Coverage Analysis of Spectrum-Shared Directional Networks: Exclusion Zone and Antenna Radiation

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***Abstract*—We mathematically characterize the coverage probability of spectrum-shared wireless networks based on stochastic geometry over Nakagami-m fading channel, where the exclusion zone is adopted for protecting the primary network and all communication nodes are equipped with directional antennas. In particular, the actual antenna radiation pattern of practical directional antennas is first modeled via a step-wise approximation method. Based on the model, we obtain a closed-form mathematical expression on the coverage probability at a (typical) primary receiver. Also, it is approximated by a simple expression.**

***Index Terms*—Spectrum sharing, coverage probability, directional antenna, stochastic geometry, exclusion zone.**

I. Introduction

The lack of radio spectrum has been considered as a main limiting factor for satisfying performance requirements in future 6G [1]. Spectrum sharing has been received much interest as a promising technology to improve spectral efficiency [2]. An *exclusion zone* is known to be one of the most effective and practical techniques in spectrum-shared wireless networks [3]. Recently, a *directional* antenna has been exploited not only for improving the received signal quality but also for suppressing the co-channel interference between the two networks [4].

In [5], the coverage probability of the interference network was also analyzed by using multi-cosine antenna pattern to improve the accuracy of analysis. As the above studies investigated, the actual radiation patterns of directional antennas are extremely difficult to mathematically analyze. In networks with the exclusive zone, only a simple *flat-top* two-level radiation pattern was assumed for the analytical tractability [3].

In this paper, we mathematically characterize the coverage probability of spectrum-shared directional wireless network based on the stochastic geometry framework, where the exclusion zone is exploited. We assume that a fading channel model is Nakagami-m fading model. However, the Gauss hypergeometric function in the derived analytical expression incurs a high computational complexity, and we also propose a simple approximated expression of coverage probability.

II. System Model

We consider a spectrum-shared directional network which consists of a single primary transmitter–receiver (PT–PR) pair and multiple secondary transmitters (ST). The PR is located at the origin of $\mathbb{R}^2$, and the exclusion zone, $\mathcal{S}_e$, is a circular shape with a radius R_e. The locations of STs and the corresponding secondary receiver (SR) are modeled by an inhomogeneous Poisson Point Process $\Phi_e = \{x_i\} \subset \mathbb{R}^2$ of intensity $\lambda \mathbf{1}_{\mathbb{R}^2 \setminus \mathcal{S}_e}$ ($\lambda > 0$), where x_i denotes the position of the i-th ST.

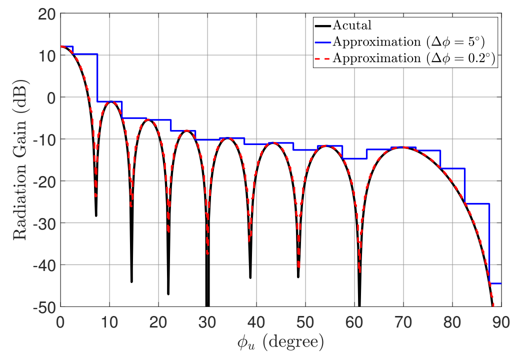

Fig. 1: Actual and approximated radiation gain of ULA.

We assume that all nodes are equipped with an uniform linear array (ULA) antenna with n antenna elements. The actual antenna radiation pattern gain when $n = 16$ and $d_a = L_a/2$ ($a \in \{t, r\}$) is shown in Fig. 1 where ϕ_u denotes the angle of arrival for PR, and the angle of departure for the STs and PT. The spacing between antenna elements and wavelength are represented as d_a and L_a, respectively. The symbol a indicates either transmitter (t) or receiver (r).

We apply a *step-wise approximation* method to approximate the actual radiation pattern model. For a given number of steps $2N_s$ ($N_s \in \mathbb{N}$), the constant radiation gain on a certain angle step is determined as the maximum actual radiation gain over the angle step $\Delta\phi = \frac{\pi}{N_s}$. A simple example is visualized in Fig. 1. We assume that all transmitter-receiver pairs perfectly steer their antenna orientation towards each other. The orientations of the STs and PR follow the uniform distribution on $[-\pi, \pi]$ as in [3], [5].

The received signal power at the PR from the i-th ST is

$$P_i = P_t |h_i|^2 G_t(\phi_{i,\mathsf{PR}}) G_r(\phi_{\mathsf{PR},i}) d_i^{-\alpha}, \tag{1}$$

where P_t denotes the transmit power of the ST and the channel gain $|h_i|^2$ follows the identically and independently distributed

$$P_c=\exp\left\{-\frac{msN_0}{P_t}-\lambda\pi\sum_{j_t=-N_s}^{N_s-1}\sum_{j_r=-N_s}^{N_s-1}\frac{g(\alpha-2)}{g(\alpha)+1}\left(\frac{{}_2F_1\left(1,2;2-\frac{2}{\alpha};\frac{g(\alpha)}{g(\alpha)+1}\right)}{\left(1-\frac{2}{\alpha}\right)(g(\alpha)+1)}-1\right)(\Delta\phi)^2\right\}\left[\sum_{n=1}^{m-1}\sum_{k=1}^{n}\sum_{l=1}^{k}\sum_{n_1+\cdots+n_l=k}\left(\frac{2\pi\lambda}{\alpha}\right)^l\right.$$
$$\left.\times\left\{\frac{(msN_0/P_t)^{n-k}}{l!(n-k)!}\prod_{i=1}^{l}\left(\sum_{j_t=-N_s}^{N_s-1}\sum_{j_r=-N_s}^{N_s-1}\frac{1}{n_i-\frac{2}{\alpha}}\frac{R_e^2\{g(\alpha)\}^{n_i}}{(g(\alpha)+1)^{n_i+1}}{}_2F_1\left(1,n_i+1;n_i+1-\frac{2}{\alpha};\frac{g(\alpha)}{g(\alpha)+1}\right)(\Delta\phi)^2\right)\right\}+\sum_{n=0}^{m-1}\frac{1}{n!}\left(\frac{msN_0}{P_t}\right)^n\right]. \quad (4)$$

exponential distribution with unit mean for all i. The symbols $\phi_{\mathsf{PR},i}$ and $\phi_{i,\mathsf{PR}}$ denote the position angles of the i-th ST based on the orientation of PR and $\phi_{i,\mathsf{PR}}$ of PR based on the orientation of the i-th ST, respectively. The term d_i denotes the distance between the i-th ST and the PR. The path-loss exponent is represented as α ($\alpha \geq 2$). In addition, the received signal power of the PT at the PR, P_o, is given by

$$P_o = P_t|h_o|^2 G_t(0) G_r(0) d_o^{-\alpha}, \quad (2)$$

where the channel gain of Nakagami-m fading, $|h_o|^2$, is known as the gamma distribution with a shape parameter m and a scale parameter $\frac{1}{m}$. The distance d_o is between PT and PR.

III. Coverage Probability Analysis

In this section, we mathematically characterize the coverage probability of the primary network with exclusion zone in a spectrum-sharing directional wireless network. The coverage probability, P_c, of primary network for a given signal-to-interference-plus-noise ratio (SINR) threshold β is given as

$$P_c = \Pr\{\mathsf{SINR} \geq \beta\} \quad (3)$$
$$= \Pr\left\{\frac{P_t|h_o|^2 G_t(0)G_r(0)d_o^{-\alpha}}{\sum_{\mathbf{x}_i\in\Phi_e} P_t|h_i|^2 G_t(\phi_{i,\mathsf{PR}})G_r(\phi_{\mathsf{PR},i})d_i^{-\alpha}+N_0} \geq \beta\right\}$$

where $g(x) = msG_tG_rR_e^{-x}$, N_0 denotes the noise power, and $s = \frac{\beta}{G_t(0)G_r(0)d_o^{-\alpha}}$ is constant. The final derived expression result of (3) is given as (4) where $G_a = G_a(j_a\Delta\phi)$ $(a \in \{t, r\})$.

In addition, we can derive a relatively simple expression using the appropriate inequality in (4).

$$P_c \approx \sum_{k=1}^{m}(-1)^{k+1}\binom{m}{k}\exp\left(-\sum_{j_t=-N_s}^{N_s-1}\sum_{j_r=-N_s}^{N_s-1}\left(-\frac{k\tau sG_tG_rR_e^{2-\alpha}}{k\tau sG_tG_rR_e^{-\alpha}+1}\right.\right.$$
$$\left.\left.+\frac{k\tau sG_tG_rR_e^{2-\alpha}}{\left(1-\frac{2}{\alpha}\right)\left(k\tau sG_tG_rR_e^{-\alpha}+1\right)^{1-2/\alpha}}\right)\lambda\pi(\Delta\phi)^2-\frac{k\tau sN_0}{P_t}\right). \quad (5)$$

IV. Numerical Results

TABLE I: Simulation Parameters

Parameter	Value	Parameter	Value
R_e	200 m	β	10
d_o	150 m	P_t	50 mW
α	4	m	4
N_s	4, 16	N_0	−174 dBm/Hz

Fig. 2 shows the coverage probability for varying the intensity of STs when the number of antenna elements of ULA is equal to 4 and 16. The closed-form coverage probability for a given approximated step-wise radiation pattern in (4) and a more simplified analytical expression in (5) matches well with computer simulation results. Although (5) is an approximated expression, it with $\Delta\phi = 0.2°$ is more accurate than (4) with $\Delta\phi = 1°$ while consuming less computation time.

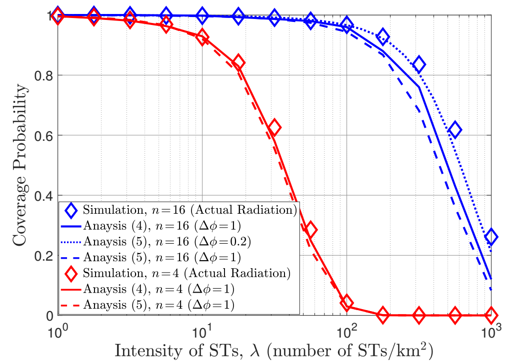

Fig. 2: Coverage probability of spectrum-shared directional wireless network with ULA antenna.

V. Conclusion

In this paper, we mathematically characterized the coverage probability of the spectrum-shared directional wireless network with exclusion zone based on stochastic geometry perspective over Nakagami-m fading channel. We proposed a generalized analytical framework which can be applied to arbitrary radiation pattern of directional antennas with the step-wise radiation pattern approximation method. Through computer simulations, the derived results were validated.

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