On the Joint Power and Rate Optimization in Multihop Relay Networks with HARQ

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Abstract—We consider a linear multihop relay network adopting Chase-combining type Hybrid ARQ strategy in Rayleigh fading channels. The total number of (re)transmissions is assumed to be not limited, i.e., we consider delay-tolerant applications. We aim at maximizing the average throughput by jointly optimizing the transmit powers and data rates over all hops, while limiting the sum of transmit powers of all nodes. However, it is hard to find the jointly optimal power allocation and rate selection strategy in this problem. In this paper, we propose an iterative algorithm in order to obtain the optimal transmit powers and transmission rates for all the nodes including source and relays. Numerical results show that the proposed algorithm which controls both power and rate yields the best throughput performance among the conventional link adaptation schemes including the schemes that adapt either power or rate.

Index Terms—Multihop relay networks, HARQ, Chase combining, power control, rate selection

I. INTRODUCTION

Relay-assisted communications have been considered as promising technologies for the next-generation wireless communication systems owing to their advantages of coverage extension and spatial diversity gain [1], [2]. Recently, commercial wireless communication systems such as WiMAX and 3GPP long-term evolution (LTE) also have adopted multihop relay techniques [3], [4]. Among various structures of relay networks, the linear topology in which multiple relays are serially connected from a source node to a destination node has been vastly studied in wireless communications [5]–[8]. The linear multihop relay network can also be regarded as a group of linearly deployed nodes of an ad-hoc network whose route is already given. Most studies on the multihop relay networks have focused on the optimal resource management such as rate adaptation and power control, while guaranteeing a given reliability constraint like bit-error probability.

Meanwhile, hybrid automatic-repeat-request (HARQ) provides time diversity gain and power gain by utilizing retransmissions when previous (re)transmissions were not successfully decoded. In particular, the HARQ based on Chase Combining (CC) retransmits the same replica of the packet in case of a decoding error and uses a maximum ratio combining at the receiver [9]. This CC-type HARQ scheme is widely used in practical wireless communication systems because it is easily implemented, compared with other HARQ schemes such as incremental redundancy (IR). There have been many studies related to link adaptation with HARQ techniques including an optimal rate selection problem of packets at initial transmission over various fading channel models - slow fading channel [10], fast fading channel [11], and time-correlated fading channel [12]. Kim et al. [13] also proposed the suboptimal rate selection in a continuous rate domain while reducing the complexity of numerical search over Rayleigh block fading channels. Recently, the effect of heterogeneous mobility on link adaptation and user scheduling in HARQ-based cellular networks has been investigated in [14], [15] and a promising rate adaptation and user scheduling strategy considering HARQ has been proposed for the interference-limited cellular networks [16].

Recently, several studies have dealt with cooperative relay networks exploiting HARQ in the three-node relay network [17]–[20] and in multihop relay networks [21]–[23]. In particular, Stanojev et al. studied the optimal design of the multihop relay network employing HARQ in a quasi-static fading environment for maximizing the end-to-end throughput by optimizing the number of hops [20]. Zhao and Valenti considered a multihop relay network with HARQ where multiple relays transmit exclusively over time and proposed the position-based relay selection method [21]. In [21], the authors did not consider the rate adaptation. However, the transmission rates of different relays can be varied according to the channel states. Kim et al. [22] optimized the transmission rates of a two-hop relay network by using exhaustive searches for delay-limited applications. In addition, the transmit powers at relays can also be adjusted in order to optimize performance of the multihop relay networks. Lau et al. [24] considered an energy-constrained multihop relay network where the total power consumption is minimized for a given end-to-end bit error rate (BER). Hasna et al. [25] investigated the optimal power allocation strategy in the multihop relay network for a given power budget, while the outage constraint is satisfied. The power allocation of the multihop relay network with HARQ protocol has not been studied.

In this paper, we investigate the joint power and rate adaptation to maximize the average throughput in a multihop relay network with the CC-type HARQ protocol. We consider the delay-tolerant applications so that we relax the limitations both on the number of retransmissions and on the outage constraint\(^1\). We formulate an optimization problem to maximize the average throughput of a multihop relay network adopting the CC-type HARQ for delay-tolerant applications. We also provide the closed-form solution of the optimal rate for a fixed transmit power at nodes as well as the closed-form solution of the transmit powers for a fixed transmission rate, while limiting the sum of transmit powers of all nodes. In addition, we propose an efficient algorithm to jointly control

\(^1\)If the number of retransmissions goes to \(\infty\), the outage probability from the source to destination becomes zero.
the transmission rates and transmit powers.

II. SYSTEM MODEL

As shown in Fig.1, we consider a $M$-hop relay network consisting of $M + 1$ nodes: a source ($N_1$), a destination ($N_{M+1}$), and $M − 1$ relay nodes ($N_2, \ldots, N_M$) where relays are numbered in the order of proximity to the source. We consider a linear network where relays are serially connected from the source to the destination. Moreover, we assume that the signal reached at the neighbor’s neighbor is negligible since the distance between neighbor nodes is far enough. All nodes have a single antenna and use half-duplex transmission. Each relay node employs decode-and-forward (DF) mode where a relay tries to decode the received packet and forwards the re-encoded packet only when decoding is successful. We adopt Chase combining (CC) type HARQ in the $M$-hop relay network.

In the first hop, $N_1$ transmits a packet to $N_2$. If $N_2$ successfully decodes the packet, $N_2$ forwards the packet to $N_3$. However, if $N_2$ fails to decode the packet, $N_1$ retransmits the same packet until $N_2$ successfully decodes the packet. This process repeats until $N_{M+1}$ successfully decodes the packet. In the CC-type HARQ, the receiver uses maximum ratio combining (MRC).

We assume a Rayleigh block-fading channel where the channel gain is constant during a single HARQ round but the channel gains of different HARQ rounds are independent and identically distributed (i.i.d.). Let $h_{nm}^k$ denote the channel coefficient at the $k$-th HARQ round (i.e. ($k-1$)-th retransmission) of the $m$-th hop. The channel coefficient $h_{nm}^k$ is modeled as an independent, zero-mean complex Gaussian random variable with variance $\sigma_m^2$, i.e., $h_{nm}^k \sim CN(0, \sigma_m^2)$. Let $P_i$ denote the transmit power of the $N_i$.

We assume that each packet has $b$ information bits and $T_m$ symbols are consumed in a HARQ round of the $m$-th hop. Then, the transmission rate of the $m$-th hop in a HARQ round becomes $R_m = \frac{b}{T_m}$ (bits/symbol or bps/Hz). Since the channel statistics are not identical for each link and simultaneous transmission of different nodes to the same node is not allowed, $R_m$ can be different for different $m$. A random variable $S_m^k$ denotes the number of HARQ rounds used for the $i$-th packet at the $m$-th hop. Fig 2 describes the retransmission protocol in the multihop relay network.

The average transmission rate over $K$ packets is expressed by $\sum_{m=1}^{M+1} \frac{b}{T_m} \sum_{n=1}^{N_m} \frac{b}{T_m} \sum_{m=1}^{N_m} S_m^k$. If $K$ goes to infinity, we obtain the long-term average throughput as

$$ T(R) = \frac{1}{K} \sum_{m=1}^{M+1} \left[ \frac{b}{T_m} \sum_{n=1}^{N_m} S_m^k \right] \quad [\text{bps/Hz}], $$

where $R = (R_1, \ldots, R_M)$.

III. PROBLEM STATEMENT

A. Single-hop Case

We consider single-hop case first. If a CC-type HARQ is applied for a sufficiently long packet, the mutual information after the $k$-th HARQ round can be expressed as

$$ I_k = \log_2 \left( 1 + \sum_{i=1}^{M} |h_i|^2 \frac{P_i}{N_0} \right), $$

where $N_0$ denotes the one-sided noise spectral density. The outage probability after the $k$-th HARQ round is given by

$$ p_k(R) = \Pr [I_k < R] = \Pr \left[ \sum_{i=1}^{M} |h_i|^2 \frac{P_i}{N_0} < (2^R - 1) \right]. \quad (2) $$

Let $X = \sum_{i=1}^{M} |h_i|^2 \frac{P_i}{N_0}$ denote an Erlang-distributed random variable whose CDF is given by $F_X(x, k, \rho_k) = 1 - \sum_{n=0}^{k-1} e^{-x}/\rho_k x^n/n!$ where $\rho_k = \sigma_m^2 P_i / N_0$. Then, $p_k(R) = F_X(2^R - 1, k, \rho_k)$. In addition, the probability that a packet is successfully decoded after the $k$-th HARQ round is written as $q_k(R) = p_{k-1}(R) - p_k(R)$. The expected number of HARQ rounds per packet for a given $R$ is expressed as

$$ \mathbb{E}[S|R] = \sum_{k=1}^{\infty} k \cdot q_k(R) = \sum_{k=1}^{\infty} p_k(R) = \frac{2^R - 1}{\rho_1} + 1. \quad (3) $$

B. Multihop Relay Case

We now consider an $M$-hop relay network. If we assume that the $m$-th hop experiences independent fading, the outage probability of the $m$-th hop after the $k$-th HARQ round is expressed by $p_{m,k}(R_m) = \Pr \left[ \sum_{i=1}^{M} |h_i|^2 \frac{P_i}{N_0} < (2^{R_m} - 1) \right]$. The probability that a packet is successfully decoded after the
k-th HARQ round at the m-th hop is denoted by $q_{m,k}(R_m) = p_{m,k-1}(R_m) - p_{m,k}(R_m)^2$.

$E_M[S_m|R_1, \ldots, R_M]$ denotes the expected number of HARQ rounds used at the m-th hop of the M-hop relay network. In order to derive the average throughput in (1), we have to derive $E_M[S_m|R_1, \ldots, R_M]$ for 1 ≤ m ≤ M. It is obvious that $E_M[S_m|R_1, \ldots, R_M] = E_M[S_m|R_1, \ldots, R_m]$ since $R_{m+1}, \ldots, R_M$ do not affect the HARQ action of the m-th hop. Meanwhile, $E_M[S|R_m]$ denotes the expected number of HARQ rounds in a single hop which has the same channel statistics with the m-th hop. For a given $R_1$, the expectation of $S_1$ is given by

$$E_M[S_1|R_1] = \sum_{l=1}^{\infty} l q_{1,l}(R_1) = \frac{2R_1 - 1}{\rho_1} + 1. \quad (4)$$

Note that $p_{1,\infty}(R_1) = 0$ means the probability that a packet is retransmitted at the first hop forever is zero. Then, every packet is surely forwarded to the next hop and to the destination in the future. The expectation of $S_2$ is expressed by

$$E_M[S_2|R_1, R_2] = \sum_{l_1=1}^{\infty} q_{1,l_1}(R_1) E_M[S|R_2] = \sum_{l_1=1}^{\infty} q_{1,l_1}(R_1) \sum_{l_2=0}^{\infty} p_{2,l_2}(R_2), \quad (5)$$

where (6) follows since there is no limitation on the number of retransmissions in each hop. Likewise, $E_M[S_m|R_1, \ldots, R_m]$ can be derived as

$$E_M[S_m|R_1, \ldots, R_m] = \sum_{k=1}^{m-1} \left[ \prod_{k=1}^{m-1} q_{k,l_k}(R_k) \left( \sum_{l_m=0}^{\infty} p_{m,l_m}(R_m) \right) \right] \quad (7)$$

$$= \left( \sum_{l_m=0}^{\infty} p_{m,l_m}(R_m) \right) \sum_{k=1}^{m-1} \prod_{k=1}^{m-1} q_{k,l_k}(R_k) \quad (8)$$

$$= \sum_{l_m=0}^{\infty} p_{m,l_m}(R_m) \quad (9)$$

$$= 2R_m - 1 + \frac{1}{\rho_m} \quad (10)$$

where (9) is given by

$$\sum_{k=1}^{m-1} \prod_{k=1}^{m-1} q_{k,l_k}(R_k) = 1$$

and (10) follows the result of (3). Therefore $T^\infty(R) = \frac{\sum_{m=1}^{M} \frac{2R_m^m - 1}{\rho_m^m} + 1}{R_m}$.

Finally, the average throughput maximization problem with total power constraint is formulated as

$$\max_{\mathbf{R}, \mathbf{P}} \quad \frac{1}{\sum_{m=1}^{M} \frac{2R_m^m - 1}{\sigma_m^m P_m/N_0} + 1} / R_m \quad \text{s.t.} \quad \sum_{m=1}^{M} P_m \leq P^\text{total} \quad (11)$$

where $\mathbf{R} = \{R_1, \ldots, R_M\}$ and $\mathbf{P} = \{P_1, \ldots, P_M\}$.

IV. PROPOSED LINK ADAPTATION STRATEGIES

A. Rate Control with Fixed Power

We first find the $\mathbf{R}$ which maximizes $T^\infty(\mathbf{R})$ for a fixed $\mathbf{P}$. This problem can be replaced with the minimization of

$$D(\mathbf{R}) = \sum_{m=1}^{M} \left[ \frac{2R_m^m - 1}{\sigma_m^m R_m/N_0} + 1 \right] / R_m. \quad (12)$$

where $W(x)$ is the Lambert W function which is the inverse relationship of the function $x = W e^W$. For $x \geq e^{-1}$, $W(x)$ becomes always a real value. The function $W$ can be divided into two branches $W \geq -1$ and $W \leq -1$, denoted as $W_0$ and $W_{-1}$, respectively. Since $\rho_m \geq 0$ and $R_m^*$, our case matches with $W_0(x)$ which has a unique value for $x$. Note that $R_m^*$ does not depend on the channel statistics of other hops.

B. Power Control with Fixed Rates

We now find the $\mathbf{P}$ which maximizes the average throughput for a fixed $\mathbf{R}$. To find the $\mathbf{P}$, the problem in (11) can be reformulated as

$$\min_{\mathbf{P}} \quad f_0(\mathbf{P}) = \sum_{m=1}^{M} \left[ \frac{2R_m^m - 1}{\sigma_m^m R_m/N_0} \right] \frac{1}{P_m} \quad (13)$$

$$\text{s.t.} \quad f_1(\mathbf{P}) = \sum_{m=1}^{M} P_m - P^\text{total} \leq 0. \quad (13)$$

$f_0(\mathbf{P})$ is strictly convex and $f_1(\mathbf{P})$ is also convex. Therefore, we can define the Lagrangian:

$$L(\mathbf{P}, \lambda) = f_0(\mathbf{P}) + \lambda f_1(\mathbf{P}), \quad (14)$$

where $\lambda$ denotes a Lagrange multiplier. By finding a point $\mathbf{P}$ satisfying $\frac{dL(\mathbf{P}, \lambda)}{dP_1} = \cdots = \frac{dL(\mathbf{P}, \lambda)}{dP_M} = 0$, we obtain the optimizer:

$$P_m^* = \frac{A_m}{\lambda}, \quad \forall m. \quad (15)$$

where $A_m = \frac{2R_m^m - 1}{\sigma_m^m R_m/N_0}$. Using $f_1(\mathbf{P}) = P^\text{total}$, we can obtain

$$\lambda = \frac{\sum_{m=1}^{M} \sqrt{A_m}}{P^\text{total}}. \quad (15)$$

Then, we obtain the solution expressed...
C. Joint Power and Rate Control

We propose an iterative algorithm to jointly control transmit powers and rates to maximize the average throughput based on results in previous subsections.

1) Let \( P^0_m \) where \( P^0_m = P^{total}/M, \quad \forall m. \)

2) Obtain \( R^0 \) where \( R^0_m = W \left( \frac{e^{P^0_m/M}}{\sigma_i^2} \right) + 1, \quad \forall m. \)

3) Substituting \( R^0 \), we obtain \( P^1 \) where \( P^1_m = \sqrt{\frac{2^{R^0_m - 1}}{\sigma_i^2 R^0_m/N_0}} \).

4) Substituting \( P^1 \), we obtain \( R^1 \) where \( R^1_m = \frac{W \left( \frac{e^{P^1_m/M}}{\sigma_i^2} \right) + 1}{\sqrt{\frac{2^{R^1_m - 1}}{\sigma_i^2 R^1_m/N_0}}}, \quad \forall m. \)

5) Repeat until \( |T(R^i, P^1) - T(R^{i-1}, P^{i-1})| \leq \epsilon. \)

V. NUMERICAL RESULTS

Fig. 3 shows the relative performance loss of the proposed scheme for varying the number of iterations for two different cases: \( M = 2, (\gamma_1, \gamma_2) = (0, 20) \text{[dB]}, P^{total} = 10 \) and \( M = 3, (\gamma_1, \gamma_2, \gamma_3) = (0, 20, 5) \text{[dB]}, P^{total} = 10 \) where \( \gamma_i = \sigma_i^2/N_0. \) We set \( N_0 = 1 \) and the relative loss as \( |T(R^{100}, P^{100}) - T(R^{N^{iter}}, P^{N^{iter}})|/T(R^{100}, P^{100}) \) where \( N^{iter} \) denotes the iteration number and \( T(R^{100}, P^{100}) \) is assumed to be very close to the optimal average throughput. We can observe that the relative loss of two cases decreases exponentially and the relative loss reaches \( 10^{-5} \) only within \( N^{iter} = 4. \)

Fig. 4 shows the average throughput performance in the two-hop relay network for varying the distance between the source and relay nodes. We use the normalized distance by setting the distance between source and destination, \( d_{SD} \), equals 1 and the distance between source and relay, \( d_{SR} \), varies from 0 to 1. The path loss exponent is set to 3 and \( N_0 = 1. \) The proposed joint power and rate control scheme uses 4 iterations and the equal power scheme (Eq-Power) uses \( P^0 \) and \( R^0 \). Then, the received SNR of the \( i \)-th hop is expressed as \( \gamma_i = \frac{P_i}{d_{SR}^\alpha N_0}. \)

We consider the case that \( P^{total} = 1 \) and \( P^{total} = 0.5. \) In Fig. 4, the proposed scheme outperforms the Eq-Power scheme in terms of the average throughput for \( P^{total} = 1 \) and 0.5. The average throughput gap between the two schemes increases as the relay is far from the midpoint (\( d_{SR} = 0.5 \)). We additionally consider the single hop case where the relay is not used. As the total transmit power decreases which means the average received SNR between nodes is low, the region where the proposed scheme outperforms the single hop case becomes dominant.

Fig. 5 shows the average throughput according to \( \gamma_1 \) for the three-hop relay network when \( P^{total} = 10 \), while maintaining the ratio among \( (\gamma_1, \gamma_2, \gamma_3) \) are maintained as \( (1, 10, 0.1). \) We compare the proposed joint power and rate control strategy with the equal power and adaptive rate (EP&AR) scheme, the equal rate and adaptive power scheme (AP&ER), and the equal power and equal rate scheme (EP&ER). For the AP&ER and EP&ER schemes, we assumed that \( R_m = 2 \text{ bps/Hz.} \) The joint strategy outperforms the other schemes. The joint strategy yields about 22% gain over the AP&ER scheme when \( \gamma_1 = -5 \text{dB}, \) and about 12% gain over the EP&AR scheme when \( \gamma_1 = 10 \text{dB}, \) respectively.

Fig. 6 shows the average throughput according to \( \gamma_1 \) for the same environment as Fig. 5, but the ratio among \( (\gamma_1, \gamma_2, \gamma_3) \) are maintained as \( (1, 2, 0.5). \) The joint strategy still outperforms the other schemes. However, the EP&AR scheme yields a similar average throughput performance with the joint strategy. The proposed joint strategy yields larger gain in terms of average throughput than other schemes as there exists significant difference in average channel gains over multiple hops. If the average channel gains over multiple hops are similar each other, the EP&AR results in similar performance with the proposed joint strategy in terms of the average throughputs.

VI. CONCLUSIONS

In this paper, we investigated the joint power and rate adaptation strategy for linear multihop networks with Chase-combining HARQ. We first formulated the optimization problem of maximizing the average throughput for a given total
power consumption. We provided the closed-form solution on the rate allocation rule for each node when the transmit power of each node is fixed and the closed-form solution on the power allocation rule of each node when the transmission rate of each node is fixed. Then, we proposed an iterative algorithm jointly controlling the power and rate of each node. Numerical results show that the proposed strategy outperforms the conventional link adaptation techniques for various communication scenarios.

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