n, \tag{1}$$

where $y_i(t) \in \mathbb{C}^T$ consists of T received symbols, $x(t) \in \mathbb{C}^T$ is the T transmitted symbols, $h_i(t) \in \mathbb{C}$ is the channel gain from the BS to the i-th user, and $z_i(t) \in \mathbb{C}^T$ is a zero-mean circular-symmetric Gaussian random vector ($z_i(t) \sim CN(0, \sigma^2 I_T)$). The transmit power constraint is set to P, i.e., $E[|x(t)|^2] \leq P$. We assume a block-fading channel where the channel gain is constant during the T symbols in a time slot and independently changes between time slots. Different users may have different channel gain statistics. The received SNR of the i-th user is given by $\gamma_i(t) = P|h_i(t)|^2/\sigma^2$. Let $F_i(\gamma)$ denote the CDF of the SNR of the i-th user, which can be obtained from long-term observations. It is easy to prove that $U_i = F_i(\gamma)$ is uniformly distributed between $[0, 1]$ and the CDF is given by

$$F_{U_i}(u) = u, \quad u \in [0, 1]. \tag{2}$$

In this paper, we assume that all users' channels are stationary and the channel statistics of each user are assumed to be independent from those of other users. While different users may have different CDFs, the values of all users' CDFs have the same uniform distribution.

Let $w_i (> 0)$ denote the weight of the i-th user. The weight indicates the user's channel access ratio compared to other users, which means that the ratio between the i-th and j-th users' channel access opportunities is given by w_i/w_j. If there are n users in the system, the i-th user's channel access ratio is $\alpha_i = \frac{w_i}{\sum_{j=1}^n w_j}$. With CDF-based scheduling, the feedback information of the i-th user is $[F_i(\gamma_i(t))]^{\frac{1}{w_i}}$ at time slot t and the index of the user selected at the BS is given by

$$\arg \max_{i \in \{1,2,...,n\}} [F_i(\gamma_i(t))]^{\frac{1}{w_i}}. \tag{3}$$

It has been shown in [4] that this scheduling yields a channel access ratio of α_i for the i-th user.

III. Feedback Reduction for CDF-based Scheduling

A. Threshold Design and Channel Access Ratio

For *equally weighted users* in a cell, since all users send the feedback information that is identically and uniformly distributed between $[0, 1]$, we can simply set the same threshold η_{th} for all users to achieve the identical channel access ratio. If the feedback information of the i-th user, U_i, is larger than η_{th}, the i-th user sends U_i to BS. If no user satisfies the condition, the BS does not receive any feedback information from the users and it selects a user in a round-robin manner where the probability of selecting the user is equal to the user's channel access ratio. When no feedback happens in the slot, we call such a slot a no-feedback (NFB) slot. We further define a slot in which more than one users send feedback to BS as a feedback (FB) slot. For *unequally weighted users*, the difficulty in determining the thresholds is to satisfy the channel access ratios in both FB and NFB slots. Different users may have different threshold values due to their different weights. However, we show in the following Theorem that it is possible to maintain the channel access ratios of different users using the same threshold η_{th} for all users.

Theorem 1: The channel access ratios of the users with CDF-FR is maintained if the threshold of all user is set to $p^{1/\sum_{j=1}^n w_j}$, where p denotes the NFB probability.

Proof: Given the threshold η_{th} for all users, the i-th user feeds back the value $U_i^{\frac{1}{w_i}}$ if it is larger than η_{th}. With this setting, we show that the channel access ratio of the i-th user in the NF slots is equal to $\alpha_i = \frac{w_i}{\sum_{j=1}^n w_j}$.

With the proposed threshold setting for CDF-FR, the NFB probability is given by:

$$\begin{aligned} p &= \Pr\{U_j^{\frac{1}{w_j}} < \eta_{\text{th}}, \forall j \in \{1, 2, \cdots, n\}\} \\ &= \prod_{j=1}^n \eta_{\text{th}}^{w_j} = \eta_{\text{th}}^{\sum_{j=1}^n w_j}. \end{aligned} \tag{4}$$

For a given NFB constraint p, the threshold η_{th} can be set to $p^{1/\sum_{j=1}^n w_j}$. Hence, the selection probability for the i-th user in each FB slot is

$$\begin{aligned} &\Pr\{\text{user } i \text{ is selected}|\text{FB slot}\} \\ &= \frac{\Pr\{\text{user } i \text{ is selected, the slot is FB slot}\}}{\Pr\{\text{the slot is FB slot}\}} \\ &= \frac{\Pr\{U_i^{\frac{1}{w_i}} > \eta_{\text{th}} \ \& \ U_i^{\frac{1}{w_i}} > U_j^{\frac{1}{w_j}}, \forall j \in \{1, 2, \cdots, i-1, i+1, \cdots n\}\}}{1 - \Pr\{U_j^{\frac{1}{w_j}} < \eta_{\text{th}}, \forall j \in \{1, 2, \cdots, n\}\}} \\ &= \frac{\int_{\eta_{\text{th}}^{w_i}}^1 \prod_{j=1, j\neq i}^n \Pr\{U_j < u^{\frac{w_j}{w_i}}\} f_{U_i}(u) du}{1-p} \\ &= \frac{\int_{\eta_{\text{th}}^{w_i}}^1 u^{\sum_{j=1, j\neq i}^n \frac{w_j}{w_i}} du}{1-p} = \frac{w_i}{\sum_{j=1}^n w_j} = \alpha_i. \end{aligned} \tag{5}$$

In the NFB slots, the users are selected with the round-robin scheduling (or random scheduling) so that the channel access ratio α_i for the i-th user is still maintained. Thus, the total channel access ratio for the i-th user is

$$\alpha_i \Pr\{\text{FB slot}\} + \alpha_i \Pr\{\text{NFB slot}\} = \alpha_i. \tag{6}$$

■

Note that we do not assume a specific channel distribution in *Theorem 1* and it can be applied to any channel distributions. Notably, selecting the same threshold value for all users who have different channel access ratios substantially simplifies the system design and implementation. The BS calculates the threshold of $p^{1/\sum_{j=1}^n w_j}$ and informs all the users.

B. Feecback Overhead Reduction

Theorem 2: With CDF-FR, the average feedback overhead in each slot is upper-bounded by $n\left(1 - p^{\frac{1}{n}}\right)$, where p denotes the NFB probability. The equality holds when all users are equally weighted. Another upper-bound of feedback overhead is given by $-\ln p$, which is valid regardless of the number of users and the weight of users.

Proof: For the i-th user, the average feedback overhead in each slot is given as:

$$\begin{aligned}\mu_i &= \Pr\{U_i^{\frac{1}{w_i}} \geq \eta_{\text{th}}\} = \Pr\{U_i \geq \eta_{\text{th}}^{w_i}\} = 1-\eta_{\text{th}}^{w_i} \\ &= 1 - p^{\frac{w_i}{\sum_{j=1}^{n} w_j}} = 1-p^{\alpha_i}.\end{aligned} \tag{7}$$

The average feedback overhead in each slot in a cell is given as:

$$\mu = \textstyle\sum_{i=1}^{n} \mu_i = n\left(1-\frac{1}{n}\sum_{i=1}^{n} p^{\alpha_i}\right). \tag{8}$$

Since $f(x)=p^x$ is a convex function of x in a region $0 < p < 1$, we have

$$\mu \leq n\left(1-p^{\frac{1}{n}\sum_{i=1}^{n}\alpha_i}\right) = n\left(1-p^{\frac{1}{n}}\right). \tag{9}$$

The equality holds when $\alpha_1 = \alpha_2 = ... = \alpha_n$, i.e., all users have the same weight. Using the fact that $x(1-p^{\frac{1}{x}})$ is an increasing function over x for $x>0$ and $0<p<1$, and $\lim_{n\to\infty}(1-\frac{x}{n})^n = e^{-x}$, we have

$$\mu \leq \lim_{n\to\infty} n\left(1-p^{\frac{1}{n}}\right) = -\ln p. \tag{10}$$

■

C. Throughput Analysis

The SNR distribution for a user given it is selected is provided by the following theorem:

Theorem 3: With CDF-FR, if a user's SNR distribution is $F(\gamma)$, its channel access ratio is $\alpha \in [0,1]$, and the NFB probability is p, the SNR distribution given this user is selected is obtained as

$$F_{\text{Sel}}(\gamma) = \begin{cases} p^{(1-\alpha)}F(\gamma), & \text{if } 0<\gamma<F^{-1}(p^\alpha), \\ F(\gamma)^{\frac{1}{\alpha}}, & \text{if } \gamma \geq F^{-1}(p^\alpha). \end{cases} \tag{11}$$

Proof: See Appendix. ■

To express the throughput, we also define the following function:

Definition 1:

$$\begin{aligned} S(x,\alpha) &= \textstyle\int_{F^{-1}(x)}^{\infty} R(\gamma)d[F(\gamma)]^{\frac{1}{\alpha}}, \\ S_L(x,\alpha) &= \textstyle\int_{0}^{F^{-1}(x)} R(\gamma)d[F(\gamma)]^{\frac{1}{\alpha}} = S(0,\alpha)-S(x,\alpha). \end{aligned}$$

Then, $S(x,\alpha)$ and $S_L(x,\alpha)$ have the following properties[1]:

Property 1: $\alpha S_L(x,\alpha)$ is an increasing function of α.

Property 2: $\frac{S(x,\alpha)}{1-x^{\frac{1}{\alpha}}}$ is an increasing function of x.

Property 3: $S(x^\alpha,\alpha) + x^{1-\alpha}S_L(x^\alpha,1)$ is a decreasing function of x.

Based on (11), the throughput of CDF-FR is calculated as

$$\begin{aligned} S_{\text{CDF-FR}}(\alpha,p) &= \alpha \textstyle\int_{F^{-1}(p^\alpha)}^{\infty} R(\gamma)d[F(\gamma)]^{\frac{1}{\alpha}} \\ &\quad + \alpha p^{1-\alpha}\textstyle\int_{0}^{F^{-1}(p^\alpha)} R(\gamma)dF(\gamma) \\ &= \alpha S(p^\alpha,\alpha) + \alpha p^{1-\alpha}S_L(p^\alpha,1). \end{aligned} \tag{12}$$

[1]We skip the proof of the properties due to the page limitation. Interested readers can refer the journal version of this paper.

We can observe that the throughput of any user depends on its channel access ratio α and the NFB probability p and is independent from other users. From *Property 3*, we can conclude that $S_{\text{CDF-FR}}$ is an increasing function of p. Hence, there is no optimal threshold for CDF-FR and, in order to obtain a higher throughput, we should reduce the value of p. When $p=0$, CDF-FR is identical to CDF-based scheduling while CDF-FR is identical to round-robin when $p=1$. Thus, CDF-FR always shows better throughput performance than round-robin and worse throughput performance than CDF-based scheduling. Compared to CDF-based scheduling, the lower- and upper-bound throughput of CDF-FR are characterized with the following theorem:

Theorem 4: The lower and upper bounds of $S_{\text{CDF-FR}}(\alpha,p)$ are given as

$$1-p \leq 1-p+\alpha p^{2-\alpha} \leq \frac{S_{\text{CDF-FR}}(\alpha,p)}{S_{\text{CDF}}(\alpha)} \leq 1, \tag{13}$$

where $S_{\text{CDF}}(\alpha)$ is the throughput of CDF-based scheduling and can be calculated as $S_{\text{CDF}}(\alpha) = S_{\text{CDF-FR}}(\alpha,0)$.

Proof: The upper-bound can be obtained from *Property 3* where the case of $p=0$ stands for $S_{\text{CDF}}(\alpha)$. For the lower-bound, we have the following derivation:

$$\begin{aligned} \tfrac{1}{\alpha}S_{\text{CDF-FR}}(\alpha,p) &= S(p^\alpha,\alpha) + p^{1-\alpha}S_L(p^\alpha,1) \\ &\geq S(p^\alpha,\alpha) + p^{1-\alpha}\alpha S_L(p^\alpha,\alpha) \\ &= (1-\alpha p^{1-\alpha})S(p^\alpha,\alpha) + \alpha p^{1-\alpha}S(0,\alpha) \\ &\geq (1-\alpha p^{1-\alpha})(1-p)S(0,\alpha) + \alpha p^{1-\alpha}S(0,\alpha) \\ &= (1-p+\alpha p^{2-\alpha})\tfrac{1}{\alpha}S_{\text{CDF}}(\alpha) \\ &\geq (1-p)\tfrac{1}{\alpha}S_{\text{CDF}}(\alpha), \end{aligned} \tag{14}$$

where *Property 1* and *Property 2* have been applied to obtain the first and second inequalities, respectively. ■

From the lower bound, we can conclude that the throughput loss ratio of CDF-FR to CDF-based scheduling is smaller than the NFB probability p. Note that *Theorem 4* is applicable to any data rate function and channel statistics. *Theorem 2* and *Theorem 4* indicate the following remarks for CDF-FR:

Remark 1: 1) There is a tradeoff between throughput and feedback overhead. A larger feedback overhead gives a higher throughput because they are both decreasing functions of p. 2) The feedback overhead is upper-bounded by the negative natural logarithm of the throughput loss ratio, i.e., if each user can tolerate a throughput loss of at most p compared to CDF-based scheduling, we can design CDF-FR with average feedback overhead smaller than $-\ln p$.

In the remainder of this section, we analyze the throughput performance with general Nakagami-m fading channels and a data rate function of $R(\gamma) = \log_2(1+\gamma)$ which is the Shannon capacity. In Nakagami-m fading channels with an integer shape parameter m, the received SNR distribution shows the Gamma distribution whose CDF is given as

$$F_{m,\overline{\gamma}}(\gamma) = 1 - \textstyle\sum_{j=0}^{m-1} \frac{1}{j!}\left(\frac{m}{\overline{\gamma}}\right)^j \gamma^j e^{-\frac{m}{\overline{\gamma}}\gamma}, \tag{15}$$

where $\overline{\gamma}$ is the average SNR. If $\frac{1}{\alpha} = K$ is an integer value, with extending the analysis in [12], $S(x,\alpha)$ can be obtained as

$$
\begin{aligned}
S(x, \tfrac{1}{K}) \quad &= \log_2(1+\gamma_{\rm th})\{1-[F_{m,\overline{\gamma}}(\gamma_{\rm th})]^K\} \\
&\quad + \log_2(e)\textstyle\sum_{k=1}^{K}\sum_{j=0}^{k(m-1)}(-1)^{k+1} \cdot \\
&\quad \binom{K}{k}\underline{c(j,k)}\left(\frac{m}{\overline{\gamma}}\right)^j \underline{T(\gamma_{\rm th}, j, \tfrac{\overline{\gamma}}{km})},
\end{aligned} \tag{16}
$$

where $\gamma_{\rm th}$ is the value satisfying $F_{m,\overline{\gamma}}(\gamma_{\rm th}) = x$, $c(j,k)$ is defined as

$$
\begin{aligned}
&c(0,k) = 1, \qquad c(1,k) = k, \\
&c(k(m-1),k) = [(m-1)!]^{-k}, \\
&c(j,k) = \tfrac{1}{j}\textstyle\sum_{l=1}^{\min(j,m-1)} \frac{l(k+1)-j}{l!} c(j-l,k), \\
&\qquad\qquad \text{for } 2 \le j \le k(m-1),
\end{aligned} \tag{17}
$$

and $T(\gamma_{\rm th}, j, \theta)$ is defined as

$$
\begin{aligned}
&T(\gamma_{\rm th}, j, \theta) = e^{\frac{1}{\theta}}\left\{(-1)^j E_1\left(\tfrac{1+\gamma_{\rm th}}{\theta}\right)\right. \\
&\left.+\textstyle\sum_{i=1}^{j}\binom{j}{i}(-1)^{j-i}\theta^i(i-1)!\left[\sum_{l=0}^{i-1}\frac{1}{l!}\left(\frac{1+\gamma_{\rm th}}{\theta}\right)^l e^{-\frac{1+\gamma_{\rm th}}{\theta}}\right]\right\},
\end{aligned} \tag{18}
$$

where the exponential integral function of the first kind is defined as $E_1(y) = \int_y^\infty \frac{e^{-t}}{t}dt$. Thus, applying (16) to (12), $S_{\rm CDF-FR}(\alpha, p)$ can be calculated.

IV. Numerical Results

In the figures shown in this section, the solid liens show the analytical results while the symbols show the simulation results. We can observe the analytical results match well with the simulation results. Fig. 1 shows the feedback overhead ratios with equally weighted users when the NFB probability is varied from 0 to 1. Note that this equally weighted case yields an upper-bound for the unequally weighted case as discussed in Section III-B. The average feedback ratio represents the ratio of the average number of users sending the feedback information to BS with CDF-FR over the total number of users. From the figure we can observe that a larger NFB probability reduces the feedback overhead more significantly. If the NFB probability is 2%, i.e $p = 0.02$, the average feedback ratio is equal to 54.3%, 32.4%, and 3.8% when $n = 5, 10, 100$, respectively. Therefore, for given a NFB probability, CDF-FR reduces the feedback overhead significantly as the number of users increases. This is mainly because the feedback overhead is bounded by $-\ln p$ regardless of the number of users as shown in *Theorem 2*.

We define throughput gain as the ratio between the throughput of CDF-FR and the throughput of round-robin scheduling in this paper. Fig. 2 shows the throughput gain of CDF-based scheduling and CDF-FR when the reciprocal of channel access ratio is varied. For equally weighted users, the reciprocal of channel access ratio is equal to the number of users in the system. The average SNR of the user being observed is set to 0dB. We can observe that the throughput gain of CDF-FR increases as the reciprocal of channel access ratio increases and a larger NFB probability reduces the throughput gain with CDF-FR. In Nakagami-m fading channels, CDF-FR yields a larger throughput gain with small m since user experiencing more fluctuations in the channel gain may obtain a higher throughput gain compared to a user with less fluctuations.

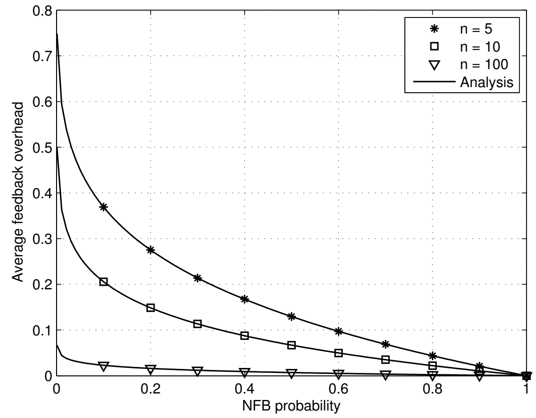

Fig. 1. Average feedback ratio vs. NFB probability

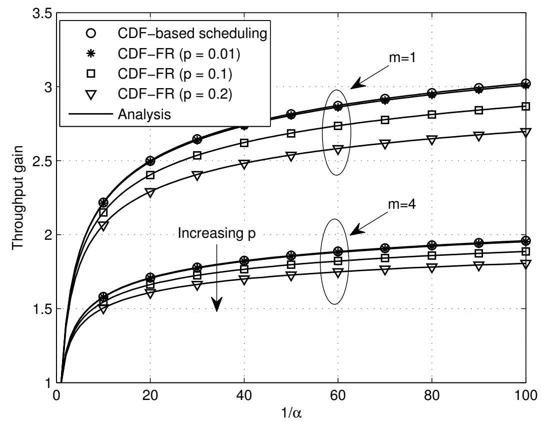

Fig. 2. Throughput gain vs. $1/\alpha$

Fig. 3 shows the throughput gains of CDF-FR for various NFB probabilities, when the channel is Rayleigh fading, which is a special case of Nakagami-m fading channels with $m = 1$ and the average SNR is 0dB. We can observe that a smaller channel access ratio and a smaller NFB probability yield a larger throughput gain. Fig. 4 shows the throughput ratio between CDF-FR and CDF-based scheduling in the same environment. We can observe that a smaller channel access ratio yields a smaller value of throughput ratio. Thus, if CDF-FR is applied, a user with a smaller channel access ratio is more prone to a throughput loss compared to a user with a larger channel access ratio. A similar trend can also be observed from the lower-bound throughput of CDF-FR shown in (13) since the formula, $1 - p + \alpha p^{1-\alpha}$, is an increasing function of α.

V. Conclusions

In this paper, we have proposed a novel feedback reduction technique for cellular downlink employing CDF-based scheduling, and analyzed its performance in terms of feedback overhead and throughput. With the proposed feedback technique, a single threshold is sufficient to maintain the channel

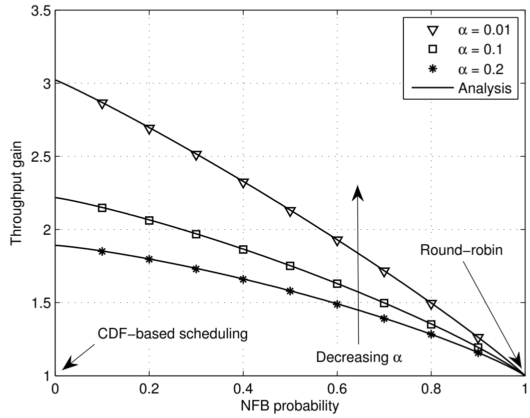

Fig. 3. Throughput gain of CDF-FR vs. NFB probability (p).

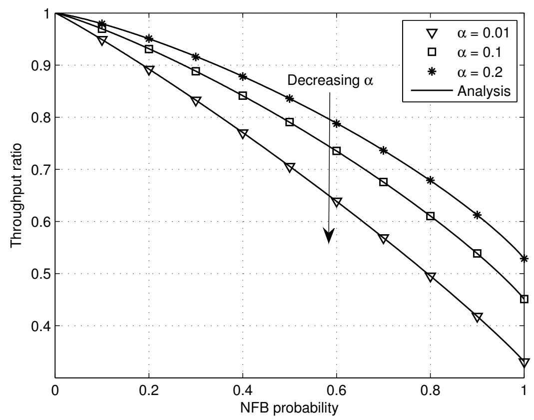

Fig. 4. Ratio between the throughput values with CDF-FR and CDF-based scheduling.

access ratio requirements of all users. The feedback overhead of the proposed feedback technique is upper-bounded by $-\ln p$ where p represents the probability that no user satisfies the threshold condition. We have also investigated fundamental tradeoff between throughput and feedback overhead. A larger feedback overhead yields a higher throughput. The throughput loss due to feedback reduction relative the throughput with full feedback is upper-bounded by $-\ln p$.

Appendix

Given the i-th user is selected, its SNR distribution in the NFB slots is derived as

$$
\begin{aligned}
&F_{i,\mathrm{Sel,NFB}}(\gamma)\\
&= \Pr\{\gamma_i < \gamma | \text{user } i \text{ is selected, the slot is a NFB slot}\}\\
&= \frac{\Pr\{\gamma_i<\gamma, \text{user } i \text{ is selected, the slot is a NFB slot}\}}{\Pr\{\text{user } i \text{ is selected, the slot is a NFB slot}\}}\\
&= \frac{\alpha_i \Pr\left\{\gamma_i<\gamma,\ U_j^{\frac{1}{w_j}}<\eta_{\mathrm{th}},\ \forall j\in\{1,2,\cdots,n\}\right\}}{\alpha_i p}\\
&= \frac{\alpha_i \frac{p}{p^{\alpha_i}} \Pr\left\{\gamma_i<\gamma,\ \gamma_i<F_i^{-1}(p^{\alpha_i})\right\}}{\alpha_i p}\\
&= \begin{cases} p^{-\alpha_i} F_i(\gamma), & \text{if } 0<\gamma<F_i^{-1}(p^{\alpha_i}),\\ 1, & \text{if } \gamma \geq F_i^{-1}(p^{\alpha_i}), \end{cases}
\end{aligned}
\tag{19}
$$

where we used the fact $\eta_{\mathrm{th}}^{w_i} = p^{\alpha_i}$ from (4). The SNR distribution in the FB slots is derived as

$$
\begin{aligned}
&F_{i,\mathrm{Sel,FB}}(\gamma)\\
&= \Pr\{\gamma_i < \gamma | \text{user } i \text{ is selected, the slot is a FB slot}\}\\
&= \frac{\Pr\{\gamma_i<\gamma, \text{user } i \text{ is selected, the slot is a FB slot}\}}{\Pr\{\text{user } i \text{ is selected, the slot is a FB slot}\}}\\
&= \frac{\Pr\{\gamma_i<\gamma,\ U_i^{\frac{1}{w_i}}>\eta_{\mathrm{th}},\ U_i^{\frac{1}{w_i}}>U_j^{\frac{1}{w_j}}, \forall j\in\{1,2,\cdots,i-1,i+1,\cdots n\}\}}{\alpha_i(1-p)}\\
&= \frac{\Pr\{\gamma_i<\gamma,\ \gamma_i>F_i^{-1}(p^{\alpha_i}),\ U_j<[F_i(\gamma_i)]^{\frac{w_j}{w_i}}, \forall j\in\{1,2,\cdots,n\}\&j\neq i\}}{\alpha_i(1-p)}\\
&= \frac{\int_{F_i^{-1}(p^{\alpha_i})}^{\gamma} [F_i(\gamma_i)]^{\sum_{j=1,j\neq i}^{n}\frac{w_j}{w_i}} dF_i(\gamma_i)}{\alpha_i(1-p)}\\
&= \begin{cases} 0, & \text{if } 0<\gamma<F_i^{-1}(p^{\alpha_i}),\\ \frac{[F_i(\gamma)]^{\frac{1}{\alpha_i}}-p}{1-p}, & \text{if } \gamma \geq F_i^{-1}(p^{\alpha_i}). \end{cases}
\end{aligned}
\tag{20}
$$

Finally, the SNR distribution given the i-th user is selected is derived as

$$
\begin{aligned}
F_{i,\mathrm{Sel}}(\gamma) &= F_{i,\mathrm{Sel,NFB}}(\gamma)\Pr\{\text{NFB slot}\}\\
&\quad + F_{i,\mathrm{Sel,FB}}(\gamma)\Pr\{\text{FB slot}\}\\
&= F_{i,\mathrm{Sel,NFB}}(\gamma)p + F_{i,\mathrm{Sel,FB}}(\gamma)(1-p)\\
&= \begin{cases} p^{1-\alpha_i} F_i(\gamma), & \text{if } 0<\gamma<F_i^{-1}(p^{\alpha_i}),\\ F_i(\gamma)^{\frac{1}{\alpha_i}}, & \text{if } \gamma \geq F_i^{-1}(p^{\alpha_i}). \end{cases}
\end{aligned}
\tag{21}
$$

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