A Message Passing Algorithm for Compressed Sensing in Wireless Random Access Networks

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***Abstract*—In this paper, we consider the joint multiuser detection and channel estimation problem in wireless random access networks which consists of a large number of users with a single antenna and a base station (BS) multiple antennas. We first convert the problem into the compressed sensing framework and then propose a message passing (MP) algorithm which reduces the implementation complexity at BS. Simulation results show that the performance of the proposed MP algorithm becomes improved as the number of antennas at BS.**

***Index Terms*—Random access networks, compressed sensing, message passing algorithm, multiple measurement vector problem, channel estimation, multiuser detection**

I. Introduction

There exist two technical issues for implementing multi-packet reception (MPR) in practical wireless random access networks (RANs): identification of active users and channel estimation of them [1]–[3]. Although many studies on MPR have ignored these challenges, they must be seriously considered to implement MPR in practice. By the way, compressed sensing (CS) has been known as a revolutionary technique for efficiently acquiring and reconstructing a *sparse* signal by finding solutions of underdetermined linear systems [4], [5]. The problem of identifying active users and estimating their channels in the RAN can be regarded as the sparsity pattern detection and the sparse signal recovery in the CS framework, respectively [6]. Recently, message passing algorithms for a sparse signal recovery in CS framework have been proposed for reducing the implementation complexity. Especially, the approximate message passing (AMP) proposed in [7] has been received much attention because it yields the optimal performance in terms of the sparsity-undersampling trade-off, and Rangan generalized the AMP algorithm in [8], which is called generalized AMP (GAMP).

In this paper, we proposed an MP algorithm for wireless random access network in which modifies the conventional GAMP algorithm for jointly peforming the active user detection among users in the network and the channel estimation of the active users when multiple antennas exist at base station (BS). We utilize the *joint sparsity* in multiple measurement vectors (MMV) in CS framework for designing the MP algorithm for the case of multiple antennas at BS.

II. System Model

Consider the wireless RANs where there exist N users sharing a wireless channel to a single BS. The n-th user is assigned a unique and dedicated codeword represented as $\mathbf{a}_n \in \mathbb{R}^M$, $n = 1, 2, ..., N$, where M denotes the degrees-of-freedom of the codeword, i.e., the length of the codeword. The BS is assumed to have L antennas and let I_K be the *active user* set with $|I_K| = K$ [1]. The received signal at the l-th antenna at the BS, $\mathbf{y}^{(l)} \in \mathbb{C}^M$, is given as:

$$\mathbf{y}^{(l)} = \sum_{i=1}^{N} \sqrt{P_i} h_{i,l} \mathbf{a}_i + \mathbf{w}_i, \tag{1}$$

where P_i denotes the transmit power of the i-th user ($P_i > 0$ if $i \in I_K$) and $h_{i,l} \in \mathbb{C}$ denotes the channel coefficient from the i-th user to the l-th antenna at the BS, which is assumed to follow as an i.i.d. complex Gaussian distribution with zero mean and unit variance, i.e., $h_{i,l} \sim \mathcal{CN}(0, 1)$. $\mathbf{w}_l \in \mathbb{C}^M$ represents the noise vector in the l-th antenna at the BS, which is assumed that $\mathbf{w}_l \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_M)$. Then, (1) can be converted in the sparse signal recovery problem with MMV in CS framework with the measurement matrix $\mathbf{A}$ and $2L$ observation vectors consisting of real and imagenary parts in $\mathbf{y}^{(l)}$ ($\mathbf{y}_R^{(l)}$ and $\mathbf{y}_I^{(l)}$), $l = 1, 2, ..., L$.

$$\mathbf{y}_R^{(l)} = \Re(\mathbf{y}^{(l)}) = [\mathbf{a}_1 \cdots \mathbf{a}_N] \begin{bmatrix} \sqrt{P_1}\Re(h_{1,l}) \\ \vdots \\ \sqrt{P_N}\Re(h_{N,l}) \end{bmatrix} + \Re(\mathbf{w}^{(l)}) \tag{2}$$

$$= \mathbf{A} \begin{bmatrix} x_{1,R}^{(l)} \\ \vdots \\ x_{N,R}^{(l)} \end{bmatrix} + \mathbf{w}_R^{(l)} = \mathbf{A}\mathbf{x}_R^{(l)} + \mathbf{w}_R^{(l)}, l = 1, \cdots, L \tag{3}$$

$$\mathbf{y}_I^{(l)} = \Im(y^{(l)}) = [\mathbf{a}_1 \cdots \mathbf{a}_N] \begin{bmatrix} \sqrt{P_1}\Im(h_{1,l}) \\ \vdots \\ \sqrt{P_N}\Im(h_{N,l}) \end{bmatrix} + \Im(\ \mathbf{w}^{(l)}) \tag{4}$$

$$= \mathbf{A} \begin{bmatrix} x_{1,I}^{(l)} \\ \vdots \\ x_{N,R}^{(l)} \end{bmatrix} + \mathbf{w}_I^{(l)} = \mathbf{A}\mathbf{x}_I^{(l)} + \mathbf{w}_I^{(l)}, l = 1, \cdots, L \tag{5}$$

Let the probability that the i-th user is active be p and let $b_i \in \{0, 1\}$ be the random variable indicating the user activity, which is expressed as $b_i \sim Bern(p)$. Then, we have $\mathsf{Pr}\left(x_{i,R}^{(1)}, x_{i,I}^{(1)}, \cdots, x_{i,R}^{(L)}, x_{i,I}^{(L)} | b_i\right) = \prod_{l=1}^{L} \mathsf{Pr}\left(x_{i,R}^{(l)} | b_i\right) \mathsf{Pr}\left(x_{i,I}^{(l)} | b_i\right)$. If the BS estimates b_i and $\mathbf{x}_R^{(l)}, \mathbf{x}_I^{(l)}$ from $\mathbf{y}_R^{(l)}$, $\mathbf{y}_I^{(l)}$ for a given P_i,

[1]The active user indicates the user who send its data packet in a specific time-slot and the probability that a user is active is assumed to be known to the BS in this paper. We also assume that relatively small number of users among all users transmit their data.

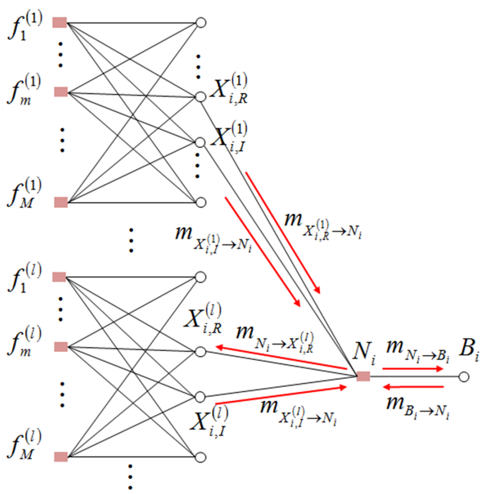

Fig. 1. Factor graph model for the proposed message passing algorithm

then the active users and the corresponding channels can be estimated, respectively.

III. Proposed Message Passing Algorithm

Fig. 1 shows the factor graph for designing the MP algorithm, which is similar to GAMP in [8]. In the GAMP, the output nodes provide both estimates $(\widehat{r^{(l)}_{i,R}}, \widehat{r^{(l)}_{i,I}})$ and variances $(\mu^{\tau,(l)}_{i,R}, \mu^{\tau,(l)}_{i,I})$ on the random variables $(\widehat{R^{(l)}_{i,R}}, \widehat{R^{(l)}_{i,I}})$ to the input nodes $X^{(l)}_{i,R}, X^{(l)}_{i,I}$, where $\widehat{R^{(l)}_{i,E}}=X^{(l)}_{i,E}+V^{(l)}_{i,E}, V\sim N\left(0,\mu^{\tau,(l)}_{i,E}\right), E\in\{R,I\}$ and $\mathsf{Pr}\left(\widehat{R^{(l)}_{i,E}}=\widehat{r^{(l)}_{i,E}}|X^{(l)}_{i,E}=x^{(l)}_{i,E}\right)\sim\mathcal{N}\left(x^{(l)}_{i,E},\mu^{\tau,(l)}_{i,E}\right), E\in\{R,I\}$. In the proposed MP algorithm, we compute the following posterioi distributions on the factor graph in Fig. 1, which are $\mathsf{Pr}\left(x^{(l)}_{i,E}|\widehat{r^{(1)}_{i,R}},\widehat{r^{(1)}_{i,I}},\cdots,\widehat{r^{(L)}_{i,R}},\widehat{r^{(L)}_{i,I}}\right)$ and $\mathsf{Pr}\left(b_i|\widehat{r^{(1)}_{i,R}},\widehat{r^{(1)}_{i,I}},\cdots,\widehat{r^{(L)}_{i,R}},\widehat{r^{(L)}_{i,I}}\right)$. The messages of the proposed algorithm in Fig. 1 are defined as:

$$m_{N_i\to X^{(l)}_{i,E}} \equiv \mathsf{Pr}\left(x^{(l)}_{i,E}|\widehat{r^{(1)}_{i,R}},\widehat{r^{(1)}_{i,I}},\cdots,\widehat{r^{(L)}_{i,R}},\widehat{r^{(L)}_{i,I}}\right), \quad (6)$$

$$m_{N_i\to B_i} \equiv \mathsf{Pr}\left(b_i|\widehat{r^{(1)}_{i,R}},\widehat{r^{(1)}_{i,I}},\cdots,\widehat{r^{(L)}_{i,R}},\widehat{r^{(L)}_{i,I}}\right), \quad (7)$$

$$m_{X^{(l)}_{i,E}\to N_i} \equiv \mathsf{Pr}\left(\widehat{r^{(l)}_{i,D}}|x^{(l)}_{i,D}\right), E\in\{R,I\}, \quad (8)$$

$$m_{B_i\to N_i} \equiv \mathsf{Pr}\left(b_i\right). \quad (9)$$

By iteratively computing the messages defined above, we can obtain more exact estimates on the active users and their channels. After the pre-defined number of iterations, we determine the active users by the maximum posteriori probability (MAP) criterion and estimate the channel coefficient of the active users by the minimum mean squared error (MMSE) criterion.

IV. Performance Evaluation

Fig. 2 and Fig. 3 show the detection error probability of the active users of the proposed MP algorithm and the normalized MMSE between the original channel coefficient and the estimated channel coefficient by proposed MP algorithm for varying SNR values, respectively. We assume that $N = 100$, $M = 50$, and $L = 1,..,4$ in the simulations. The number of iterations is set to 8. The SNR is defined as: $\mathsf{SNR} = \mathbb{E}[\| \mathbf{A}\mathbf{x}^{(l)}_E \|^2_2]/MN\sigma^2$, where $E \in \{R,I\}$

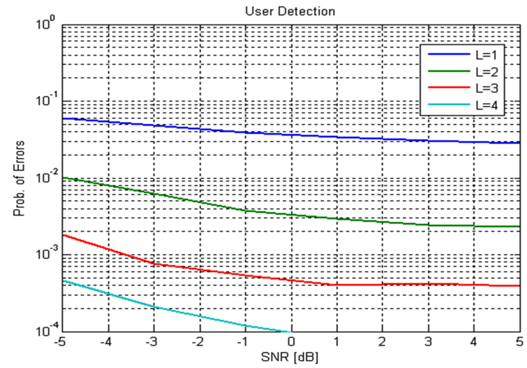

Fig. 2. User detection performance of the proposed message passing algorithm according to SNR values for a given L

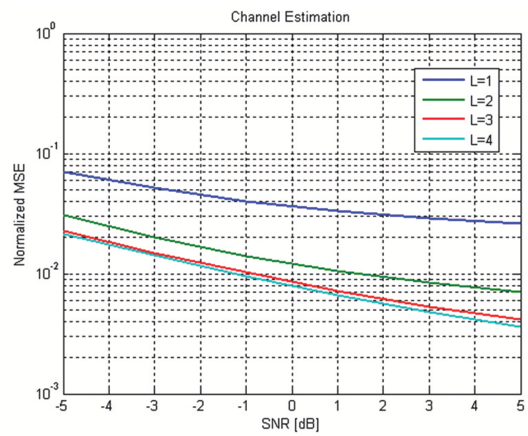

Fig. 3. Channel estimation performance of the proposed message passing algorithm according to SNR values for a given L

and $l = 1, ..., L$. The performances of the proposed MP algorithm becomes improved as the SNR values increase. Note that the performances also becomes enhanced as the number of antennas at the BS increases because the proposed MP algorithm adopts the joint sparsity among antennas at the BS efficiently.

Acknowledgement

This research was supported by the ICT Standardization program of MISP (The Ministry of Science, ICT & Future Planning).

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