# Achievable Degrees-of-Freedom by Distributed Scheduling in an ( $n, K$ )-user Interference Channel 

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#### Abstract

In this paper, we study the achievable degree-offreedom (DoF) of an ( $n, K$ )-user interference network where $n$ transmitter-receiver pairs are randomly distributed but only $K$ transmitter-receiver pairs are allowed to communicate ( $n \gg K$ ). We propose a distributed user scheduling method to achieve the maximum DoF (i.e., $K$ ), which sequentially adds a transmitterreceiver pair causing/receiving interference to/from the previously selected transmitter-receiver pairs below a certain threshold level. It is proven that the maximum $K$ DoF is achievable if the total number of communication pairs $n$ scales $\omega\left(\operatorname{SNR}^{K(K-1)}\right)$ where SNR denotes the received signal-to-noise ratio. In addition, the total amount of the required feedback for the worst case and the feedback overhead per user are investigated in interference limited environments.


## I. Introduction

Opportunistic scheduling using multi-user diversity has received much attention for better utilizing fading phenomenon in wireless networks. The multi-user diversity gain comes from taking advantage of characteristic of the time-varying fading channel across different users. There have been many works on opportunistic scheduling to obtain multi-user diversity in centralized networks [1], [2]. In addition, opportunistic distributed scheduling has also been proposed in decentralized networks [3]-[5]. In decentralized networks, however, it is not easy to obtain channel state information (CSI) of other users since the infrastructure like base station does not exist. Accordingly, the design of a distributed opportunistic scheduling algorithm is considered very challenging compared with centralized scheduling.

Recently, the DoF of the $(n, K)$-user interference channel embedded in a dense network $(n \rightarrow \infty)$ was studied in [6]. For a single-input single-output (SISO) case, it was shown that DoF of $d \in[0, K]$ is achievable by user-group scheduling if the network size scales like $n=\omega\left(\operatorname{SNR}^{d(K-1)}\right)$ without power allocation ${ }^{1}$. The adopted scheduling is centralized; the whole network is divided into $\left\lfloor\frac{n}{K}\right\rfloor$ disjoint user-groups and the user-group achieving the maximum rate among all usergroups is selected after computing the achievable rate of each user-group. However, the centralized scheduling is practically prohibited in infra-less ad hoc networks.

[^0]In this paper, we propose a user distributed scheduling method which achieves the maximum DoF of the $(n, K)$ user interference channel. In the proposed scheduling, the transmitter-receiver pair which causes/receives interference to/from previously selected transmitter-receiver pairs below a certain threshold level is sequentially selected. We show that the maximum DoF $K$ for SISO case is achievable by the proposed scheduling method if total number of transmitterreceiver pairs (i.e., network size) scales at least as $n=$ $\omega\left(\mathrm{SNR}^{K(K-1)}\right)$ in the $(n, K)$-user interference channel. Interestingly, the proposed scheduling method achieves the scaling law which is obtained via the centralized scheduling method of [6] although it operates in a distributed manner. We also show that the feedback overhead per user is marginal as the network becomes more interference limited by analyzing the total amount of the required feedback.

## II. System Model

For given positive integers $n$ and $K(n \gg K)$, we consider an ( $n, K$ )-user interference channel in which $n$ user pairs (i.e., transmitter-receiver pairs) are randomly distributed in a dense network as shown in Fig. 1. Each transmitter is assumed to communicate with only its designated receiver without help of relays. In the $(n, K)$-user interference channel, only $K$ transmitters among $n$ transmitter-receiver pairs are allowed to transmit independent message to their corresponding receivers simultaneously (active user pairs) at each time slot. Consequently, the selected $K$ transmitter-receiver pairs construct the $K$ user interference channel and the remaining $n-K$ user pairs do not transmit (inactive user pairs). We assume that the wireless channel is time-invariant and each user is equipped with single antenna. It is assumed a timesynchronized network where the signal transmitted from a user interferes with other users.
We define $U$ as the set of indices of all user pairs in the network and $S_{k}$ as the set of indices of the $k$ user pairs selected until $k$ th steps, where $1 \leq k \leq K$. Hence, $\left|S_{k}\right|=k$ and $S_{k} \subset S_{p}$ for all $p \geq k$. Similarly, we define $S_{k}^{c}=U / S_{k}$ as the set of indices of remaining user pairs after $k$-th user pair is selected. In the proposed scheduling protocol, the user pairs are sequentially selected. After selecting $K$ user pairs, $K$ transmitters in the set $S_{K}$ send their own data simultaneously and construct a $K$-user interference channel. Without loss of


Fig. 1. For the selection step of the third user pair, interference that a node $s \in S_{2}^{c}$ causes to and receives from the previously selected user pairs (solid circles) are shown, where $S_{2}^{c}$ is denoted by dotted circles. The interference denoted by the dotted arrow is pre-calculated by overhearing probing signals in the previous selection step of the Tx-Rx pair 1. To calculate $I_{2}^{s, c}$ and $I_{2}^{s, r}$, the caused/received interference to/from Tx-Rx 2 selected at the second selection step is required to be measured.
generality, we denote the index of $K$-selected user pair as $1,2, \ldots, K$ for mathematical simplicity. After $K$ user pairs are selected at time $t$, the received signal at the selected $j$-th receiver is given as:

$$
\begin{equation*}
y_{j}[t]=\sqrt{\gamma_{j, j}} h_{j, j}[t] x_{j}[t]+\sum_{i=1, i \neq j}^{K} \sqrt{\gamma_{i, j}} h_{i, j}[t] x_{i}[t]+z_{j}[t] \tag{1}
\end{equation*}
$$

where $\sqrt{\gamma_{i, j}}\left(i, j \in S_{K}\right)$ represents the path-loss between the active $i$-th transmitter and $j$-th receiver. $\gamma_{i, j}$ is modeled by $\left(\frac{d}{d_{0}}\right)^{\alpha}$, where $d_{0}$ is a reference distance, $d$ is the distance between nodes, and $\alpha(>2)$ is the path-loss exponent. $h_{i, j}[t]$ indicates the fading channel between the active $i$-th transmitter and $j$-th receiver that is modeled by an independent and identically distributed (i.i.d.) complex Gaussian random variable with zero mean and unit variance. $z_{j}[t]$ is an i.i.d. additive complex Gaussian noise with zero mean and unit variance at the $j$-th receiver. $y_{j}[t]$ is the received signal at the $j$-th receiver. $x_{i}[t]$ represents the transmitted signal of the $i$-th transmitter. Each transmitter $i$ satisfies the average power constraint $\mathbb{E}\left[x_{i} x_{i}^{H}\right]=$ SNR, where SNR denotes the signal-to-noise ratio. For convenience, we will omit the time index $t$ in the following sections.

## III. Distributed, Opportunistic and Sequential User Scheduling

In this section, we propose a distributed user pair scheduling protocol to achieve maximum DoF in the $(n, K)$-user interference channel (i.e., $K$ ). In the proposed protocol, $K$ active user pairs are opportunistically and sequentially selected in a distributed manner by utilizing pilot signal (or reference signal) to estimate channel at each user pair. We assume that time duration for exchanging the pilot signal is short enough to be negligible compared to data-packet transmission time.

The first transmitter-receiver pair is randomly selected among $n$ user pairs and the index of this randomly selected
user pair belongs to $S_{1}$. This can be implemented by random number generation for determining contentional window size in each transmitter. We define the caused interference from an arbitrary transmitter $s$ in $S_{k}^{c}$ to $k$ selected receivers in the $k$-th selection step (i.e., after $k$ user pairs are selected) as

$$
\begin{equation*}
I_{k}^{s, c}=\sum_{j=1}^{k} \gamma_{s, j}\left|h_{s, j}\right|^{2} \tag{2}
\end{equation*}
$$

Similarly, we also define the received interference from $k$ selected transmitters to an arbitrary receiver $s$ in $S_{k}^{c}$ in the $k$-th selection step as

$$
\begin{equation*}
I_{k}^{s, r}=\sum_{i=1}^{k} \gamma_{i, s}\left|h_{i, s}\right|^{2} \tag{3}
\end{equation*}
$$

Note that the caused and received interferences are calculated in the second user pair selection step for the first time since the first user pair is assumed to be randomly selected without any consideration of interference. Hence, (2) and (3) can be used as a metric for the $(k+1)$-th user pair selection in our proposed scheduling protocol, where $1 \leq k \leq K-1$.

At each user pair selection step in the proposed protocol, all candidate user pairs in $S_{k}^{c}$ compute (2) and (3) and the user pair that both transmitter and receiver simultaneously satisfy a specific threshold condition is selected and added to the set $S_{k}$. The user selection procedure is described in detail below.

- Step 1: A randomly selected transmitter of the first user pair sends pilot signal to its designated receiver and the receiver sends back the pilot signal.
- Step 2: Each transmitter in $S_{1}^{c}$ calculates its causing interference $I_{1}^{s, c}$ to the receiver of the first user pair by overhearing the reference signal in Step 1 based on channel reciprocity. Similarly, each receiver in $S_{1}^{c}$ can also calculate the received interference $I_{1}^{s, r}$ from the first transmitter by overhearing the broadcast reference signal in Step 1.
- Step 3: Each receiver of user pair in $S_{1}^{c}$ examines whether its received interference is lower than a predefined threshold $\epsilon_{1}$. In other words, each receiver checks whether the threshold condition is satisfied or not. If a receiver satisfies the threshold condition, then it sets a contention window size according to the amount of the received interference by well-organized setting to avoid long waiting time for transmitting ${ }^{2}$. Similarly, each transmitter in $S_{1}^{c}$ also examines whether its causing interference to the receiver of the first user pair $I_{1}^{s, c}$ is lower than the threshold $\epsilon_{1}$.
- Step 4: According to the contention window size in Step 3, the receiver with the minimum contention window size sends an indicating signal bearing the information

[^1]whether the receiver satisfies the threshold condition or not, to its transmitter. If a transmitter receives the indicating signal from its receiver and it satisfies the threshold condition, it immediately sends back a probing signal to the receiver to notify that the user pair is selected as the second user pair and its index belongs to $S_{2}$. Then, the remaining candidate receivers which satisfy the threshold condition in $S_{1}^{c}$, stop the waiting process for sending the indicating signal. Note that the second user pair satisfies the threshold condition so that both the caused and the received interference are at most $\epsilon_{1}$. If the transmitter which receives the indicating signal from its receiver does not satisfy the threshold condition, then it does not send any signal (i.e., being silent) and the receiver with the second minimum contention window size sends the indicating signal to its transmitter. This process is repeated until both transmitter and receiver satisfy the threshold condition. If there is no selected user pair, an outage is declared, all nodes defer transmission until the next transmission time, and the protocol is reset. However, if $n$ satisfies certain scaling law, a user pair must be selected in each selection step, which will be shown in the next section.

- Step 5: The receiver of the selected second user pair in Step 4 broadcasts a reference signal.
- Step 6: Similar to Step 2, each transmitter and receiver in $S_{2}^{c}$ calculates $I_{2}^{s, c}$ and $I_{2}^{s, r}$, respectively.
- Step 7: Through the same feedback operation of the indicating signal as Step 4, the third user pair is selected and then its index belongs to $S_{3}$.
- Step 8: The same user selection processes are repeated until $K$ user pairs are selected. Then, the $K$ user pairs transmit their data packet simultaneously.
Note that this user selection is opportunistic, sequential and distributed. It is noteworthy that, since each user pair is selected sequentially, the pre-calculated values of the interference $I_{k-1}^{s, c}$ and $I_{k-1}^{s, r}$ in the selection of the $k$-th user pair can be reused to calculate the $I_{k}^{s, c}$ and $I_{k}^{s, r}$ in the selection of the $(k+1)$-th user pair. Therefore, the received interference value from the $k$-th selected user pair and the causing interference value to the $k$-th selected user pair are only required to be calculated, i.e., $I_{k}^{s, c}$ and $I_{k}^{s, r}$. To help understand, see Fig. 1.


## IV. Achievable Degrees-of-Freedom

In this section, we analyze the achievable DoF of the proposed user scheduling protocol. It is proven that the required number of user pairs in a network is sufficient to be scaled as $\omega\left(\mathrm{SNR}^{K(K-1)}\right)$ to achieve the maximum number of DoF $K$. If $K$ user pairs are selected by the proposed protocol in Section III, then the total achievable sum rate of $K$-user SISO network is given as

$$
\begin{equation*}
\sum_{j=1}^{K} R_{j}=\sum_{j=1}^{K} \log \left(1+\frac{\gamma_{j, j}\left|h_{j, j}\right|^{2} \mathrm{SNR}}{1+\sum_{i=1, i \neq j}^{K} \gamma_{i, j}\left|h_{i, j}\right|^{2} \mathrm{SNR}}\right) \tag{4}
\end{equation*}
$$

The achievable DoF by the proposed protocol in the $(n, K)$ user interference channel is given by

$$
\begin{equation*}
\sum_{j=1}^{n} d_{j}=\lim _{\mathrm{SNR} \rightarrow \infty} \frac{\sum_{j=1}^{K} R_{j}}{\log (\mathrm{SNR})} \tag{5}
\end{equation*}
$$

where $d_{j}$ is achievable DoF at the $j$-th user pair. We also define the caused and the received interference from the $s\left(\in S_{k}^{C}\right)$-th user pair without consideration of the path-loss term in (2) and (3), respectively, as $J_{k}^{s, c}=\sum_{j=1}^{k}\left|h_{s, j}\right|^{2}$ and $J_{k}^{s, r}=\sum_{i=1}^{k}\left|h_{i, s}\right|^{2}$, where $1 \leq k \leq K-1$. Note that $h_{i, s}$ has complex Gaussian elements with zero-mean and unit-variance. Therefore, $J_{k}^{s, c}$ and $J_{k}^{s, r}$ have the chi-square distributions with $2 k$ degrees of freedom for each $k \in\{1,2, \cdots, K-1\}$. Since $J_{k}^{s, c}$ and $J_{k}^{s, r}$ have the same distribution, if we represents $J_{k}^{s, c}$ and $J_{k}^{s, r}$ as a unified random variable $J_{k}^{s}$ for simplification, then the cumulative distribution function (CDF) of $J_{k}^{s}$ is given by

$$
\begin{equation*}
F_{J_{k}^{s}}\left(l_{k}\right)=\frac{\gamma\left(k, l_{k} / 2\right)}{\Gamma(k)} \tag{6}
\end{equation*}
$$

where $\Gamma(z)=\int_{0}^{\infty} t^{z-1} e^{-t} d t$ is the Gamma function and $\gamma(z, x)=\int_{0}^{x} t^{z-1} e^{-t} d t$ is the lower complete Gamma function.

Lemma 1 ( [7]). When $J_{k}^{s}$ has chi-square distribution with $2 a$ degrees of freedom, the CDF of $J_{k}^{s}$ is given by $F_{J_{k}^{s}}\left(l_{k}\right)=$ $\frac{\gamma\left(a, l_{k} / 2\right)}{\Gamma(a)}$. For any $0 \leq l_{k}<2$, the $C D F F_{J_{k}^{s}}$ of $J_{k}^{s}$ is lowerand upper-bounded by

$$
\begin{equation*}
C_{k}^{1}\left(l_{k}\right)^{a} \leq F_{J_{k}^{s}}\left(l_{k}\right) \leq C_{k}^{2}\left(l_{k}\right)^{a} \tag{7}
\end{equation*}
$$

where $C_{k}^{1}=\frac{2^{-a}}{a \Gamma(a)} e^{-\frac{l_{k}}{2}}$ and $C_{k}^{2}=\frac{2^{-a}}{a \Gamma(a)}\left(1+\frac{\frac{l_{k}}{2}}{a+1-\frac{l_{k}}{2}}\right)$.
Theorem 1. It is assumed that $K$ user pairs are selected by the proposed opportunistic and sequential user scheduling protocol in ( $n, K$ )-user interference channel where single antenna is equipped at each receivers. Then, $\sum_{j=1}^{n} d_{j}=K$ is achievable if $n=\omega\left(\operatorname{SNR}^{K(K-1)}\right)$.
Proof. From (4), our proposed protocol achieves the maximum number of DoF $K$, if the interference

$$
\begin{equation*}
\sum_{i=1, i \neq j}^{K} \gamma_{i, j}\left|h_{i, j}\right|^{2} \mathrm{SNR} \tag{8}
\end{equation*}
$$

has a finite value $\epsilon>0$ which is independent of SNR for given all $1 \leq j \leq K$. The number of the achievable DoF can be written as

$$
\begin{equation*}
\sum_{j=1}^{n} d_{j}=\mathrm{P}_{\mathrm{prop}} \cdot K \tag{9}
\end{equation*}
$$

where $\mathrm{P}_{\text {prop }}=$
$\lim _{\mathrm{SNR} \rightarrow \infty} \operatorname{Pr}\left\{\sum_{i=1, i \neq j}^{K} \gamma_{i, j}\left|h_{i, j}\right|^{2} \mathrm{SNR} \leq \epsilon\right.$ for all $\left.j \in\{1, \cdots, K\}\right\}$.

The probability $\mathrm{P}_{\text {prop }}$ in (10) is lower bounded by

$$
\begin{align*}
& \geq \lim _{\mathrm{SNR} \rightarrow \infty} \operatorname{Pr}\left\{\sum_{j=1}^{K} \sum_{i=1, i \neq j}^{K} \gamma_{i, j}\left|h_{i, j}\right|^{2} \mathrm{SNR} \leq \epsilon\right\}  \tag{11}\\
& \geq \lim _{\mathrm{SNR} \rightarrow \infty} \operatorname{Pr}\left\{\sum_{j=1}^{K} \sum_{i=1, i \neq j}^{K} \gamma_{\max }\left|h_{i, j}\right|^{2} \mathrm{SNR} \leq \epsilon\right\}  \tag{12}\\
& \geq \lim _{\mathrm{SNR} \rightarrow \infty} \operatorname{Pr}\left\{\sum_{j=1}^{K} \sum_{i=1, i \neq j}^{K}\left|h_{i, j}\right|^{2} \leq \epsilon \mathrm{SNR}^{-1}\right\}  \tag{13}\\
& \geq \lim _{\mathrm{SNR} \rightarrow \infty} \prod_{k=1}^{K-1} \operatorname{Pr}\left\{\left(\sum_{j=1}^{k}\left|h_{k+1, j}\right|^{2}+\sum_{i=1}^{k}\left|h_{i, k+1}\right|^{2}\right) \leq \frac{2 \epsilon_{k}}{\mathrm{SNR}}\right\}  \tag{14}\\
& \geq \lim _{\mathrm{SNR} \rightarrow \infty} \prod_{k=1}^{K-1} \operatorname{Pr}\left\{\sum_{j=1}^{k}\left|h_{k+1, j}\right|^{2} \leq \frac{\epsilon_{k}}{\mathrm{SNR}}, \sum_{i=1}^{k}\left|h_{i, k+1}\right|^{2} \leq \frac{\epsilon_{k}}{\mathrm{SNR}}\right\} \tag{15}
\end{align*}
$$

where $\gamma_{\text {max }}=\max _{i, j \in\{1, \cdots, K\}, i \neq j} \gamma_{i, j}$ and $\epsilon=2\left(\epsilon_{1}+\epsilon_{2}+\right.$ $\left.\cdots+\epsilon_{K-1}\right)$.

The inequality (11) holds because the constraint (10) that each antenna at the selected users has the interference lower than $\epsilon$ is relaxed into the constraint that the sum of interference of all selected users is lower than $\epsilon$ (i.e., $\operatorname{Pr}\{A \leq \epsilon\} \operatorname{Pr}\{B \leq$ $\epsilon\} \geq \operatorname{Pr}\{A+B \leq \epsilon\})$. The inequality (13) comes from the fact that each path-loss term is less than 1 . Note that each probability term in the last inequality corresponds to each step of user pair selection in our proposed scheduling protocol.

Each probability term in (15) is equivalent to the probability that at least one transmitter-receiver pair satisfies the threshold condition in each user selection step. Therefore, the probability in (15) is rewritten by

$$
\begin{align*}
& \lim _{\mathrm{SNR} \rightarrow \infty} \prod_{k=1}^{K-1}\left[1-\left\{1-\left(F_{J_{k}^{s}}\left(\epsilon_{k} \mathrm{SNR}^{-1}\right)\right)^{2}\right\}^{n-k}\right]  \tag{16}\\
& \stackrel{(a)}{\geq} \lim _{\mathrm{SNR} \rightarrow \infty} \prod_{k=1}^{K-1}\left[1-\left\{1-\left(\bar{C}_{k}^{1}\right)^{2} \mathrm{SNR}^{-2 k}\right\}^{n-k}\right] \tag{17}
\end{align*}
$$

where $\bar{C}_{k}^{1}=C_{k}^{1} \epsilon_{k}{ }^{k}$ and $C_{k}^{1}=\frac{2^{-k}}{k \Gamma(k)} e^{-\epsilon_{k} \mathrm{SNR}^{-1} / 2}$ for $1 \leq k \leq$ $K-1$. Since the $J_{k}^{s}$ has chi-square distribution with $2 k$ degrees of freedom, the inequality (a) is easily derived by using the lowerbound in Lemma 1 . Since $\bar{C}_{k}^{1}$ is considered as a constant when SNR goes to infinity, $1-\left\{1-\left(\bar{C}_{k}^{1}\right)^{2} \mathrm{SNR}^{-2 k}\right\}^{n-k}$ goes to 0 as SNR goes to infinity for a finite value of $n$. Consequently, the right hand side of the inequality (a) goes to zero for a finite value of $n$.

We now show that if the total number of transmitter-receiver pairs $n$ scales at least as $\omega\left(\mathrm{SNR}^{2 k}\right)$ for each $k, \mathrm{P}_{\text {prop }}$ goes to 1 . To make each probability in (17) be one implicates that at least one transmitter-receiver pair surely satisfies the threshold conditions at both the transmitter and the receiver in each user pair selection step and one transmitter-receiver pair is surely selected in each selection step by our proposed scheduling protocol. For any constant $c>0$, it is easily derived by using the relation of $\lim _{x \rightarrow \infty}\left(1-\frac{c}{x}\right)^{x}=\frac{1}{e^{c}}$ that if the
total number of transmitter-receiver pairs $n$ grows as fast as $\omega\left(\mathrm{SNR}^{2 k}\right)$, then $\lim _{\mathrm{SNR} \rightarrow \infty}\left\{1-\left(\bar{C}_{k}^{1}\right)^{2} \mathrm{SNR}^{-2 k}\right\}^{n-k}$ goes to 0 . Consequently, each probability term in (17) can be 1 if the required transmitter-receiver pairs scale at least as $\omega\left(\mathrm{SNR}^{2 k}\right)$ for $1 \leq k \leq K-1$. Since the channel coefficients are mutually independent, the scaling law of the required number of transmitter-receiver pairs to achieve the maximum number of DoF $K$ is obtained by $n=\omega\left(\operatorname{SNR}^{2+4+\cdots+2(K-1)}\right)=$ $\omega\left(\mathrm{SNR}^{K(K-1)}\right)$.

Note that even though user pairs are selected in a distributed manner via the proposed protocol, the resulting user scaling law is the same as that of [6] which is based on centralized scheduling.

## V. Feedback Overheads

In the previous section, we analyzed the achievable DoF of the proposed protocol without consideration of signaling overheads (i.e., feedback of the indicating signal). If the codeword length of user data is long enough compared to the signaling message length, the signaling overheads can be negligible in DoF analysis. In this section, to understand how relatively long should the codeword length be compared to the feedback overheads, we investigate the total amount of feedback overheads in the proposed protocol. We also quantify the burden of feedback overhead on each user.

Theorem 2. When the entire $K$ user pairs are selected via the proposed scheduling protocol which uses the finite threshold value $\epsilon_{k}$ independent of SNR for $1 \leq k \leq K-1$, the worst case feedback overheads are upper-bounded as

$$
\begin{equation*}
\sum_{k=1}^{K-1} F_{k} \leq \sum_{k=1}^{K-1}(n-k) \frac{2^{-k}}{k \Gamma(k)}\left(1+\frac{\tilde{l}_{k} / 2}{k+1-\tilde{l}_{k} / 2}\right)\left(\tilde{l}_{k}\right)^{k} \tag{18}
\end{equation*}
$$

and the scaling of feedback in the high SNR is written as

$$
\begin{equation*}
\sum_{k=1}^{K-1} F_{k} \leq \frac{n}{\mathrm{SNR}} \tag{19}
\end{equation*}
$$

where $\tilde{l}_{k}=\tilde{\epsilon}_{k} \mathrm{SNR}^{-1}, \tilde{\epsilon}_{k}=\epsilon_{k} / \gamma_{\min }$, and $\gamma_{\min }=$ $\min _{1 \leq i \leq k, s \in S_{k}^{c}} \gamma_{i, s}$.

Proof. The detailed proof of this theorem is omitted due to page limit.

Note that (18) shows the number of feedback occurrence in the proposed protocol for the worst case. If we assume that each feedback of indicating signal consumes one time slot, the number of feedback occurrence can be interpreted as total consumed time slots for feedback. However, if data transmission time is enough longer than the total consumed time for feedback, then DoF loss can be negligible.
Note also that the scaled amount of total feedback overheads in the proposed protocol in the high SNR region where the network becomes strongly interference-limited is given by (19) for the worst case scenario. However, the typical amount of


Fig. 2. Comparison of leakage interference among proposed scheduling, centralized scheduling [6], and random scheduling for both pathloss and fixed pathloss $=1$


Fig. 3. The burden of feedback overhead per user for varying $K$ and $\rho$ are shown.
total feedback overheads might be much less than (19) since all remaining receivers do not send the indicating signal in the sequential feedback mechanism once a transmitter-receiver pair is selected. In the optimistic scenario, the total feedback overheads are only $K-1$ time slots since only 1 time slot is required in each user selection step.

The overhead burden per user can be quantified by dividing the total feedback overheads in the network by the required number of users for achieving the maximum DoF K. From (19), the worst case feedback overhead per user is given by $\rho=\frac{\frac{n}{\mathrm{SNR}}}{n}=\frac{\left\lceil\mathrm{SNR}^{\left(K^{2}-K-1\right)}\right\rceil}{\left[\mathrm{SNR}^{K(K-1)}\right]} \approx \frac{1}{\mathrm{SNR}}$ when SNR is high. Note that as the network becomes more interference limited, the feedback overhead per user becomes smaller.

## VI. Numerical results

In this section, we compare the performance of the proposed protocol with that of centralized scheduling [6]. The
centralized scheduling selects the best user group which has the largest number of DoF once after the achievable DoFs of each group consisting of $K$ user pairs are computed.

Fig. 2 shows that leakage interference amount of the proposed scheduling, centralized scheduling, and random scheduling for both random pathloss and fixed pathloss $(=1)$. As $n$ increases, the proposed scheduling more efficiently controls the leakage interference than the centralized scheduling and random scheduling methods. This is because our proposed scheduling method selects the user pair causing/receiving interference to/from the previously selected user pairs below a certain level.
Fig. 3 shows the burden of feedback overhead on each user for varying $K$. As SNR increases, the worst case feedback overhead burden per user behaves like $\rho=\frac{1}{\mathrm{SNR}}$. The result shows that feedback overhead incurred from the second user pair selection is dominant over those in other user pair selections. While the sum of interference at each user selection step increases, the threshold condition remains as a small constant value and thus the number of feedback occurrence reduces.

## VII. Conclusion

In this paper, we proposed a distributed scheduling protocol to achieve the maximum DoF $K$ in an $(n, K)$-user interference network. It was proven that the maximum $\operatorname{DoF} K$ is achievable through the proposed scheduling method if total user pairs $n$ scales at least as $\omega\left(\mathrm{SNR}^{K(K-1)}\right)$. We analyzed the required total feedback amount of the proposed protocol and showed that the feedback overhead per user becomes negligible as the network becomes more interference limited.

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[^0]:    ${ }^{1}$ As in standard notation, for any two real-valued functions $f$ and $g$, we write $f(n)=\omega(g(n))$ if $\lim _{n \rightarrow \infty} \frac{f(n)}{g(n)}=\infty$.

[^1]:    ${ }^{2}$ The collision and resultant latency are not considered in the achievable DoF analysis because the data transmission time for a codeword is assumed to be long enough to neglect the contention time. Furthermore, the contention widow size determined by both pathloss and channel gain might well resolve contention and result in smaller collision probability compared to conventional WLAN.

