Opportunistic Downlink Interference Alignment

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***Abstract*— We introduce an opportunistic downlink interference alignment (ODIA) for interference-limited cellular downlink, which intelligently combines user scheduling and downlink IA techniques. The proposed ODIA not only efficiently reduces the effect of inter-cell interference from other-cell base stations (BSs) but also eliminates intra-cell interference among spatial streams in the same cell. We show that compared to the existing downlink IA schemes, the minimum number of users required to achieve a target degrees-of-freedom (DoF) can be fundamentally reduced, i.e., the fundamental user scaling law can be improved, by using the ODIA. In addition, we introduce a limited feedback strategy in our ODIA framework, and then analyze the minimum number of feedback bits required to obtain the same performance as that of the ODIA assuming perfect feedback.**

I. Introduction

In multiuser cellular environments, each user or base station (BS) may suffer from intra-cell and/or inter-cell interference. Interference alignment (IA) was proposed by fundamentally solving the interference problem when there are multiple communication pairs [1]. It was shown that the IA scheme can achieve the optimal degrees-of-freedom (DoF) in the multiuser interference channel with time-varying channel coefficients. In particular, IA techniques [2], [3] for cellular uplink and downlink networks, also known as the interfering multiple-access channel (IMAC) or interfering broadcast channel (IBC), respectively, have received a great attention. The existing IA framework for cellular networks, however, still has several practical challenges: the scheme in [3] requires an arbitrarily large frequency/time-domain dimension extension, and the scheme in [2] is based on iterative optimization of processing matrices and cannot be optimally extended to an arbitrary cellular network setup in terms of DoF.

Recently, the concept of opportunistic IA (OIA) was introduced in [4]–[6] for the multi-cell uplink network (i,e., the IMAC model), where there are one M-antenna BS and N users in each cell. The OIA scheme incorporates user scheduling into the classical IA framework by opportunistically selecting S ($S \leq M$) users amongst the N users in each cell in the sense that inter-cell interference is aligned at a pre-defined interference space. It was shown in [5], [6] that one can asymptotically achieve the optimal DoF if the number of users in a cell is beyond a certain value, i.e., if a certain *user scaling* condition is guaranteed. For the multi-cell downlink network (i.e., the IBC model) assuming one M-antenna BS and N per-cell users, studies on the OIA have been conducted in [7]–[12]. More specifically, the user scaling condition for obtaining the optimal DoF was derived for the K-cell multiple-input single-output (MISO) IBC [7], and then such a study on the DoF achievability was extended to the K-cell multiple-input multiple-output (MIMO) IBC with L receive antennas at each user [8]–[12]—full DoF can be achieved asymptotically, provided that N scales faster than SNR^{KM-L}, for the K-cell MIMO IBC model using OIA [11], [12], where SNR denotes the received signal-to-noise ratio.

In this paper, we propose an *opportunistic downlink IA (ODIA)* as a promising interference management technique for interference-limited K-cell downlink networks, where each cell consists of one BS with M antennas and N users having L antennas each. The proposed ODIA jointly takes into account user scheduling and downlink IA issues. The main contribution of this paper is three-fold as follows.

- We first show that the *minimum number of users* required to achieve S DoF ($S \leq M$) can be fundamentally reduced to $\mathsf{SNR}^{(K-1)S-L+1}$ by using the ODIA, compared to the existing downlink IA schemes requiring the user scaling law $N = \omega(\mathsf{SNR}^{KS-L})$ [11], [12],[1] where S denotes the number of spatial streams per cell.
- We introduce a *limited feedback strategy* in the ODIA framework, and then analyze the *minimum number of feedback bits* required to obtain the same DoF performance as that of the ODIA assuming perfect feedback, which is given by $\omega(\log_2 \mathsf{SNR})$.
- To verify the ODIA schemes, we perform numerical evaluation via computer simulations. Simulation results show that the proposed ODIA significantly outperforms existing interference management and user scheduling techniques in terms of sum-rate in realistic cellular environments.

We refer to our full paper [13] for more detailed description and all the rigorous proofs.

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[1] $f(x) = \omega(g(x))$ implies that $\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$.

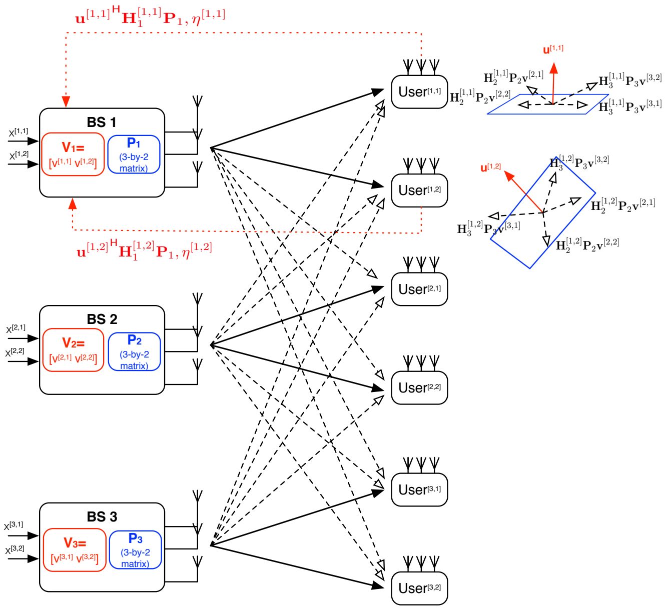

Fig. 1. The MIMO IBC model, where $K = 3$, $M = 3$, $S = 2$, $L = 3$, and $N = 2$.

II. System and Channel Models

We consider a K-cell MIMO IBC where each cell consists of a BS with M antennas and N users with L antennas each. The number of selected users in each cell is denoted by $S(\leq M)$. It is assumed that each selected user receives a single spatial stream. To consider nontrivial cases, we assume that $L < (K-1)S + 1$ since all inter-cell interference can be completely canceled at the receivers (i.e., users) otherwise. The channel matrix from the k-th BS to the j-th user in the i-th cell is denoted by $\mathbf{H}_k^{[i,j]} \in \mathbb{C}^{L \times M}$, where $i, k \in \mathcal{K} \triangleq \{1, \ldots, K\}$ and $j \in \mathcal{N} \triangleq \{1, \ldots, N\}$. Each element of $\mathbf{H}_k^{[i,j]}$ is assumed to be independent and identically distributed (i.i.d.) according to $\mathcal{CN}(0, 1)$. In addition, quasi-static frequency-flat fading is assumed, i.e., channel coefficients are constant during one transmission block and change to new independent values for every transmission block. Owing to the channel reciprocity of time-division duplexing systems, the j-th user in the i-th cell can estimate the channels $\mathbf{H}_k^{[i,j]}$, $k = 1, \ldots, K$, using pilot signals sent from all the BSs, i.e., the local channel state information (CSI) at the transmitters is available. Figure 1 shows an example of the MIMO IBC model, where $K = 3$, $M = 3$, $S = 2$, $L = 3$, and $N = 2$. The details in the figure will be described in the subsequent section.

III. Proposed ODIA

We first describe the overall procedure of our proposed ODIA scheme for the MIMO IBC, and then define its achievable sum-rate and DoF.

A. Overall Procedure

The ODIA scheme is described according to the following four steps.

1) Initialization (Broadcast of Reference Beamforming Matrices): First, as illustrated in Fig. 1, the precoding matrix at each BS is composed of the product of a predetermined reference beamforming matrix, denoted by $\mathbf{P}_k$, and a user-specific beamforming matrix, denoted by $\mathbf{V}_k$. In this step, we mainly focus on the design of $\mathbf{P}_k$. Specifically, the reference beamforming matrix at the BS in the k-th cell is given by $\mathbf{P}_k = [\mathbf{p}_{1,k}, \ldots, \mathbf{p}_{S,k}]$, where $\mathbf{p}_{s,k} \in \mathbb{C}^{M \times 1}$ is an orthonormal basis for $k \in \mathcal{K}$ and $s = 1, \ldots, S$. Each BS independently generates $\mathbf{p}_{k,s}$ according to the isotropic distribution over the M-dimensional unit sphere. If the reference beamforming matrix is generated in a pseudo-random fashion, BSs do not need to broadcast them to users. Then, the j-th user in the i-th cell obtains $\mathbf{H}_k^{[i,j]}$ and $\mathbf{P}_k$ for $k = 1, \ldots, K$.

2) Receive Beamforming & Scheduling Metric Feedback: In the second step, we explain how to decide a user scheduling metric at each user along with given receive beamforming, where the design of receive beamforming will be explained in Section IV. Let $\mathbf{u}^{[i,j]} \in \mathbb{C}^{L \times 1}$ denote the unit-norm weight vector at the j-th user in the i-th cell, i.e., $\left\|\mathbf{u}^{[i,j]}\right\|^2 = 1$. Since the user-specific beamforming $\mathbf{V}_k$ will be utilized only to cancel intra-cell interference out, $\mathbf{V}_k$ does not change the inter-cell interference level at each user, which will be specified later. Thus, from the notion of $\mathbf{P}_k$ and $\mathbf{H}_k^{[i,j]}$, the j-th user in the i-th cell can compute the quantity of received interference from the k-th BS while using its receive beamforming vector $\mathbf{u}^{[i,j]}$, which is given by

$$\tilde{\eta}_k^{[i,j]} = \left\|\mathbf{u}^{[i,j]^{\mathsf{H}}}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\right\|^2, \qquad (1)$$

where $i \in \mathcal{K}$, $j \in \mathcal{N}$, and $k \in \mathcal{K} \setminus i = \{1, \ldots, i-1, i+1, \ldots, K\}$. Using (1), the scheduling metric at the j-th user in the i-th cell, denoted by $\eta^{[i,j]}$, is defined as the sum of received interference power from other cells. That is,

$$\eta^{[i,j]} = \sum_{k=1, k \neq i}^{K} \tilde{\eta}_k^{[i,j]}. \qquad (2)$$

As illustrated in Fig. 1, each user feeds the metric in (2) back to its home-cell BS. In addition to the scheduling metric in (2), each user needs to feed its effective channel vector back, so that the user-specific beamforming $\mathbf{V}_k$ is designed at each BS. The effective channel vector of the j-th user in the i-th cell is given by

$$\mathbf{u}^{[i,j]^{\mathsf{H}}}\mathbf{H}_i^{[i,j]}\mathbf{P}_i. \qquad (3)$$

3) User Scheduling: Upon receiving N users' scheduling metrics in the serving cell, each BS selects S users having the metrics up to the S-th smallest one. Without loss of generality, the indices of selected users in every cell are assumed to be $(1, \ldots, S)$. In this and subsequent sections, we focus on how to simply design a user scheduling method to guarantee the optimal DoF.

4) Transmit Beamforming & Downlink Data Transmission: The signal vector at the i-th BS transmitted to the j-th user in the i-th cell is given by $\mathbf{v}^{[i,j]}x^{[i,j]}$, where $x^{[i,j]}$ is the transmit symbol with power of $1/S$, and the user-specific beamforming matrix for S users is given by $\mathbf{V}_i = \left[\mathbf{v}^{[i,1]}, \ldots, \mathbf{v}^{[i,S]}\right]$, where

$\mathbf{v}^{[i,s]} \in \mathbb{C}^{S\times 1}$ for $i \in \mathcal{K}$. Denoting the transmit symbol vector of the i-th cell by $\mathbf{x}_i = \left[x^{[i,1]}, \ldots, x^{[i,S]}\right]^T$, the received signal vector at the j-th user in the i-th cell is then written as

$$\begin{aligned}\mathbf{y}^{[i,j]} &= \mathbf{H}_i^{[i,j]}\mathbf{P}_i\mathbf{V}_i\mathbf{x}_i + \sum_{k=1,k\neq i}^{K}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{V}_k\mathbf{x}_k + \mathbf{z}^{[i,j]}\\ &= \underbrace{\mathbf{H}_i^{[i,j]}\mathbf{P}_i\mathbf{v}^{[i,j]}x^{[i,j]}}_{\text{desired signal}} + \underbrace{\sum_{s=1,s\neq j}^{S}\mathbf{H}_i^{[i,j]}\mathbf{P}_i\mathbf{v}^{[i,s]}x^{[i,s]}}_{\text{intra-cell interference}}\\ &\quad + \underbrace{\sum_{k=1,k\neq i}^{K}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{V}_k\mathbf{x}_k}_{\text{inter-cell interference}} + \mathbf{z}^{[i,j]},\end{aligned}$$

where $\mathbf{z}^{[i,j]} \in \mathbb{C}^{L\times 1}$ denotes the additive white Gaussian noise vector, each element of which is i.i.d. complex Gaussian with zero mean and the variance of SNR^{-1}. The received signal vector at the j-th user in the i-th cell, postprocessed by receive beamforming, can be rewritten as:

$$\begin{aligned}&\tilde{y}^{[i,j]} = \mathbf{u}^{[i,j]\mathsf{H}}\mathbf{y}^{[i,j]}\\ &= \mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_i^{[i,j]}\mathbf{P}_i\mathbf{v}^{[i,j]}x^{[i,j]} + \mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_i^{[i,j]}\mathbf{P}_i\sum_{s=1,s\neq j}^{S}\mathbf{v}^{[i,s]}x^{[i,s]}\\ &+ \mathbf{u}^{[i,j]\mathsf{H}}\sum_{k=1,k\neq i}^{K}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{V}_k\mathbf{x}_k + \mathbf{u}^{[i,j]\mathsf{H}}\mathbf{z}^{[i,j]}. \qquad (4)\end{aligned}$$

By selecting the users having small $\eta^{[i,j]}$ in (2), $\mathbf{H}_k^{[i,j]}\mathbf{P}_k$ tends to be orthogonal to the receive beamforming vector $\mathbf{u}^{[i,j]}$; thus, inter-cell interference channel matrices $\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{V}_k$ in (4) also tend to be orthogonal to $\mathbf{u}^{[i,j]}$, as illustrated in Fig. 1.

To cancel out intra-cell interference, the user-specific beamforming matrix $\mathbf{V}_i \in \mathbb{C}^{S\times S}$ is given by

$$\begin{aligned}\mathbf{V}_i &= [\mathbf{v}^{[i,1]}, \mathbf{v}^{[i,2]}, \ldots, \mathbf{v}^{[i,S]}]\\ &= \begin{bmatrix}\mathbf{u}^{[i,1]\mathsf{H}}\mathbf{H}_i^{[i,1]}\mathbf{P}_i\\ \mathbf{u}^{[i,2]\mathsf{H}}\mathbf{H}_i^{[i,2]}\mathbf{P}_i\\ \vdots\\ \mathbf{u}^{[i,S]\mathsf{H}}\mathbf{H}_i^{[i,S]}\mathbf{P}_i\end{bmatrix}^{-1} \cdot \begin{bmatrix}\sqrt{\gamma^{[i,1]}} & 0 & \cdots & 0\\ 0 & \sqrt{\gamma^{[i,2]}} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \sqrt{\gamma^{[i,S]}}\end{bmatrix},\end{aligned} \qquad (5)$$

where $\sqrt{\gamma^{[i,j]}}$ denotes a normalization factor that satisfies the unit-transmit power constraint. In consequence, the received signal does not contain the intra-cell interference term of (4).

B. Achievable Sum-Rate and DoF

Let $R^{[i,j]}$ denote the achievable rate of the j-th user in the i-th cell. Then, from (4) and (5), the achievable total DoF can be defined as

$$\text{DoF} = \lim_{\mathsf{SNR}\to\infty}\frac{\sum_{i=1}^{K}\sum_{j=1}^{S}R^{[i,j]}}{\log\mathsf{SNR}},$$

where

$$\begin{aligned}&R^{[i,j]}\\ &= \log_2\left(1 + \frac{\gamma^{[i,j]}\cdot|x^{[i,j]}|^2}{\left|\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{z}^{[i,j]}\right|^2 + \sum_{k=1,k\neq i}^{K}\left|\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{V}_k\mathbf{x}_k\right|^2}\right)\\ &= \log_2\left(1 + \frac{\gamma^{[i,j]}}{\frac{S}{\mathsf{SNR}} + \sum_{k=1,k\neq i}^{K}\sum_{s=1}^{S}\left|\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{v}^{[k,s]}\right|^2}\right). \qquad (6)\end{aligned}$$

IV. DoF Achievability

In this section, we characterize the DoF achievability in terms of the *user scaling law* along with the optimal receive beamforming technique. To this end, we start from the design of receive beamforming that maximizes the achievable DoF. For given channel instance, from (6), each user can attain the maximum DoF of one if and only if the interference term $\sum_{k=1,k\neq i}^{K}\sum_{s=1}^{S}\left|\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\mathbf{v}^{[k,s]}\right|^2\mathsf{SNR}$ remains constant with increasing SNR. Note that $R^{[i,j]}$ can be bounded by

$$\begin{aligned}&R^{[i,j]}\\ &\geq \log_2\left(1 + \frac{\gamma^{[i,j]}}{\frac{S}{\mathsf{SNR}} + \sum_{k=1,k\neq i}^{K}\sum_{s=1}^{S}\left\|\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\right\|^2\left\|\mathbf{v}^{[k,s]}\right\|^2}\right)\\ &\geq \log_2\left(1 + \frac{\gamma^{[i,j]}\cdot\mathsf{SNR}}{\frac{S}{\mathsf{SNR}} + \sum_{k=1,k\neq i}^{K}\sum_{s=1}^{S}\left\|\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\right\|^2\left\|\mathbf{v}_i^{(\max)}\right\|^2}\right)\\ &= \log_2(\mathsf{SNR}) + \log_2\left(\frac{1}{\mathsf{SNR}} + \frac{\gamma^{[i,j]}/\left\|\mathbf{v}_i^{(\max)}\right\|^2}{S/\left\|\mathbf{v}_i^{(\max)}\right\|^2 + I^{[i,j]}}\right),\end{aligned}$$

where $\mathbf{v}_i^{(\max)}$ and $I^{[i,j]}$ are given by

$$\begin{aligned}\mathbf{v}_i^{(\max)} = \arg\max\Big\{&\left\|\mathbf{v}^{[i',j']}\right\|^2 :\\ &i' \in \{1,\ldots,i-1,i+1,\ldots,K\}, j' \in \{1,\ldots,S\}\Big\}\end{aligned}$$

and

$$I^{[i,j]} \triangleq \sum_{k=1,k\neq i}^{K}\sum_{s=1}^{S}\left\|\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_k^{[i,j]}\mathbf{P}_k\right\|^2\mathsf{SNR},$$

respectively. Here, $\mathbf{v}_i^{(\max)}$ is fixed for given channel instance since $\mathbf{v}^{[i,j]}$ is determined by $\mathbf{H}_i^{[i,j]}$ for $j = 1, \ldots, S$. Recalling that the indices of the selected users are $(1, \ldots, S)$ for all the cells, we can expect the DoF of one for each user if and only if for some $\epsilon \in [0, \infty)$,

$$I^{[i,j]} < \epsilon, \quad \forall j \in \mathcal{S}, i \in \mathcal{K}.$$

We now aim to minimize the sum of interference, $\sum_{i=1}^{K} \sum_{j=1}^{S} I^{[i,j]}$, through receive beamforming at the users. Since $I^{[i,j]} = \sum_{s=1}^{S} \eta^{[i,j]}\mathsf{SNR}$, it follows that

$$\sum_{i=1}^{K}\sum_{j=1}^{S} I^{[i,j]} = S \sum_{i=1}^{K}\sum_{j=1}^{S} \eta^{[i,j]}\mathsf{SNR}.$$

This implies that the total amount of distributed effort to minimize $\eta^{[i,j]}$ at each user eventually reduces the sum of received interference. Thus, each user finds the beamforming vector that minimizes $\eta^{[i,j]}$ from

$$\begin{aligned}\mathbf{u}^{[i,j]} &= \arg\min_{\mathbf{u}} \eta^{[i,j]} = \arg\min_{\mathbf{u}} \sum_{k=1, k\neq i}^{K} \left\| \mathbf{u}^{\mathsf{H}} \mathbf{H}_k^{[i,j]} \mathbf{P}_k \right\|^2 \\ &= \arg\min_{\mathbf{u}} \left\| \mathbf{G}^{[i,j]} \mathbf{u} \right\|^2,\end{aligned}$$

where

$$\mathbf{G}^{[i,j]} \triangleq \Bigg[\left(\mathbf{H}_1^{[i,j]}\mathbf{P}_1\right), \ldots, \left(\mathbf{H}_{i-1}^{[i,j]}\mathbf{P}_{i-1}\right), \left(\mathbf{H}_{i+1}^{[i,j]}\mathbf{P}_{i+1}\right), \ldots, \left(\mathbf{H}_K^{[i,j]}\mathbf{P}_K\right) \Bigg]^{\mathsf{H}} \in \mathbb{C}^{(K-1)S\times L}.$$

Let us denote the singular value decomposition of $\mathbf{G}^{[i,j]}$ as

$$\mathbf{G}^{[i,j]} = \mathbf{\Omega}^{[i,j]} \mathbf{\Sigma}^{[i,j]} {\mathbf{V}^{[i,j]}}^{\mathsf{H}},$$

where $\mathbf{\Omega}^{[i,j]} \in \mathbb{C}^{(K-1)S\times L}$ and $\mathbf{V}^{[i,j]} \in \mathbb{C}^{L\times L}$ consist of L orthonormal columns, and $\mathbf{\Sigma}^{[i,j]} = \mathrm{diag}\left(\sigma_1^{[i,j]}, \ldots, \sigma_L^{[i,j]}\right)$ for $\sigma_1^{[i,j]} \geq \cdots \geq \sigma_L^{[i,j]}$. The optimal $\mathbf{u}^{[i,j]}$ is then given by

$$\mathbf{u}^{[i,j]} = \mathbf{v}_L^{[i,j]},$$

where $\mathbf{v}_L^{[i,j]}$ is the L-th column of $\mathbf{V}^{[i,j]}$. With this choice, the scheduling metric can be simplified to

$$\eta^{[i,j]} = {\sigma_L^{[i,j]}}^2. \tag{7}$$

Since each column of $\mathbf{P}_k$ is isotropically and independently distributed, each element of the effective interference channel matrix $\mathbf{G}^{[i,j]}$ is i.i.d. complex Gaussian with zero mean and unit variance.

The following theorem establishes the DoF achievability of the proposed ODIA.

Theorem 1 (DoF/User scaling law): The ODIA scheme with the scheduling metric (7) asymptotically achieves the DoF of KS for given $S \in \{1, \cdots, M\}$ if

$$N = \omega\left(\mathsf{SNR}^{(K-1)S-L+1}\right). \tag{8}$$

Compared to the previous results leading to $N = \omega\left(\mathsf{SNR}^{KS-L}\right)$ [7], [11], [12], the exponent of SNR gets reduced significantly using the proposed ODIA. The essential of the ODIA is that the design of the precoder $\mathbf{V}_i$ can be decoupled from the design of the receive beamforming vector $\mathbf{u}^{[i,j]}$, because the scheduling metric $\eta^{[i,j]}$ is calculated at the user side in a distributed fashion without the knowledge of $\mathbf{V}_i$. Even with this decoupled approach, interference can still be minimized due to the cascaded precoder design. As a result, it is possible to achieve the optimal DoF without any iterative precoder and receive beamforming vector optimization as done in [2]. In addition, the proposed ODIA operates with any system parameters M, L, and K, whereas the optimal achievable DoF is guaranteed only for some special cases in the IA scheme in [2].

Remark 1 (Uplink-downlink duality): The same user scaling condition $N = \omega\left(\mathsf{SNR}^{K(S-1)-L+1}\right)$ was achieved to obtain KS DoF in the MIMO IMAC model [6]. Hence, Theorem 1 implies that a duality holds for the uplink and downlink OIA frameworks in terms of the achievable DoF and required user scaling law.

The user scaling law also characterizes the trade-off between the achievable DoF and the required number of users, i.e., the more the number of users, the higher achievable DoF.

V. ODIA WITH LIMITED FEEDBACK

In the proposed ODIA scheme, the effective channel vectors (${\mathbf{u}^{[i,j]}}^{\mathsf{H}}\mathbf{H}_i^{[i,j]}\mathbf{P}_i$) in (3) can be fed back to the corresponding BS using pilots rotated by the effective channels [14]. However, this analog feedback requires two consecutive pilot phases for each user: regular pilot for uplink channel estimation and analog feedback for effective channel estimation. For this reason, pilot overhead grows with the number of users. As a result, in practical systems with massive users, it is more preferable to follow the widely-used limited feedback approach [15], in which effective channels are fed back using codebooks after quantization.

For limited feedback of effective channel vectors, we define the codebook as

$$\mathcal{C}_f = \left\{\mathbf{c}_1, \ldots, \mathbf{c}_{N_f}\right\},$$

where N_f is the codebook size and $\mathbf{c}_k \in \mathbb{C}^{S\times 1}$ is a unit-norm codeword, i.e., $\|\mathbf{c}_i\|^2 = 1$. Hence, the number of used feedback bits is given by

$$n_f = \lceil \log_2 N_f \rceil (\text{bits}).$$

Let us denote the effective channel as

$${\mathbf{f}^{[i,j]}}^{\mathsf{H}} = {\mathbf{u}^{[i,j]}}^{\mathsf{H}} \mathbf{H}_i^{[i,j]} \mathbf{P}_i.$$

Each user quantizes the normalized effective channel for given $\mathcal{C}_f$ from

$$\tilde{\mathbf{f}}^{[i,j]} = \arg\max_{\{\mathbf{w}=\mathbf{c}_k: 1\leq k\leq N_f\}} \frac{\left|{\mathbf{f}^{[i,j]}}^{\mathsf{H}}\mathbf{w}\right|^2}{\left\|\mathbf{f}^{[i,j]}\right\|^2}.$$

Now, each user feeds back three types of information: 1) the index of $\tilde{\mathbf{f}}^{[i,j]}$, 2) the channel gain $\left\|\mathbf{f}^{[i,j]}\right\|^2$, and 3) the scheduling metric $\eta^{[i,j]}$. Then, BS i constructs the quantized effective channel vectors $\hat{\mathbf{f}}^{[i,j]}$ from

$$\hat{\mathbf{f}}^{[i,j]} \triangleq \left\|\mathbf{f}^{[i,j]}\right\|^2 \cdot \tilde{\mathbf{f}}^{[i,j]}, \quad i = 1, \ldots, S,$$

and the precoding matrix $\hat{\mathbf{V}}_i$ from

$$\hat{\mathbf{V}}_i = \hat{\mathbf{F}}_i^{-1}\mathbf{\Gamma}_i,$$

where $\mathbf{\Gamma}_i = \operatorname{diag}\left(\sqrt{\gamma^{[i,1]}}, \ldots, \sqrt{\gamma^{[i,S]}}\right)$ and $\hat{\mathbf{F}}_i = \left[\hat{\mathbf{f}}^{[i,1]}, \ldots, \hat{\mathbf{f}}^{[i,S]}\right]^{\mathsf{H}}$.

With limited feedback, residual intra-cell interference becomes non-zero due to the quantization error in $\hat{\mathbf{V}}_i$ existing in the received signal vector after receive beamforming. The following theorem establishes the achievability result for the ODIA with limited feedback.

Theorem 2 (DoF/User and feedback bit scaling laws): The ODIA using either a Grassmannian or a random codebook achieves the same DoF and user scaling law as the ODIA case with perfect CSI in Theorem 1 if

$$n_f = \omega\left(\log_2 \mathsf{SNR}\right). \tag{9}$$

That is, KS DoF is asymptotically achievable if $N = \omega\left(\mathsf{SNR}^{(K-1)S-L+1}\right)$ and (9) holds.

From Theorem 2, the minimum number of feedback bits, n_f, is characterized to achieve the optimal KS DoF, which scales as $\log_2(\mathsf{SNR})$. It is worth noting that our achievability results are the same for the Grassmannian and random codebook cases. More specifically, our analysis focuses basically on the asymptotic performance for given channel instance with increasing SNR, and it turns out that this asymptotic result follows the same trend for the considered two codebooks. In fact, this result is consistent with the previous work in the literature (e.g., [16]), where as n_f increases, the performance gap between the two codebook-based methods was shown to be negligible via computer simulations.

VI. Numerical Results

In this section, we compare the performance of the proposed ODIA with two conventional schemes that also utilize a multi-cell random beamforming method at the BSs. First, we use the "max-SNR" scheme, where each user designs the receive beamforming vector in the sense of maximizing the desired signal power and feeds back the maximized signal power to the corresponding BS. Each BS selects a set of S users who feed back the values up to the Sth largest one. Second, the "min-INR" scheme is used, where each user performs the receive beamforming in the sense minimizing the sum of inter-cell interference and intra-cell interference [11], [12].

As illustrated in Fig. 2, we evaluate the sum-rates for varying SNR values when $K = 3$, $M = 4$, $L = 2$, $S = 2$, and $N = 50$. The proposed ODIA outperforms the conventional schemes for almost all SNR regimes due to the combined effort of 1) transmit beamforming perfectly eliminating intra-cell interference and 2) receive beamforming effectively reducing inter-cell interference. The sum-rate performance of the ODIA with limited feedback (ODIA-LF) gets improved as n_f increases, as expected. In practice, $n_f = 6$ nearly achieves the sum-rate performance of the ODIA with full feedback for the codebook dimension of two (i.e., $S = 2$).

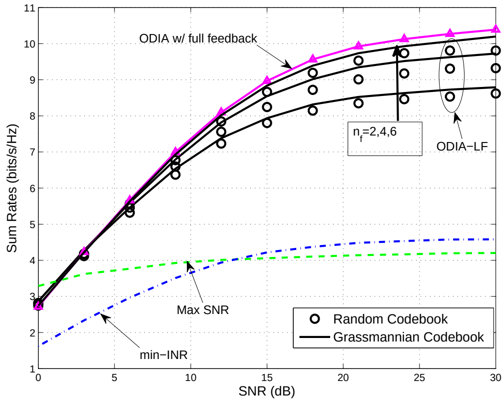

Fig. 2. Sum-rates versus SNR when $K = 3$, $M = 4$, $L = 2$, $S = 2$, and $N = 50$.

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