# Opportunistic Network Decoupling in Multi-Source Interfering Relay Networks 

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#### Abstract

We introduce a new achievability scheme, termed opportunistic network decoupling (OND), where a novel relay scheduling strategy is utilized in the $K \times N \times K$ channel with interfering relays, consisting of $K$ source-destination pairs and $N$ half-duplex relays in-between them. A subset of relays using alternate relaying is opportunistically selected in terms of producing the minimum total interference level, thereby resulting in network decoupling. As our main result, it is shown that under a certain relay scaling condition, the OND protocol with alternate half-duplex relaying achieves $K$ degrees-of-freedom even in the presence of interfering links among relays. Numerical evaluation is also shown to validate the performance of the proposed OND scheme.


## I. Introduction

Interference between wireless links has been taken into account as a critical problem in communication systems. Recently, interference alignment (IA) was proposed for fundamentally solving the interference problem when there are multiple communication pairs [1]. It was shown that the IA scheme can achieve the optimal degrees-of-freedom (DoF), which is equal to $K / 2$, in the $K$-user interference channel with timevarying channel coefficients. Since then, interference management schemes based on IA have been further developed and analyzed in various wireless network environments: multipleinput multiple-output (MIMO) interference network [2], [3], X network [4], and cellular network [5]-[7].

Following up on these successes for single-hop networks, more recent and emerging work has studied multihop networks with multiple source-destination (S-D) pairs. For the 2 -user 2-hop network with 2 relays (referred to as the $2 \times 2 \times 2$ interference channel), it was shown in [8] that interference neutralization combining with symbol extension achieves the optimal DoF. A more challenging network model is to consider $K$-user two-hop relay-aided interference channels, consisting of $K$ source-destination (S-D) pairs and $N$ helping relay nodes located in the path between S-D pairs, termed the $K \times N \times K$ channel. Several achievability schemes have been known for the network, but a detailed understanding is still in progress. By applying the result from [9] to the $K \times N \times K$ channel, one can show that $K / 2 \mathrm{DoF}$ is achieved by using orthogonalize-and-forward relaying, which completely neutralizes interference at all destinations if $N$ is greater than or equal to $K(K-1)+1$. Another achievable scheme, called aligned network diagonalization, was intro-
duced in [10] and was shown to achieve the optimal DoF in the $K \times N \times K$ channel while tightening the required number of relays. The scheme in [10] is based on using the real interference alignment framework [6]. In [8], [10], however, the system model under consideration assumes that there is no interfering signal between relays and the relays are full-duplex. Moreover, in [11], the $2 \times 2 \times 2$ interference channel with full-duplex relays interfering with each other was characterized and its DoF achievability was shown using aligned interference neutralization. ${ }^{1}$ Another interesting idea of interference management using overhearing relays was introduced in [13].

In this paper, we study the $K \times N \times K$ channel with interfering relays, which can be taken into account as one of multi-source interfering relay networks, and introduce an opportunistic network decoupling (OND) protocol that achieves full DoF with comparatively easy implementation under the channel model. This work thus focuses on the $K \times N \times K$ channel with one additional assumption that $N$ half-duplex relays interfere with each other, which is a more feasible scenario. The scheme adopts the notion of multiuser diversity gain for performing interference management over two hops. In the literature, there are some results on the usefulness of fading in broadcast channels, where one can obtain a multiuser diversity gain: opportunistic scheduling [14], opportunistic BF [15], and random BF [16]. Scenarios exploiting the multiuser diversity gain have also been extended in cooperative networks by applying an opportunistic routing [17]. In our scheme, a scheduling strategy is presented in time-division duplexing (TDD) two-hop environments with time-invariant channel coefficients, where a subset of relays is opportunistically selected in terms of producing the minimum total interference level. To improve spectral efficiency, the alternate relaying protocol in [18] is employed with a modification. As our main result, it turns out that in a high signal-to-noise ratio (SNR) regime, the OND protocol with alternate halfduplex relaying still achieves the min-cut upper bound of $K$ DoF even in the presence of inter-relay interference and half-duplex assumption, provided the number of relays, $N$, scales faster than $\mathrm{SNR}^{3 K-2}$, which is the minimum number of

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Fig. 1. The $K \times N \times K$ channel model with interfering relays where $K=3$ and $N=10$.
relays required to guarantee our achievability result. Numerical evaluation also indicates that the OND scheme has higher sumrates than those of the other relaying methods under realistic network conditions (e.g., finite $N$ and SNR) since inter-relay interference is significantly reduced owing to the opportunistic gain. Note that unlike the case of [11], our protocol basically operates with local channel state information (CSI) at the transmitter and thus is suitable for distributed/decentralized networks.

The rest of this paper is organized as follows. In Section II, we describe the system and channel models. In Section III, the proposed OND protocol with alternate relaying is specified and its achievable DoF is analyzed. Numerical results of the OND scheme are provided in Section IV. Finally, we summarize the paper with some concluding remarks in Section V.

We refer to our full paper [19] for more detailed description and all the proof.

## II. System and Channel Models

Consider the $K \times N \times K$ channel model with interfering relays to describe practical multiuser relaying networks for the case where each S-D pair is geographically far apart, thus creating a huge challenge for spectral efficiency. As illustrated in Fig. 1, there are $K$ S-D pairs, where each receiver is the destination of exactly one source node and is interested only in traffic demands of the source. As in the typical cooperative relaying setup, $N$ relay nodes are located in the path between S-D pairs so as to help to reduce path-loss attenuations. Each source transmits its own message to the corresponding destination only through one of $N$ relays, and thus it is assumed that there is no direct path between an S-D pair. The example for $K=3$ and $N=10$ is shown in Fig. 1. Suppose that each node is equipped with a single transmit antenna. Each relay node is assumed to operate in half-duplex mode and to fully decode, re-encode, and retransmit the source message i.e., decode-and-forward protocol is taken into account. Unlike the work in [8], [10], $N$ relays are assumed to interfere with each other. ${ }^{2}$ With alternate relaying, each selected relay node toggles between the transmit and listen modes for alternate

[^1]time slots of message transmission of the sources. If $N$ is sufficiently large, then it is possible to exploit the channel randomness for each hop and thus to obtain the opportunistic gain in multiuser environments. We assume that each node (either a source or a relay) has an average transmit power constraint $P$.

Now, let us turn to channel modeling. Let $\mathcal{S}_{k}, \mathcal{D}_{k}$, and $\mathcal{R}_{i}$ denote the $k$ th source, the corresponding destination, and the $i$ th relay node, respectively, where $k \in\{1, \cdots, K\}$ and $i \in\{1, \cdots, N\}$. The terms $h_{i k}^{(1)}, h_{k i}^{(2)} \in \mathbb{C}$ denote the channel coefficients from $\mathcal{S}_{k}$ to $\mathcal{R}_{i}$ and from $\mathcal{R}_{i}$ to $\mathcal{D}_{k}$, corresponding to the first and second hops, respectively. The term $h_{\text {in }}^{(r)} \in \mathbb{C}$ indicates the channel coefficient between two relays $\mathcal{R}_{i}$ and $\mathcal{R}_{n}$. All the channels are assumed to be Rayleigh, having zeromean and unit variance, and to be independent across different $i, k, n$, and hop index. We assume a block-fading model, i.e., the channels are constant during one block (e.g., frame), consisting of one scheduling time slot and $L$ data transmission time slots, and changes to a new independent value for every block.

## III. Achievability Result

In this section, we propose the OND protocol in the $K \times$ $N \times K$ channel with interfering relays. Then, its performance is analyzed in terms of achievable DoF.

## A. Opportunistic Network Decoupling in the $K \times N \times K$ Channel With Interfering Relays

In this subsection, we introduce an OND protocol as the achievable scheme to guarantee the optimal DoF of the $K \times N \times K$ channel with inter-relay interference, where $2 K$ relay nodes among $N$ candidates are opportunistically selected for data forwarding in the sense of having a sufficiently small amount of interference level. The proposed scheme is basically performed by utilizing the channel reciprocity of TDD systems.

Suppose that $\pi_{1}(k)$ and $\pi_{2}(k)$ denote the indices of two relays communicating with the $k$ th S-D pair for $k \in$ $\{1, \cdots, K\}$. In this case, without loss of generality, assuming that $L$ is an odd number, the specific steps of each node during one block are described as follows:

- Time slot 1: Sources $\mathcal{S}_{1}, \cdots, \mathcal{S}_{K}$ transmit their first encoded symbols $x_{1}^{(1)}(1), \cdots, x_{K}^{(1)}(1)$, where $x_{k}^{(1)}(l)$ represents the $l$ th transmitted symbol of the $k$ th source node. ${ }^{3}$ A set of $K$ selected relay nodes, $\Pi_{1}=$ $\left\{\pi_{1}(1), \cdots, \pi_{1}(K)\right\}$, operating in receive mode at each odd time slot, listens to $x_{1}^{(1)}(1), \cdots, x_{K}^{(1)}(1)$ (note that a relay selection strategy will be specified in a later section). Other $N-K$ relay nodes and destinations $\mathcal{D}_{1}, \cdots, \mathcal{D}_{K}$ remain idle.
- Time slot 2: The $K$ sources transmit their encoded symbols $x_{1}^{(1)}(2), \cdots, x_{K}^{(1)}(2)$. The $K$ relays in

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Fig. 2. The overall procedure of our OND scheme with alternate relaying in the $K \times N \times K$ channel with interfering relays.
the set $\Pi_{1}$ forward their first re-encoded symbols $x_{\pi_{1}(1)}^{(2)}(1), \cdots, x_{\pi_{1}(K)}^{(2)}(1)$ to the corresponding $K$ destinations. If the relays in $\Pi_{1}$ successfully decode the corresponding symbols, then $x_{\pi_{1}(k)}^{(2)}(1)$ is the same as $x_{k}^{(1)}(1)$. Another set of $K$ selected relay nodes, $\boldsymbol{\Pi}_{2}=$ $\left\{\pi_{2}(1), \cdots, \pi_{2}(K)\right\}$, operating in receive mode at each even time slot, listens to and decodes $x_{1}^{(1)}(2), \cdots, x_{K}^{(1)}(2)$ while being interfered with by $\mathcal{R}_{\pi_{1}(1)}, \cdots, \mathcal{R}_{\pi_{1}(K)}$. The $K$ destinations receive from $\mathcal{R}_{\pi_{1}(1)}, \cdots, \mathcal{R}_{\pi_{1}(K)}$ and decode $x_{\pi_{1}(1)}^{(2)}(1), \cdots, x_{\pi_{1}(K)}^{(2)}(1)$. The remaining $N-2 K$ relays keep idle.

- Time slot 3: The $K$ sources transmit their encoded symbols $\quad x_{1}^{(1)}(3), \cdots, x_{K}^{(1)}(3)$. The $K$ relays $\pi_{2}(1), \cdots, \pi_{2}(K)$ forward their re-encoded symbols $x_{\pi_{2}(1)}^{(2)}(2), \cdots, x_{\pi_{2}(K)}^{(2)}(2)$ to the corresponding $K$ destinations. Another $K$ relays in $\boldsymbol{\Pi}_{1}$ receive and decode $x_{1}^{(2)}(3), \cdots, x_{K}^{(2)}(3)$ while being interfered with by $\mathcal{R}_{\pi_{2}(1)}, \cdots, \mathcal{R}_{\pi_{2}(K)}$. The $K$ destinations receive from $\mathcal{R}_{\pi_{2}(1)}, \cdots, \mathcal{R}_{\pi_{2}(K)}$ and decode $x_{\pi_{2}(1)}^{(2)}(2), \cdots, x_{\pi_{2}(K)}^{(2)}(2)$. The remaining $N-2 K$ relays keep idle.
- The processes in time slots 2 and 3 are repeated to the $(L-1)$ th time slot.
- Time slot $L$ : The $K$ relays in $\Pi_{2}$ forward their reencoded symbols $x_{\pi_{2}(1)}^{(2)}(L-1), \cdots, x_{\pi_{2}(K)}^{(2)}(L-1)$ to the corresponding $K$ destinations. The $K$ sources and the other $N-K$ relays remain idle.
At each odd time slot $l$ (i.e., $l=1,3, \cdots, L$ ), let us consider the received signal at each selected relay for the first hop and the received signal at each destination for the second hop, respectively. For the first hop (Phase 1), the received signal at $\mathcal{R}_{\pi_{1}(i)}$ is given by

$$
\begin{align*}
y_{\pi_{1}(i)}^{(1)}(l) & =\sum_{k=1}^{K} h_{\pi_{1}(i) k}^{(1)} x_{k}^{(1)}(l)+\sum_{n=1}^{K} h_{\pi_{1}(i) \pi_{2}(n)}^{(r)} x_{\pi_{2}(n)}^{(2)}(l-1) \\
& +z_{\pi_{1}(i)}^{(1)}(l) \tag{1}
\end{align*}
$$

where $x_{k}^{(1)}(l)$ and $x_{\pi_{2}(n)}^{(2)}(l-1)$ are the $l$ th and the $(l-$ 1)th transmit symbol of $\mathcal{S}_{k}$ and transmit symbol of $\mathcal{R}_{\pi_{2}(n)}$, respectively. As addressed earlier, if relay $\mathcal{R}_{\pi_{2}(k)}$ successfully decode the received symbol, then it follows that $x_{\pi_{2}(k)}^{(2)}(l-$ $1)=x_{k}^{(1)}(l-1)$. The received signal $y_{\pi_{1}(i)}^{(1)}(l)$ at $\mathcal{R}_{\pi_{1}(i)}$ is corrupted by the independent and identically distributed (i.i.d.) and circularly symmetric complex additive Gaussian noise $z_{\pi_{1}(i)}^{(1)}(l)$ having zero-mean and variance $N_{0}$. Note that the second term in the right-hand side (RHS) of (1) indicates inter-relay interference, which occurs when the $K$ relays in the set $\Pi_{1}$, operating in receive mode, listen to the sources, the relays are interfered with by the other set $\Pi_{2}$, operating in transmit mode. Note that when $l=1$, relays have no symbols to transmit, and the second term in the RHS of (1) becomes zero. Similarly when $l=L$, sources do not transmit symbols, and the first term in the RHS of (1) becomes zero. For the second hop (Phase 2), assuming that the $K$ selected relay nodes transmit their data packets simultaneously, the received signal at $\mathcal{D}_{k}$ is given by

$$
y_{k}^{(2)}(l)=\sum_{n=1}^{K} h_{k \pi_{2}(n)}^{(2)} x_{\pi_{2}(n)}^{(2)}(l-1)+z_{k}^{(2)}(l)
$$

where $z_{k}^{(2)}(l)$ is the i.i.d. Gaussian noise having zero-mean and variance $N_{0}$. We also note that when $l=1$, there are no signals from relays.

Likewise, at each even time slot (i.e., $l=2,4, \cdots, L-1$ ), the received signals at $\mathcal{R}_{\pi_{2}(i)}$ and $\mathcal{D}_{k}$ (i.e., the first and second hops) are given by

$$
\begin{aligned}
y_{\pi_{2}(i)}^{(1)}(l) & =\sum_{k=1}^{K} h_{\pi_{2}(i) k}^{(1)} x_{k}^{(1)}(l)+\sum_{n=1}^{K} h_{\pi_{2}(i) \pi_{1}(n)}^{(r)} x_{\pi_{1}(n)}^{(2)}(l-1) \\
& +z_{\pi_{2}(i)}^{(1)}(l)
\end{aligned}
$$

and

$$
y_{k}^{(2)}(l)=\sum_{n=1}^{K} h_{k \pi_{1}(n)}^{(2)} x_{\pi_{1}(n)}^{(2)}(l-1)+z_{k}^{(2)}(l)
$$

respectively. The overall procedure of the aforementioned OND scheme with alternate half-duplex relaying is illustrated in Fig. 2 (two terms $\tilde{L}_{\pi_{1}(k), k}$ and $L_{\pi_{2}(k), k}$ are specified later in the following relay selection steps).

Now, let us describe how to choose two types of relay sets, $\Pi_{1}$ and $\Pi_{2}$ among $N$ relay nodes, where $N$ is sufficiently large (the minimum $N$ required to guarantee the DoF optimality will be analyzed in Section III-B).

1) Step 1 (The First Relay Set Selection): Let us first focus on selecting the set $\Pi_{1}=\left\{\pi_{1}(1), \cdots, \pi_{1}(K)\right\}$, operating in receive and transmit modes in odd and even time slots, respectively. For every scheduling period, it is possible for relay $\mathcal{R}_{i}$ to obtain all the channel coefficients $h_{i k}^{(1)}$ and $h_{k i}^{(2)}$ by using a pilot signaling sent from all of the source and destination nodes before data transmission, where $i \in\{1, \cdots, N\}$ and $k \in\{1, \cdots, K\}$ (note that this is our local CSI assumption). When $\mathcal{R}_{i}$ is assumed to serve the $k$ th $\mathrm{S}-\mathrm{D}$ pair $\left(\mathcal{S}_{k}, \mathcal{D}_{k}\right)$, it then examines both i) how much interference is received from the other sources and ii) how much interference is generated by itself to the other destinations, by computing the following scheduling metric $\tilde{L}_{i, k}$ :

$$
\begin{equation*}
\tilde{L}_{i, k}=\sum_{m=1, m \neq k}^{K}\left(\left|h_{i m}^{(1)}\right|^{2}+\left|h_{m i}^{(2)}\right|^{2}\right) \tag{2}
\end{equation*}
$$

where $i \in\{1, \ldots, N\}$ and $k \in\{1, \ldots, K\}$. Note that the first term $\sum_{m=1, m \neq k}^{K}\left|h_{i m}^{(1)}\right|^{2}$ in (2) denotes the sum of interference power received at $\mathcal{R}_{i}$ for the first hop (i.e., Phase 1). On the other hand, the second term $\sum_{m=1, m \neq k}^{K}\left|h_{m i}^{(2)}\right|^{2}$ indicates the sum of interference power generating at $\mathcal{R}_{i}$, which can be interpreted as the leakage of interference to the $K-1$ receivers expect for the corresponding destination, for the second hop (i.e., Phase 2) under the same assumption.

According to the computed metrics $\tilde{L}_{i, k}$ in (2), a timerbased method is used for relay selection similarly as in [20]. Note that the method based on time is considerably suitable in distributed systems in the sense that information exchange among all relay nodes can be minimized. At the beginning of every scheduling period, the relay $\mathcal{R}_{i}$ computes the set of $K$ scheduling metrics, $\left\{\tilde{L}_{i, 1}, \cdots, \tilde{L}_{i, K}\right\}$, and then starts its own timer with $K$ initial values, which are proportional to the $K$ metrics. Thus, there exist $N K$ metrics over the whole relay nodes, and we need to compare them so as to determine who will be selected. The timer of the relay $\mathcal{R}_{\pi_{1}(\hat{k})}$ with the least one $\tilde{L}_{\pi_{1}(\hat{k}), \hat{k}}$ among $N K$ metrics will expire first, where $\pi_{1}(\hat{k}) \in\{1, \cdots, N\}$ and $\hat{k} \in\{1, \cdots, K\}$. The relay then transmits a short duration RTS (Request to Send) message, signaling its presence, to the other $N-1$ relays. Thereafter, the relay $\mathcal{R}_{\pi_{1}(\hat{k})}$ is first selected to forward the $\hat{k}$ th S-D pair's packet. All the other relays are in listen mode while waiting for their timer to be set to zero (i.e., to expire). At the stage of deciding who will send the second RTS message, it is assumed that the other relays are not allowed to communicate with the $\hat{k}$ th S-D pair, and thus the associated
metrics $\left\{\tilde{L}_{1, \hat{k}}, \cdots, \tilde{L}_{\pi_{1}(\hat{k})-1, \hat{k}}, \tilde{L}_{\pi_{1}(\hat{k})+1, \hat{k}}, \cdots, \tilde{L}_{N, \hat{k}}\right\}$ are discarded with respect to timer operation. If another relay has an opportunity to send the second RTS message in order to declare its presence, then it is selected to communicate with the corresponding S-D pair. When such $K$ RTS messages are sent out in consecutive order, i.e., the set of $K$ relays, $\boldsymbol{\Pi}_{\mathbf{1}}=$ $\left\{\mathcal{R}_{\pi_{1}(1)}, \cdots, \mathcal{R}_{\pi_{1}(K)}\right\}$ is chosen, the timer-based algorithm for the first relay set selection terminates, yielding no RTS collision with high probability.
2) Step 2 (The Second Relay Set Selection): Now let us turn to choosing the set of $K$ relay nodes (among $N-K$ candidates), $\boldsymbol{\Pi}_{\mathbf{2}}=\left\{\pi_{2}(1), \cdots, \pi_{2}(K)\right\}$, operating in receive and transmit modes in even and odd time slots, respectively. Using $K$ RTS messages broadcasted from the $K$ relay nodes in the set $\boldsymbol{\Pi}_{\mathbf{1}}$, it is possible for relay node $\mathcal{R}_{i} \in\{1, \cdots, N\} \backslash \boldsymbol{\Pi}_{\mathbf{1}}$ to compute the sum of inter-relay interference power generated from the relays in $\boldsymbol{\Pi}_{\mathbf{1}}$, denoted by $\sum_{k=1}^{K}\left|h_{i \pi_{1}(k)}^{(r)}\right|^{2}$. When $\mathcal{R}_{i}$ is again assumed to serve the $k$ th S-D pair $\left(\mathcal{S}_{k}, \mathcal{D}_{k}\right)$, it examines how much interference is received from the undesired sources and the selected relays in the set $\{1, \cdots, N\} \backslash \boldsymbol{\Pi}_{\mathbf{1}}$ for the first hop and how much interference is generated by itself to the other destinations by computing the following metric $L_{i, k}$, termed total interference level (TIL):

$$
\begin{align*}
L_{i, k} & =\tilde{L}_{i, k}+\sum_{k=1}^{K}\left|h_{i \pi_{1}(k)}^{(r)}\right|^{2} \\
& =\sum_{m=1, m \neq k}^{K}\left(\left|h_{i m}^{(1)}\right|^{2}+\left|h_{m i}^{(2)}\right|^{2}\right)+\sum_{k=1}^{K}\left|h_{i \pi_{1}(k)}^{(r)}\right|^{2} \tag{3}
\end{align*}
$$

where $i \in\{1, \ldots, N\}$ and $k \in\{1, \ldots, K\}$.
According to the computed TIL $L_{i, k}$, we also apply the timer-based method used in Step 1 for the second relay set selection. The relay $\mathcal{R}_{i} \in\{1, \cdots, N\} \backslash \boldsymbol{\Pi}_{\mathbf{1}}$ computes the set of $K$ TILs, $\left\{L_{i, 1}, \cdots, L_{i, K}\right\}$, and then starts its timer with $K$ initial values, proportional to the $K$ TILs. Thus, we need to compare $(N-K) K$ TIL metrics over the relay nodes in the set $\{1, \cdots, N\} \backslash \boldsymbol{\Pi}_{1}$ in order to determine who will be selected as the second relay set. The rest of the relay set selection protocol (i.e., RTS message exchange among relay nodes) almost follows the same line as that of Step 1. The timerbased algorithm for the second relay set selection terminates when $K$ RTS messages are sent out in consecutive order. Then, $K$ relay nodes having a sufficiently small amount of TIL $L_{i, k}$ are selected as the second relay set $\Pi_{2}$.

We remark that owing to the channel reciprocity of TDD systems, the sum of inter-relay interference power received at any relay $R_{i} \in \mathbf{\Pi}_{\mathbf{1}}, \sum_{k=1}^{K}\left|h_{i \pi_{2}(k)}^{(r)}\right|^{2}$, also turns out to be sufficiently small when $N$ is large. That is, it is also guaranteed that $K$ selected relays in the set $\Pi_{1}$ have a sufficiently small amount of TIL.
3) Step 3 (Data Transmission): The $2 K$ selected relays request data transmission to their desired source nodes. Each source $\left(\mathcal{S}_{k}\right)$ then starts to transmit data to the corresponding destination ( $\mathcal{D}_{k}$ ) via one of its two relay nodes alternately
( $\mathcal{R}_{\pi_{1}(k)}$ or $\mathcal{R}_{\pi_{2}(k)}$ ), which was specified earlier. If the TILs of the selected relays are arbitrarily small, then i) the associated undesired source-relay and relay-destination channel links and ii) the inter-relay channel links are all in deep fade. In Section III-B, we will show that it is possible to choose such relays with the help of the multiuser diversity gain.

At the receiver side, each relay or destination detects the signal sent from its desired transmitter, while simply treating interference as Gaussian noise. Thus, no multiuser detection is performed at each receiver, thereby resulting in easier implementation.

## B. Achievable DoF

In this subsection, using the scaling argument bridging between the number of relays, $N$, and the received SNR (refer to [7], [21], [22] for the details), we shall show 1) the achievable DoF of the $K \times N \times K$ channel with interfering relays as $N$ increases and 2) the minimum $N$ required to guarantee the achievability result. The total number of DoF, denoted by DoF $_{\text {total }}$, is defined as [1]

$$
\mathrm{DoF}_{\text {total }}=\sum_{k=1}^{K}\left(\lim _{\mathrm{SNR} \rightarrow \infty} \frac{T_{k}(\mathrm{SNR})}{\log _{2} \mathrm{SNR}}\right)
$$

where $T_{k}(\mathrm{SNR})$ denotes the transmission rate of source $\mathcal{S}_{k}$. The following theorem establishes our main result.

Theorem 1: Suppose that the OND scheme with alternate relaying is used for the $K \times N \times K$ channel with interfering relays. Then, for $L$ data transmission time slots,

$$
\mathrm{DoF}_{\text {total }} \geq \frac{(L-1) K}{L}
$$

is achievable if $N=\omega\left(\mathrm{SNR}^{3 K-2}\right) .{ }^{4}$
Note that the achievable DoF asymptotically approaches $K$ for large $L$. From Theorem 1, let us provide the following interesting discussions regarding the DoF achievability.

Remark 1: The proposed OND scheme with alternate relaying achieves $K$ DoF in the $K \times N \times K$ channel with interfering links among relay nodes, if the number of relay nodes, $N$ scales faster than $\mathrm{SNR}^{3 K-2}$ and the number of transmission symbols in one block, $L$, is sufficiently large. The parameter $N$ required to obtain full DoF (i.e., $K$ DoF) exponentially increase with the number of S-D pairs, $K$, in order to make the sum of $3 K-2$ interference terms in the TIL metric nonincreasing with increasing SNR at each relay, which enables that all the interfering signals are nulled out at each selected relay by exploiting the multiuser diversity gain.

Remark 2: It is not difficult to show that the centralized relay selection method that maximizes the received SINR (at either the relay or the destination) using global CSI at the transmitter, which is a combinatorial problem with exponential complexity, gives the same relay scaling result $N=\omega\left(\mathrm{SNR}^{3 K-2}\right)$ along with full DoF. However, even with our OND scheme using decentralized relay selection based

$$
{ }^{4} f(x)=\omega(g(x)) \text { means that } \lim _{x \rightarrow \infty} \frac{g(x)}{f(x)}=0
$$



Fig. 3. The TIL versus $N$ when $K=3$.
only on local CSI, the same achievability result is obtained, thus resulting in much easier implementation.

In addition, a simple upper bound on the DoF is shown and compared to the achievable DoF in the following argument.

Remark 3: Using the min-cut upper bound, it is obvious to show that the number of DoF for the MIMO channel, generated by the cut such that $K$ transmit nodes are on the left of the network and the other nodes including active relays and destination nodes are on the right, is upper-bounded by $K$. Hence, it turns out that even with the half-duplex relaying assumption, our achievable scheme is DoF-optimal with the help of the alternate relaying. Note that this upper bound is generally derived regardless of whether the number of relays, $N$, tends to infinity or not, whereas the scaling condition $N=\omega\left(\mathrm{SNR}^{3 K-2}\right)$ is necessary in the achievability proof of the OND scheme.

## IV. Numerical Evaluation

In this section, we perform computer simulations to validate the performance of the proposed OND scheme for finite parameters $N$ and SNR. For comparison, two baseline relay selection schemes operating based on local CSI in a distributed manner are also considered: 1) a random relay selection scheme and 2) a Max-Min SNR scheme, which selects the SD pair such that from relay $\mathcal{R}_{i}$ 's perspective $(i \in\{1, \cdots, N\})$, the minimum out of the channel gains of two communication links (either from $\mathcal{S}_{k}$ to $\mathcal{R}_{i}$ or from $\mathcal{R}_{i}$ to $\mathcal{D}_{k}$ ) becomes the maximum among the associated minimum gains of $K$ SD pairs. The Max-Min scheme is well-suited for relay-aided systems if interfering links are absent. Alternate relaying is also used for each of the two compared schemes.

The average amount of TIL in (3) is first evaluated as the number of relays, $N$, increases. In Fig. 3, the log-log plot of TIL versus $N$ is shown when $K=3 .{ }^{5}$ Our result indicates that the TIL of the OND scheme tends to decrease almost linearly with $N$ (which is verified analytically in [19]) while the TIL of the other two schemes is not reduced according to

[^3]

Fig. 4. The sum-rates versus $N$ when $K=3, \mathrm{SNR}=30 \mathrm{~dB}$, and $\mathrm{INR}_{\mathrm{R}}=$ 10 dB .
increasing $N$. It is further seen how many relays are required with the OND scheme to guarantee that the TIL is less than a small constant for a given parameter $K$.

In addition, Fig. 4 illustrates the achievable sum-rates versus $N$ when $K=3, \mathrm{SNR}=30 \mathrm{~dB}$, and $\mathrm{INR}_{\mathrm{R}}=10 \mathrm{~dB}$, where $\mathrm{INR}_{\mathrm{R}}$ denotes the inter-relay interference-to-noise ratio (INR). For the max-min SNR scheme, the desired channel gain grows as $N$ increases while the amount of interference remains the same. In consequence, it is examined that the rate of increase in the sum-rates of the OND scheme with respect to $N$ is much higher than that of the max-min SNR scheme owing to the significant interference suppression.

## V. Conclusion

An efficient distributed OND protocol was proposed for the $K \times N \times K$ channel with interfering relays, referred to as one of multi-source interfering relay networks. A novel relay scheduling strategy with alternate half-duplex relaying was presented in two-hop environments, where a subset of relays is opportunistically selected in terms of producing the minimum total interference level, thereby resulting in network decoupling. It was shown that the OND protocol asymptotically achieves full DoF even in the presence of inter-relay interference and half-duplex assumption, provided that the number of relays, $N$, scales faster than $\mathrm{SNR}^{3 K-2}$. Numerical evaluation was also shown to verify that our scheme outperforms the other relay selection methods under realistic network conditions (e.g., finite $N$ and SNR) with respect to sum-rates.

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[^0]:    ${ }^{1}$ The idea in [11] was later extended to the 2 -user 3-hop network with 4 relays, i.e., the $2 \times 2 \times 2 \times 2$ interference channel [12].

[^1]:    ${ }^{2}$ If we can cancel the interfering signals among multiple relays, then the existing achievable scheme of the $K \times N \times K$ channel can also be applied here.

[^2]:    ${ }^{3}$ For notational convenience, we use scalar notation instead of vector notation for each coding block from source nodes, but the size of each symbol is assumed to be sufficiently long to achieve Shannon-theoretic channel capacity.

[^3]:    ${ }^{5}$ Even if it seems unrealistic to have a great number of relays in cooperative relay networks, the range for parameter $N$ is taken into account to precisely see some trends of curves varying with $N$.

