Practical Signal Models for Orthogonal Code Hopping Multiplexing Systems

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Abstract. In this paper, we propose three received signal models in orthogonal code hopping multiplexing (OCHM) systems for exactly evaluating performance. In the conventional studies, *synergy* effect in OCHM was not considered even though it can improve the system performance. We investigate the synergy effect on the system performance through the multi-user link level simulation for each case.

Keywords: OCHM, synergy, perforation, code-collision, frame error rate

1 Introduction

An orthogonal code hopping multiplexing (OCHM) system has been proposed to accommodate more mobile station (MS) with bursty traffic than the number of orthogonal codewords in downlink [1]. It utilizes statistical multiplexing for orthogonal downlink in DS/CDMA systems. When a hopping pattern collision among MSs occurs in the conventional FH-CDMA systems, it is considered as an inevitable interference (*hit*) in case that all MUs are asynchronous with one another [2]. However, the code collision can be detected and controlled by BS in a synchronous downlink environment. The conventional code-collision control scheme did not consider path loss due to the distance from BS. Furthermore, in previous work on OCHM, the authors have not evaluated the synergy effects on performance through the multiuser link level simulations [1], [3]-[6]. In this paper, we introduce three receive signal models for realistic performance evaluations and we compare their performances.

2 Conventional OCHM System with Synergy and Perforation

In this section, we first review the conventional OCHM system with synergy and perforation. Fig. 1 shows the block diagram of the conventional OCHM system deploying the synergy (positive collision case) and perforation (negative collision

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case) schemes. Furthermore, both the transmitted and received power levels for the specific MS are shown. T_S stands for the symbol time and each user changes the orthogonal codeword (OC) according to HP at each symbol time, which may cause a code collision. However, most of users may be inactive because of low channel activities when they demand the data services. MS *b* and MS *d* are inactive in Fig. 1, but they follow their HPs during a session. In this case, code collisions between the active MSs and inactive MSs do not affect the performance of the active MSs. The shaded parts in Fig. 1 indicate this type of collision.

When an code collision among the active users occurs, a BS compares MSs' data experiencing code collision and determines whether all MSs' data with the same code collision are the same or not. If all the corresponding data are the same, the collision does not need to be controlled, which results in an energy gain at the receiver. On the contrary, if all data with the same code collision are not the same, all the corresponding data symbols are not transmitted (perforated) during the symbol time. For example, MS e experiences a synergy at (n + 2)T_S and a perforation at (n + 4)T_S.

When an MS experiences a synergy, BS allocates power for the MS without any changes, and then, the all symbols experiencing a synergy have an additional energy at the receivers because of other MSs' energy added by despreading process using the same OC. The quantity of the additional energy is determined by the distances between a BS and code-collision MSs in case that the OCHM system utilizes a power control scheme. In Fig. 1, if MS *e* is at a cell boundary and MS *c* is located near a BS, the additionally received power at MS *e* is much smaller than the normalized received power which is the power when a code collision does not occur. On the contrary, the additionally received power at MS *c* is much bigger than the normalized received power in case of a code collision. The synergy scheme results in an energy gain. However, its effect varies according to the location of MSs in a cell. Statistically, the MSs near BS have a more energy gain than those at the cell boundary. Therefore, in OCHM system, the energy gain at the receiver due to the synergy scheme is complex to analyze even though many previous papers assumed the all code-collision MSs located in the same distance from BS in their performance evaluations [1], [3]–[6].

When a user experiences a perforation, BS does not allocate power for the perforated symbol and MS detects only noise at the perforated symbol time. Hence, the perforation degrades the performance and additional energy may be required to transmit for a target frame error rate (FER).

The code collision probability of the OCHM system is expressed as:

$$P_c = 1 - \left(1 - \frac{\bar{\upsilon}}{N_{OC}}\right)^{K-1} \quad \textbf{(1)}$$

where $\bar{\upsilon}$ is the channel activity N_{OC} is the number of orthogonal codewords, and K is the number of active users in a cell. For a given channel activity $\bar{\upsilon}$, P_C increases as the number of active users increases. The perforation probability of encoded symbols in the conventional OCHM systems is written as:

$$P_p = 1 - \left(1 - \frac{m-1}{m} \cdot \frac{\bar{\upsilon}}{N_{OC}}\right)^{K-1}, \quad \textbf{(2)}$$

where *s* is the number of symbol locations in data modulation (i.e., *m* = 2 for BPSK). Hence, the synergy probability is given as:

$$P_s = P_c - P_p . \tag{3}$$

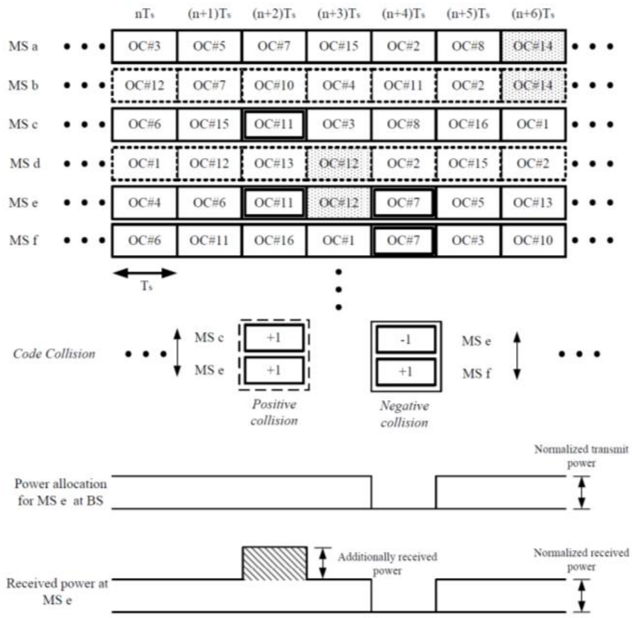

Fig. 1. Block diagram of the conventional OCHM system.

3 Proposed Signal Models

3.1 Synergy and Perforation Model (SPM)

For analysis of SPM, we need to consider the distance between BS and user as we noted before because it determines the quantity of additionally received power due to synergy at the receiver. Furthermore, a synergy between two MSs is only considered, which is a reasonable assumption since most synergies are occur between two MSs. The MSs are assumed to be uniformly distributed in a cell. Fig. 2 shows a cell layout in mobile communication systems including the OCHM system. We fix the distance between a target MU and the BS to h($0 \le h \le R$) as shown in Fig. 2. The cell radius is assumed to be R. If the synergy occurs at the receiver of an MS, the received symbol energy E_{syn} is given by

$$E_{syn} = E_s + E_s \cdot \left(\frac{r}{h}\right)^{\alpha}, \tag{4}$$

where E_S indicates the symbol energy when the code collision does not occur and r denotes the distance between the MS that induces the synergy with the target MS and the BS, respectively. In addition, α is the path-loss exponent and we assume that the system operates with a perfect power control scheme. We here neglect the fading effect. The last term of Eq. (4) represents the additionally received power due to the synergy. From Eq. (4), the probability distribution function of the received symbol energy with a synergy is expressed as:

$$F_{E_{syn}}(s) = P\{E_{syn} \le s\} = P\left\{E_s\left[1 + \left(\frac{r}{h}\right)^{\alpha}\right] \le s\right\} \tag{5}$$

where $E_s \le s \le E_s(1 + (R/h)^{\alpha})$ since $0 \le r \le R$. After basic manipulations, Eq.(5) is rewritten as:

$$F_{E_{syn}}(s) = P\left\{r \le h\left(\frac{s}{E_s} - 1\right)^{1/\alpha}\right\} \tag{6}$$

$$= \int_0^{h \cdot \sqrt[\alpha]{s/E_s - 1}} \frac{2r}{R^2} dr \tag{7}$$

$$= \frac{h^2}{R^2}(s-1)^{2/\alpha}, \tag{8}$$

where R denotes the cell radius. Hence, the received symbol energy with a synergy is determined by the distance h between an MS and a BS. If h decreases, the probability that the received symbol energy with a synergy is bigger than a given value of energy increases. The probability density function (PDF) of E_{syn} is expressed as:

$$f_{E_{syn}}(s) = \frac{2h^2}{\alpha R^2}(s-1)^{(2-\alpha)/\alpha}. \tag{9}$$

In SPM, the received signal model of the binary phase shift keying (BPSK) symbol in AWGN channel is expressed as:

$$Y = \begin{cases} t_1 \sim N\left(0, \sigma^2\right), & if\ perforation \\ t_2 \sim N\left(\sqrt{E_{syn}}, \sigma^2\right), & if\ synergy \\ t_3 \sim N\left(\sqrt{E_s}, \sigma^2\right), & otherwise \end{cases}. \tag{10}$$

where the transmitted symbol is assumed to have positive sign with a symbol energy of E_S. In addition, $x \sim N(\mu, \sigma^2)$ represents that x is a Gaussian random variable with mean μ and variance σ^2. For a given value of E_{syn} and the distance h between MS and BS, the conditional PDF of a received signal is written as:

$$f_Y(y \mid E_{syn}=s,h) = P_p \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2} + P_s \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\sqrt{s})^2/2\sigma^2}$$
$$+(1-P_c)\cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\sqrt{E_s})^2/2\sigma^2}. \tag{11}$$

where $P_S = P_C - P_P$. Hence the PDF conditioned by h is expressed as:

$$\begin{aligned} f_Y(y \mid h) &= \int_{E_s}^{E_s(1+(R/h)^{\alpha})} f_Y(y \mid E_{syn}=s)\, f_{E_{syn}}(s)\, ds \\ &\quad + P_p \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2} + (1-P_c)\cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\sqrt{E_s})^2/2\sigma^2} \\ &\quad + \frac{P_s}{\sqrt{2\pi\sigma^2}} \int_{E_s}^{E_s(1+(R/h)^{\alpha})} e^{-(y-\sqrt{s})^2/2\sigma^2}\, f_{E_{syn}}(s)\, ds, \end{aligned} \tag{12}$$

where $f_{E_{syn}}(s)$ is shown in Eq. (9). Eq. (12) shows the PDF of the received signal in SPM and it varies according to the distance h between the target MS and the BS. Hence, for a given collision probability, the system performance including BER performance also varies according to h. Therefore, the synergy effect on the system performance varies depending on not only the synergy probability (P_S) but also the distance (h). Furthermore, Eq. (12) does not have a closed-form solution and is not tractable.

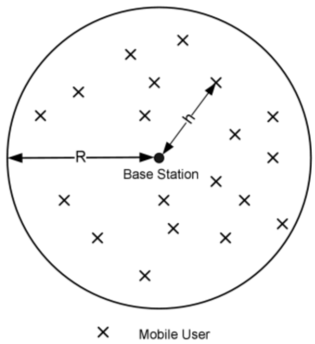

Fig. 2. Cell layout of the system

3.2 Perforation Only Model (POM)

In this model, we assume the additionally received power is set to zero. Hence, this signal model provides an upper bound of the BER performance of OCHM systems. The received signal model of the BPSK symbol in AWGN channel is expressed as:

$$Y = \begin{cases} t_1 \sim N\left(0, \sigma^2\right), & \text{if perforation} \\ t_2 \sim N\left(\sqrt{E_s}, \sigma^2\right), & \text{otherwise.} \end{cases} \tag{13}$$

In Eq. (13), we also assume that a positive symbol is transmitted and its symbol energy is E_S. We call this model a perforation only model (POM) because it considers a perforation effect when code collisions occur. The distribution function of POM is obtained as

$$F_Y(y) = G\left(\frac{y}{\sigma}\right) \cdot P_p + G\left(\frac{y - \sqrt{E_s}}{\sigma}\right) \cdot (1 - P_p) \tag{14}$$

where

$$G\left(\frac{x-\mu}{\sigma}\right) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} dx.$$

Therefore, PDF of the received signal in OCHM systems is given as

$$f_Y(y) = P_p \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2} + (1 - P_p) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\sqrt{E_s})^2/2\sigma^2}. \tag{15}$$

where the received signal follows the POM. As noted before, POM provides the lower bound of the performance of OCHM systems. Furthermore, POM maintains the consistence according to the distance between the MS and the BS since the perforation effect is not dependent on the relative distance.

3.3 Simplified Synergy and Perforation Model (S-SPM)

In S-SPM, we assume that the synergy occurs between two MSs with the same distance from BS. Previous works on OCHM performed their performance evaluations using S-SPM [1], [3]–[6]. Thus, the received signal model of the BPSK symbol in AWGN channel is expressed as:

$$Y = \begin{cases} t_1 \sim N\left(0, \sigma^2\right), & \text{if perforation} \\ t_2 \sim N\left(\sqrt{2E_s}, \sigma^2\right), & \text{if synergy} \\ t_3 \sim N\left(\sqrt{E_s}, \sigma^2\right), & \text{otherwise.} \end{cases} \tag{16}$$

The distribution function of S-SPM is obtained as:

$$F_Y(y) = G\left(\frac{y}{\sigma}\right) \cdot P_p + G\left(\frac{y - \sqrt{2E_s}}{\sigma}\right) \cdot P_s + G\left(\frac{y - \sqrt{E_s}}{\sigma}\right) \cdot (1 - P_c) \cdot \tag{17}$$

Therefore, PDF of the received signal in OCHM systems is given as

$$f_Y(y) = P_p \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2} + P_p \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\sqrt{2E_s})^2/2\sigma^2}$$

$$+ (1-P_c) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\sqrt{E_s})^2/2\sigma^2}, \tag{18}$$

where the received signal follows the S-SPM. S-SPM provides performance with synergy even though it is simplified. Furthermore, S-SPM model yields the identical BER or FER performances regardless the distance between BS and a MS.

4 Simulation Result

In this section, we perform link-level simulations to evaluate the FER performance of the proposed signal models. Simulation result can be applied to QPSK because QPSK can be characterized as two orthogonal BPSK channels. Simulation parameters are described as follows:

- Data Modulation: BPSK
- Wireless channel: AWGN
- Length of a frame : 1024bits
- Code rate: 1/3
- Channel coding: Turbo codes [7]
- Decoding algorithm : Max-Log-MAP [8]
- Number of iterations : 8
- Path-loss exponent (α): 4

We compare the FER performances of POM and S-SPM in Fig. 3. As noted before, POM provides the lower bound of FER performance in OCHM system since it does not consider the synergy effect and S-SPM provides the approximate performance of OCHM system. In both models, the FER performance degrades as the coding collision probability increases. S-SPM requires 2.02dB of the received E_b/N_o at MS when the code-collision probability is equal to 0.4, while POM requires 3.35dB. In real systems, synergy effect is determined by the distance between BS and a specific MS and S-SPM does not yield exact performances of MSs those who are located at various distances from BS.

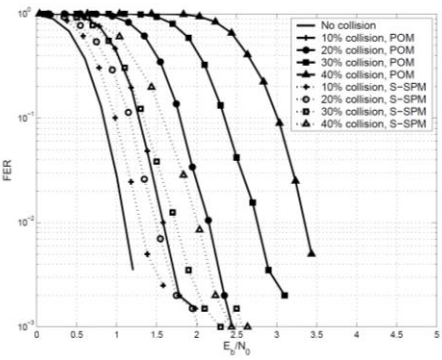

Fig. 3. FER performance comparison of POM and S-SPM

6 Conclusions

We introduced three received signal models used in OCHM systems, SPM, POM, S-SPM. POM provides a lower bound of BER or FER performance since it does not consider the additionally added energy at receiver due to synergy effect. S-SPM yields a simplified performance considering both synergy and perforation at receiver although it assumes that all MSs are located in the same distance from BS.

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