A Tradeoff Between Single-User and Multi-User MIMO Schemes in Multi-Rate Uplink WLANs

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***Abstract*—Due to high spectral-efficiency of multiple-input multiple-output (MIMO) transmission techniques, IEEE 802.11n WLAN system adopted a single-user MIMO (SU-MIMO) scheme in which multiple symbol streams are transmitted from a single station (STA) to enhance the system performance. On the other hand, recently, adoption of a multi-user MIMO (MU-MIMO) scheme for multi-packet reception (MPR) in uplink WLAN has also attracted attention. The SU-MIMO scheme achieves a MIMO multiplexing gain at physical (PHY) layer while the MU-MIMO scheme achieves a MIMO multiplexing gain at medium access control (MAC) layer. Thus, there is a fundamental question which scheme is a better solution for uplink WLANs and, in this paper, we analyze and compare these two schemes with random STA distribution scenarios. Moreover, with the adaptation of MAC layer parameters, we also analyze and compare the maximum throughput performance of both the SU- and MU-MIMO schemes in uplink WLANs and we find a proper decision criterion to select the MIMO mode in uplink WLANs.**

***Index Terms*—IEEE 802.11n, multi-user MIMO, multi-packet reception, multi-rate WLAN.**

I. INTRODUCTION

IEEE 802.11 [1] wireless local area networks (WLANs) have been widely deployed in the world due to simple use. The medium access control (MAC) technique of IEEE 802.11 WLAN is called distributed coordination function (DCF) which is a carrier sense multiple access with a collision avoidance (CSMA/CA) scheme with binary exponential backoff. The DCF employs two types of packet transmission schemes: basic access and request-to-send/clear-to-send (RTS/CTS) access mechanisms. Compared to the basic access mechanism, the RTS/CTS access mechanism exchanges RTS and CTS frames before the transmission of a data frame in order to reduce the time resource waste caused by the collision of large-sized packet transmissions. Due to high popularity of WLANs, there have been many studies on the DCF in WLAN systems [2]–[4]. However, most studies focused on the MAC performance under the assumption of a simple collision model in which frame errors occur when there are simultaneous transmissions from multiple stations(STAs). Thanks to the recent advanced signal processing techniques at physical (PHY) layer especially with multi-user multiple input multiple output (MU-MIMO) techniques, the receiver can detect all or partial of the simultaneously transmitted frames in wireless communications which is also called multi-packet reception (MPR).

Due to an inherent collision mitigation property of the MU-MIMO scheme in WLANs, there have been several studies on the MU-MIMO based WLANs [5]–[8]. For the basic access mechanism, Jin *et al.* [5] proposed a collision mitigation scheme in uplink WLANs using multiple antennas at an access point (AP). Utilizing MU-MIMO detection techniques at the AP, we showed that multiple frames can be successfully decoded even in the presence of simultaneous transmissions in uplink. For the RTS/CTS access mechanism, Zheng *et al.* [6] proposed an MU-MIMO technique to support simultaneous transmissions in uplink WLANs. Assuming the channel state information (CSI) is available at each STA, Huang *et al.* [7] [8] proposed a channel state based random access protocol for MU-MIMO based uplink WLANs. For the WLAN systems, since STAs are located with different distances away from the AP, they may operate at different average received signal-to-noise ratio (SNR) regions and, consequently, each STA may select a different data rate. In these multi-rate WLANs, a near-far interference problem occur at the AP when there are simultaneous transmissions from STAs. Despite the random STA distributions in WLANs, only the case of independent and identically distributed (i.i.d) post-detection SNR for each STA was considered in [7] even in the case of simultaneous transmissions from multiple STAs. Although this near-far interference problem was mentioned in [8], they assumed the STAs performed a power control scheme so that all the channels between different transmit antennas at the STAs and different receive antennas at the AP were eventually i.i.d and, consequently, all the STAs operate at the identical channel conditions. Thus far, the efficiency of the MU-MIMO scheme in uplink WLANs with random STA distributions has not been investigated.

On the other hand, the current IEEE 802.11n [9] standard has already adopted a single-user MIMO (SU-MIMO) technique which enables each STA to transmit multiple streams through its multiple antennas. Moreover, the downlink MU-MIMO scheme has been adopted by the ongoing IEEE 802.11ac [10] standard. With the downlink MU-MIMO scheme in IEEE 802.11ac, much higher complexity is added at the AP if the uplink MU-MIMO scheme is applied in the infrastructure mode. Although the MU-MIMO scheme was not included in standard bodies yet, as a possible candidate scheme for the future WLAN systems, it is necessary to compare the performance of the uplink WLANs with the MU-

Manuscript received November 6, 2010; revised April 10, 2011 and July 12, 2011; accepted July 16, 2011. The associate editor coordinating the review of this paper and approving it for publication was M. Morelli.

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This research was supported by the MKE (The Ministry of Knowledge Economy), Korea, under the ITRC (Information Technology Research Center) support program supervised by the NIPA (National IT Industry Promotion Agency) (NIPA-2011-(C1090-1111-0011)).

Digital Object Identifier 10.1109/TWC.2011.080311.101980

MIMO scheme to that with the SU-MIMO scheme. The SU-MIMO scheme enables high data rate for each STA at PHY layer while it does not support simultaneous transmissions from multiple STAs. However, each STA needs to transmit a single stream with a MU-MIMO scheme in order to support simultaneous transmissions and, consequently, the data rate of each STA may be lower than that of the SU-MIMO scheme. Moreover, although the MU-MIMO scheme supports multiple simultaneous transmissions from multiple STAs, a random access nature of the DCF can not always schedule the simultaneous transmissions. Thus, the performance comparison between the SU-MIMO scheme and the MU-MIMO scheme is an interesting problem. We compared the performance of uplink WLANs with SU-MIMO and MU-MIMO schemes [11] under the assumption of a single-rate WLAN environment.

Extending this work [11], we analyze and compare the throughput performance of the SU-MIMO and the MU-MIMO schemes in the uplink WLANs with random STA distribution scenarios. Instead of power control, rate adaptation is considered here since the current IEEE 802.11 series WLANs support multiple data rates. Since most current WLAN devices adopt the basic access mechanism to exchange data and acknowledgement (ACK) frames, the basic access mechanism is considered in this paper. Each STA has no information on the instantaneous CSI without exchanging RTS and CTS frames and it should select a proper data rate based on long-term statistics of the channel. Furthermore, since the system with the MU-MIMO scheme is more efficient if there are more simultaneous transmissions, we also analyze and compare the SU-MIMO and the MU-MIMO schemes when the MAC layer parameters are adapted to achieve the maximum throughput performance of each scheme.

The key contributions of this paper are summarized as follows: First, considering the random STA distributions, we analyze the performance of the MU-MIMO scheme in uplink WLANs with multi-rate support. Second, we evaluate the performance of both the SU-MIMO and the MU-MIMO schemes in uplink WLANs in terms of average throughput and investigate a tradeoff between these two schemes. Third, we analyze the maximum throughput of the SU-MIMO and the MU-MIMO schemes and compare the gains obtained by the SU-MIMO scheme at PHY layer with those obtained by the MU-MIMO scheme at MAC layer.

The rest of this paper is organized as follows: In Section II, the system models of the SU- and MU-MIMO schemes are described. In Section III, the performance of the PHY and MAC layers is analyzed for the SU- and MU-MIMO schemes. In Section IV, the maximum throughput of WLANs with the SU- and MU-MIMO schemes is analyzed and compared. In Section V, numerical results are presented for performance comparison of the SU- and MU-MIMO schemes. Finally, conclusions are presented in Section VI.

II. SYSTEM MODEL

A. SU-MIMO Scheme

The number of antennas at each STA and the AP is set to N, respectively, in this paper. In the SU-MIMO scheme, input bits are first encoded with one encoding block and are transmitted through N parallel antennas. The received signals through N antennas are first post-detected into N symbol streams and are decoded by one decoding block at the AP side. The post-detection can be done by MIMO decoding techniques, such as zero forcing (ZF), minimum mean square error (MMSE), successive interference cancellation (SIC), and maximum linkelihood (ML). Since the encoded symbol stream is directly mapped to N antennas, collisions may occur when there are multiple transmissions from multiple STAs. Since the transmission power, P, is limited at each STA, the transmit power at each antenna of the STA is set to P/N.

B. MU-MIMO Scheme with a Single Antenna Transmission (MU-MIMO-SA)

For the MU-MIMO scheme, we first consider the case of a single antenna transmission for each STA (MU-MIMO-SA). Since each STA has no CSI information due to a lack of RTS/CTS frame exchange in the basic access mechanism, the STAs cannot perform antenna selection to enhance the channel quality and they just randomly select one transmit antenna. With channel estimation at the AP, it can recover the transmitted data streams originated from different STAs with MIMO decoding techniques. If the number of simultaneous transmissions, M, is smaller than the number of antennas, N, at the AP, the AP can perform the MIMO decoding. In the case of $M \geq N$, although there is a successful decoding possibility for some data streams, we do not consider this kind of capture effect in this paper.

C. MU-MIMO Scheme with a Space-Time Block Coding (STBC) Transmission (MU-MIMO-STBC)

One way to enhance the performance of the MU-MIMO scheme is applying the space-time block code (STBC) scheme for each transmitting STA (MU-MIMO-STBC). Each STA encodes the input bits with the STBC codes such as an Alammouti scheme or a quasi-orthogonal space time block code (QOSTBC) [12], and transmits through N antennas. In the case of simultaneous transmissions from $M(\leq N)$ STAs, the AP can perform ML or the array processing scheme [13] for multi-user detection.

There are two technical issues for implementing the MU-MIMO scheme in uplink WLANs: synchronization and channel estimation. In general, there are no further transmissions from other STAs due to an inherent CSMA/CA protocol when an STA has already started its transmission. Thus, simultaneous transmissions occur only when multiple STAs start their transmission at the same time. Since each STA is located at a different distance from the AP, the time of arrival of each data stream at the AP may be slightly different. However, if the time mismatch is smaller than the guard time of an orthogonal frequency division multiplexing (OFDM) symbol, it does not degrade the signal detection performance [14]. Thus, this synchronization problem is not a critical task for IEEE 802.11a/g/n based WLAN systems since the mandatory guard time is $0.8\mu s$ and the typical WLAN coverage is smaller than 100m.

For channel estimation, there have been several studies on the channel estimation for multi-user MIMO detections. For example, Jeon *et al.* [15] proposed Walsh code based orthogonal preambles and Sung *et al.* [16] introduced superimposed training sequences for multi-user channel estimations. Furthermore, since the physical layer convergence protocol (PLCP) header of each data frame is encoded with the lowest data rate in WLAN systems, it can help the channel estimation with a blind detection technique [17] [18]. Since the channel estimation procedure is not a scope of this paper, we assume the perfect channel estimation at the AP under the same assumption in the previous work [5]–[8]. The interested reader can refer to the channel estimation procedure for the MU-MIMO based WLANs in [18] in which we can replace the RTS frame aided blind estimation with the PLCP header aided blind estimation. Here we simply compare the complexity of the channel estimation in terms of the required number of channel gains. For the SU-MIMO scheme, since there are N transmit and N receive antennas, the required number of channel gains is N^2. For the MU-MIMO-SA scheme, owing to a single-antenna transmission for each STA, the required number of channel gains is MN for simultaneous transmissions from M STAs. For the MU-MIMO-STBC scheme, since each STA transmits packets with N antennas, the required number of channel gains is MN^2 when there are simultaneous transmissions from M STAs. Thus, we can observe that the estimation complexity of the MU-MIMO-STBC scheme shows much higher complexity than that of the SU-MIMO and the MU-MIMO-SA schemes in terms of the required number of channel gains.

III. Performance Analysis of Multi-Rate Uplink WLANs

Since we consider uplink WLANs with random STA distribution, we first analyze the post-detection SNR for each STA when there exist simultaneous transmissions and then, discuss the data rate selection rules. Our key observation is that the post-detection SNR for each STA is only affected by the number of STAs with simultaneous transmissions and it is not affected by the signal strength of other STAs with Rayleigh fading channel when the AP performs the ZF decoding for the MU-MIMO-SA scheme or performs the array processing for the MU-MIMO-STBC scheme. This property enables the STAs to adapt the data rates without the knowledge about the SNR working regions of other STAs. At MAC layer, taking into account the multiple data rates for the STAs, we analyze the system throughput performance for both the SU- and the MU-MIMO schemes.

A. PHY Layer Analysis with the MU-MIMO-SA Scheme

1) ZF Decoding: For the MU-MIMO scheme, when there are M transmitting STAs and N receive antennas at the AP, channel matrix $\boldsymbol{H}$ can be expressed as $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \cdots, \boldsymbol{h}_M]$, where $\boldsymbol{h}_i = (h_{1i} h_{2i} \cdots h_{Ni})^T$ denotes the channel gain from the i-th STA to the N antennas of the receiver. With independent Rayleigh fading channel, $h_{ji} (j = 1, 2, \cdots, N)$ is an independent, zero-mean, complex Gaussian random variable with a variance of $2\sigma_i^2$. Each variance of $2\sigma_i^2 (i = 1, 2, \cdots, M)$ may have a different value because each STA is located with a different distance from the AP. The received signal at the AP can be expressed as $\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}$, where $\boldsymbol{s} = (s_1, s_2, \cdots, s_M)^T$ and $\boldsymbol{r} = (r_1, r_2, \cdots, r_N)^T$

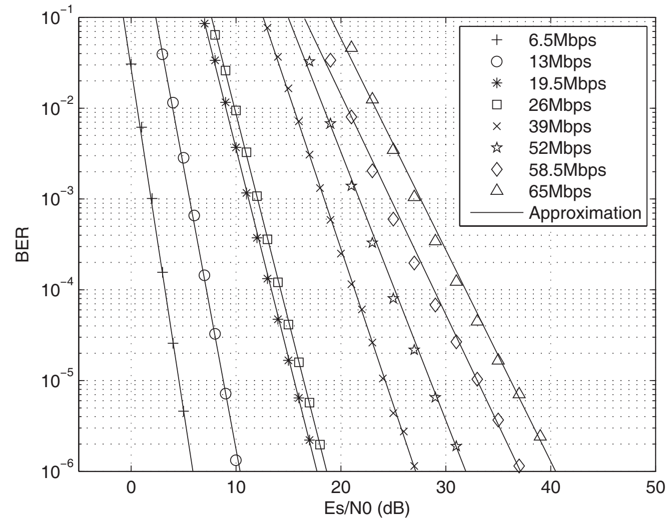

Fig. 1. BER performance of various modulation and coding schemes.

represent the transmitted and received symbol vectors, respectively. The term $\boldsymbol{n}$ is a complex Gaussian vector in which each component has zero mean and variance N_0. Using the channel gain $\boldsymbol{H}$, the AP can recover the transmitted symbols by using ZF decoding techniques. After ZF at the AP, the post-detection SNR for the i-th STA is given as [19]

$$\gamma_i = \frac{\gamma_0}{[(\boldsymbol{H}^H \boldsymbol{H})^{-1}]_{ii}}, \tag{1}$$

where γ_0 is the transmitted SNR for each antenna pair and $[A]_{ii}$ is the element in the i-th row and the i-th column of the matrix A. Although the variance of each column of $\boldsymbol{H}$ can be different, the statistical property of $1/[(\boldsymbol{H}^H \boldsymbol{H})^{-1}]_{ii}$ is not affected by other columns and it has a Gamma distribution with $2(N-M+1)$ degrees-of-freedom (DoF) and variance σ_i^2. Consequently, the post-detection SNR for the i-th transmitter, $\gamma_i (1 \le i \le M)$ has a probability density function (PDF) [20]

$$f_{2(N-M+1)}(\gamma_i) = \frac{\exp\left(-\frac{\gamma_i}{2\sigma_i^2 \gamma_0}\right)}{(N-M)!(2\sigma_i^2 \gamma_0)} \times \left(\frac{\gamma_i}{2\sigma_i^2 \gamma_0}\right)^{N-M}. \tag{2}$$

As shown in Eq. (2), the post-detection SNR for the i-th STA, γ_i, only depends on its own channel variance σ_i^2 and it is not affected by the SNR values of other STAs. However, an increase in the number of transmitting STAs, M, reduces the DoF. In the case of $M = N$, the post-detection SNR distribution is identical to that in a single-input single-output (SISO) Rayleigh fading channel case.

Since WLAN systems mostly operate in indoor environments with severe scattering, the channel gain exhibits a highly frequency-selective property. Additionally, considering the bit interleaving at the coding block, the channel gain for a data stream can be approximated to be i.i.d. The modulation and coding scheme (MCS) levels supported by IEEE 802.11n systems are introduced in Table I and Fig. 1 shows the bit error ratio (BER) performance in an i.i.d, SISO, Rayleigh-fading channel scenario in which the DoF value is 2. This BER performance also gives an upper bound in the case of $N > M$ because in this case, the channel exhibits larger DoFs. The BER performance can be approximated as $BER(\gamma) \approx \alpha\gamma^{-\beta}$

TABLE I
MODULATION AND CODING SCHEMES

Mode	Data rate	Modulation	Coding rate	DoF = 2		DoF = 4		DoF = 8	
				α	β	α	β	α	β
1	6.5 Mbps	BPSK	1/2	0.0272	7.528	0.0031	9.853	0.0008	9.540
2	13 Mbps	QPSK	1/2	2.9217	6.228	1.5946	8.221	0.5516	8.996
3	19.5 Mbps	QPSK	3/4	145.3967	4.607	635.9026	7.287	396.1361	8.784
4	26 Mbps	16QAM	1/2	309.5414	4.554	3716.732	6.844	1.3944e4	8.537
5	39 Mbps	16QAM	3/4	2120.869	3.437	1.0223e6	6.280	5.1308e7	8.439
6	52 Mbps	64QAM	2/3	2643.438	2.953	2.6722e6	5.297	3.8569e9	7.825
7	58.5 Mbps	64QAM	3/4	984.6983	2.422	8.6993e5	4.524	1.591e10	7.373
8	65 Mbps	64QAM	5/6	2663.313	2.329	8.6170e7	5.009	1.023e12	7.626

in which γ is the average received SNR and the values of α and β are listed in Table I. In Fig. 1, each mark indicates the simulation result for each MCS level and the solid line shows the approximated results. We can find the approximation results are very close to the simulation ones. For a frame with length L, the frame error ratio (FER) for the i-th STA can be approximated as

$$FER(\gamma_i) = 1 - (1 - BER(\gamma_i))^L \approx L \cdot BER(\gamma_i). \quad (3)$$

For the SU-MIMO scheme, since each STA transmits the data stream with N antennas, this corresponds to two DoFs and we can apply the FER results from Eq. (3). In the calculation of the FER, we should pay attention to the calculation of the average received SNR. As introduced in Section II-A, since the total transmit power, P, is limited to a single STA, the transmit power for each antenna of the STA is equal to P/M. This implies that the data rate does not increase linearly with an increasing number of transmit antennas at the STA in IEEE 802.11n system.

Based on the FER analysis shown in this subsection, we can find that for the data rate selection, each STA does not need to consider the signal strength of the simultaneously transmitting STAs and it only needs to consider the achieved DoF. There are several link adaptation schemes for WLANs and, in this paper, we assume that the STAs adaptively set their data rate based on their average received SNRs for each transmit and receive antenna pair. In order to support an MPR capability of N packets, each STA should select the data rate under the assumption that the achieved DoF is equal to two. The target FER is set to 0.01 in this paper.

2) MMSE Decoding: For the MMSE decoding, the post-detection SNR for the i-th STA is expressed as [19]

$$\gamma_i = \frac{1}{\left[\left(\gamma_0 \boldsymbol{H}^H \boldsymbol{H} + I_M\right)^{-1}\right]_{ii}} - 1, \quad (4)$$

where I_M is the M-array identity matrix. Gao *et al.* [21] analyzed a closed form expression of the post-detection SNR with MMSE decoding. Comparing the expression in [21] and the cumulative distribution function of the post-detection SNR with ZF decoding which can be obtained through integrating Eq. (2), we can easily conclude that the MMSE decoding shows better performance than ZF decoding.

Generally, the MMSE MIMO decoding is considered as the combination of the matched filter which shows good performance at the low interference region and the ZF receiver which shows good performance at the high interference region [22]. Taking the merits of the above two receivers, the MMSE receiver shows better performance than the ZF receiver. The ZF receiver only eliminates the interference from other streams while the MMSE receiver not only eliminates the interference but also reduces the effect of noise. Thus, if the interference STAs operate in much higher SNR regions, compared to a tagged STA, due to strong interference, the MMSE receiver yields almost the same performance as the ZF receiver. Especially, in a randomly distributed STA environment, STAs which transmit frames simultaneously may give strong interference to the other STAs and each STA has no information on the locations of other STAs. Thus, in order to guarantee the required FER, each STA should select a proper data rate in the most conservative way - assuming the ZF receiver at the AP.

3) SIC Decoding: For the SIC decoding, ordering is an important issue and the performance is affected by the ordering. In the case of a single rate transmission and a slow, flat fading channel, if there are two receive antennas, optimal ordering is to first detect the stream with the highest SNR. If there are more than two receive antennas, the optimal decoding order is not discovered yet. When it comes to the multi-rate transmission, even with two receive antennas, decoding the stream with the highest SNR first is not an optimal ordering due to different data rates and different required SNR values for successful decoding. Moreover, the channel considered in WLAN is in a selective-fading environment where the channel gains for a data stream may vary symbol by symbol. Thus, the ordering is more complicated and the optimal decoding performance with SIC can be obtained with testing all possible decoding orders.

The implementation of the SIC decoding is based on a Gram-Schmidt orthogonalization procedure, which builds a set of orthogonal vectors from a set of linearly independent vectors. The post-detection SNR for the i-th STA was analyzed in [23] for the identical variance for each channel gain. However, following the analytical procedure shown in [23], we can easily obtain the post-detection SNR distribution for the i-th STA which is Gamma distributed with a DoF of $2(N - M + i)$ and it is not affected by the SNR values of the other STAs. Note that the post-detection SNR is derived with the assumption that the previous i-1 streams are successfully decoded. Otherwise, the decoding may fail with a high probability due to strong interference. Here we can observe that the post-detection SNR for the first decoding stream has the same SNR distribution as that with the ZF decoding and, consequently, has the same FER performance. Moreover, upon successful decoding for the previous streams, the remaining streams can obtain better FER performance than that with the ZF receiver due to larger DoF values. With all possible decoding orders, each stream has opportunities to be decoded first, which yields the same FER performance with the ZF receiver. Moreover, each stream may also have opportunities to be decoded second or later if there exist decoding successes of other streams and, in this case, the FER performance may be better than that with the ZF receiver. Thus, we can conclude the ZF receiver shows lower performance than the SIC receiver with all possible decoding orders.

Since the number of simultaneously transmitting STAs varies over time and the optimal order of SIC decoding is hard to find in practice, each STA can select the data rate, assuming that its packet is decoded first. The selected data rate is identical to that with the ZF receiver.

B. PHY layer Analysis with the MU-MIMO-STBC Scheme

In the case of N=2 in which STAs and AP have two antennas respectively, each STA can transmit a data stream with an Alamouti scheme. In the case of simultaneous transmissions from two STAs, there are two kinds of decoding technique at the AP side: ML and array processing [13]. Since ML has a very high computational complexity, we consider the array processing method which eventually cancels the interference of other STAs. Following the array processing, we can find the signal strengths of other STAs do not affect the post-detection SNR statistics of the desired STA. For the array processing, Kazemitabar and jafarkhani [24] demonstrated that the post-detection SNR is Gamma distributed with a DoF value of $4(N-M+1)$ where M is the number of the simultaneously transmitted STAs. In order to support simultaneous transmissions from maximum N STAs, each STA should select a proper data rate under the assumption of DoF=4, which is the result of the assumption, $M=N$.

In the case of N=4 in which STAs and AP have four antennas each, since there does not exist orthogonal STBC for complex signal transmission [25], we consider a quasi-orthogonal STBC (QOSTBC) transmission for each STA. With the QOSTBC transmissions, the AP can cancel the interference of other STAs through array processing [12]. It was proven that the QOSTBC does not reduces the data rate while still achieving a full diversity gain of $4(N-M+1)$ [24]. Since this full diversity order is obtained through ML decoding for the rotated modulation symbol pairs of the QOSTBC scheme, the implementation complexity is very high especially with high order modulations. Thus, it is hard to obtain the exact BER performance in WLAN systems with the QOSTBC transmissions for a convolutional coding scheme. In this paper, we approximate the post-detection SNR as the Gamma distribution with a DoF of $8(N-M+1)$ which shows the same diversity gain of $4(N-M+1)$. Note that this approximation can be obtained through the complete separation of the rotated modulation symbol pairs which cannot be obtained with ML decoding for the QOSTBC scheme. Consequently, it shows better BER performance. In order to support simultaneous transmissions from maximum N STAs, each STA should select a proper data rate under the assumption of DoF=8, which is the result of the assumption, $M=N$.

The BER performance for the data rates supported by a WLAN with post-detection SNR values with DoFs of 4 and 8 also can be approximated with Eq. (3). The corresponding (α, β) values are also listed in Table. I.

C. MAC Layer Analysis

IEEE 802.11n WLAN adopts a CSMA/CA protocol with binary exponential backoff. The STAs take a backoff procedure after their shared channel is sensed idle during a DCF interframe space (DIFS) period. Each STA randomly chooses an integer value as a backoff counter value within $(0, CW-1)$, where CW is the contention window size and is initially set to a minimum value $CW_{\min}$. In the backoff procedure, the backoff counter value is decreased by one for each idle $SlotTime$ and is frozen if the channel is occupied by other STAs. An STA with a backoff counter value of 0 can transmit a frame. After the successful transmission, the STA can receive an acknowledgement (ACK) frame from the receiver after a short interframe space (SIFS) period. If the transmitted frame collides with other frames, then the STA cannot receive an ACK frame from the receiver. In order to retransmit the failed frame, the STA doubles the CW value and returns to a backoff procedure after an ACK timeout period. The CW value can be increased up to a maximum value $CW_{\max}$ and there also exists a maximum retry limit for retransmission.

Bianchi [2] proposed a Discrete Time Markov Chain (DTMC) model to compute the saturation throughput in a saturation traffic case. We extended this model to accommodate simultaneous transmissions from multiple STAs at PHY layer in a network environment where AP is located at the center of a basic service set (BSS) and n-contending STAs communicate with the AP. Thus, the multi-rate transmissions are considered. We focus on uplink performance without considering downlink transmission except the transmission of ACK frames.

For each STA, let τ be the transmission probability and p be the backoff stage transition probability that the STA retransmits the previous transmitted frame. Since we assume that the STAs select their data rates for a target FER of 0.01 which is quite low at PHY layer, there are few channel errors and, consequently, the backoff stage transition probability is nearly identical to the collision probability. If the AP can receive maximum N simultaneous transmissions, a transmission failure occurs when there are more than N simultaneous transmissions. From the viewpoints of the backoff procedure [2] and network, we can obtain the relationship between τ and p, as

$$\tau = \frac{2(1-p^{R+1})}{W(1-2^L p^{R+1}) + Wp[\sum_{i=0}^{L-1}(2p)^i] + (1-p^{R+1})}, \tag{5}$$

$$p = 1 - \sum_{m=0}^{N-1} \binom{n-1}{m} \tau^m (1-\tau)^{n-1-m}, \tag{6}$$

where W represents the minimum contention window size $CW_{\min}$. The term L is the maximum number of doublings of the CW, which is identical to $\log_2(CW_{\max}/CW_{\min})$. Numerically solving (5) and (6), the values of τ and p can be obtained. As a special case, $N = 1$ represents the SU-MIMO scheme.

The probability P_{tr}^m that there are simultaneous transmissions from $m(\geq 0)$ STAs in a time slot is expressed as

$$P_{tr}^m = \binom{n}{m}\tau^m(1-\tau)^{n-m}. \quad (7)$$

Then, the average throughput can be obtained as

$$\text{Throughput} = \frac{\mathbb{E}[\text{payload}]}{P_{tr}^0\sigma + \sum_{m=1}^{N} P_{tr}^m T_{tr}^m + \sum_{m=N+1}^{n} P_{tr}^m T_c^m}, \quad (8)$$

where σ is the backoff slot time. T_{tr}^m and T_c^m are the successful and collided transmission times for m STAs' simultaneous transmissions and they are expressed as

$$\begin{aligned} T_{tr}^m = &\ \mathbb{E}[MaxDataTime_m] + SIFS + ACKtime \\ &+DIFS, \\ T_c^m = &\ \mathbb{E}[MaxDataTime_m] + ACKtimeout + DIFS, \end{aligned}$$

where $\mathbb{E}[MaxDataTime_m]$ is the average maximum data transmission time of the m simultaneously transmitting STAs. Since we consider multi-rate WLANs and STAs select their data rates based on the average received SNR, the data transmission time may be different for each STA. For simplicity, we assume the payload size is constant. Let K and $R_k(R_1 < R_2 < ... < R_K)$ denote the number of data rates and the $k-$th data rate, respectively. m_k denotes the number of STAs with a data rate of R_k, and we set $p_k = m_k/n, (k = 1, 2, ..., K)$ to denote its portion. Let $T_k(T_1 > T_2 > ... > T_K)$ denote the data transmission time related to the data rate of R_k. Then, the probability that the channel occupation time is T_k due to m simultaneous transmissions is expressed as

$$\begin{aligned} P_{m,k} &= \textstyle\sum_{i=1}^{m} \binom{m}{i} p_k^i p_{k+}^{m-i} \\ &= \left(1 - \textstyle\sum_{i=0}^{k-1} p_i\right)^m - \left(1 - \textstyle\sum_{i=0}^{k} p_i\right)^m, \end{aligned} \quad (9)$$

where $p_{k+} = \sum_{j=k+1}^{K} p_j$ and the value of the auxiliary parameter p_0 is 0. Consequently, we can obtain the average transmission time due to m simultaneous transmissions as

$$\mathbb{E}[MaxDataTime_m] = \sum_{k=1}^{K} T_k P_{m,k}. \quad (10)$$

Thus far, we have analyzed the throughput performance of the multi-rate WLANs. This throughput performance analysis is applicable to stationary STAs each of which has a fixed data rate. However, we have more interest in a randomly distributed environment. In this case, the average throughput can be expressed as

$$\text{Throughput}_{\text{avg}} = \lim_{S\to\infty} \frac{1}{S}\sum_{l=1}^{S} \text{Throughput}_l(p_{l1}, p_{l2}, ..., p_{lK}). \quad (11)$$

where S and $\text{Throughput}_l(p_{l1}, p_{l2}, ..., p_{lK})$ denote the total number of STA location samples[1] and the throughput of the l-th sample of the STA locations where the portion of each data rate is $p_{lk}(k = 1, 2, ..., K)$. It is not easy to analyze the exact result for the average throughput. However, as proved in AppendixA, the throughput equation described in Eq. (8) is a convex function of the probability set $\{p_k(k = 1, 2, ..., K)\}$. Based on the Jensen's inequality, we can obtain the lower bound of the average throughput,

$$\text{Throughput}_{\text{avg}} \geq \text{Throughput}_l(\overline{p}_1, \overline{p}_2, ..., \overline{p}_K), \quad (12)$$

where $\{\overline{p}_1, \overline{p}_2, ..., \overline{p}_K\}$ are the probability mass function of the data rates of STAs and can be expressed as $\overline{p}_k = \mathbb{E}[\frac{m_k}{n}]$ for $k = 1, 2, ..., K$. We will show in Section V that this lower bound is very close to the exact average throughput value and, thus, it is a good approximation.

[1]For an STA location sample, each STA has a fixed distance value away from the AP and, consequently, each STA selects a fixed data rate. The locations of STAs may vary over different STA location samples.

IV. Maximum Throughput Analysis

For the MU-MIMO scheme based uplink WLANs, if there are a small number of STAs in the system, the performance enhancement is limited even though the AP can support a number of simultaneous transmissions. Thus, it is necessary to analyze the maximum throughput by exploiting the potential of the MU-MIMO scheme. There have been several papers on the maximum throughput analysis in WLANs [2] [18]. For the SU-MIMO scheme based WLANs in which a simple collision model is considered, Bianchi [2] analyzed the maximum throughput performance as a function of the number of contending STAs and average payload transmission time. For the MU-MIMO based WLANs in which MPR is available, Zhang *et al.* [18] evaluated the optimal asymptotic throughput as a function of exponential factor of the backoff algorithm and showed the 'superlinearity' of the MPR capable WLANs. In this section, for the finite population, we analyze the maximum throughput of the MU-MIMO scheme based uplink WLANs with a basic access mechanism and find a selection criterion between the SU-MIMO scheme and the MU-MIMO scheme in uplink WLANs.

In order to analyze the maximum throughput of the WLANs with the MU-MIMO scheme, we define the average channel occupancy time as

$$T_s = \frac{\sum_{m=1}^{N} P_{tr}^m T_{tr}^m + \sum_{m=N+1}^{n} P_{tr}^m T_c^m}{1-(1-\tau)^n}. \quad (13)$$

This average channel occupancy time can be calculated by each STA or AP in an infrastructure based WLAN through long term channel observations. Based on this definition, the throughput equation can be modified as

$$\text{Throughput} = \frac{\mathbb{E}[P]}{\sigma} \cdot \frac{\sum_{i=1}^{N} \binom{n}{i} i\tau^i(1-\tau)^{n-i}}{(1-\tau)^n + [1-(1-\tau)^n]T_s^*}, \quad (14)$$

where $T_s^* = T_s/\sigma$ is the normalized channel occupancy time. The throughput can be maximized when the derivative of Eq. (14) with respect to τ is equal to 0.

$$(1-\tau)^n\left(1 - \frac{1}{T_s^*}\right) + \frac{\sum_{i=1}^{N} \binom{n}{i} i\tau^i(1-\tau)^{n-i}}{\sum_{i=1}^{N} \binom{n}{i} i^2\tau^i(1-\tau)^{n-i}} n\tau - 1 = 0. \quad (15)$$

Here, we define the average number of transmitting STAs in a slot or the average transmission rate at MAC layer as $x = \tau n$

and assume that $x << n$. Then, Eq. (15) can be rewritten as

$$\left[\left(1-\frac{x}{n}\right)^{-\frac{n}{x}}\right]^{-x}\left(1-\frac{1}{T_s^*}\right)+\frac{\sum_{i=1}^{N}\binom{n}{i}i\left(\frac{x}{n-x}\right)^{i}}{\sum_{i=1}^{N}\binom{n}{i}i^{2}\left(\frac{x}{n-x}\right)^{i}}x-1$$
$$=\left[\left(1-\frac{x}{n}\right)^{-\frac{n}{x}}\right]^{-x}\left(1-\frac{1}{T_s^*}\right)+\frac{\sum_{i=1}^{N}\left[\frac{n!}{(n-i)!i!}\left(\frac{x}{n-x}\right)^{i}\right]i}{\sum_{i=1}^{N}\left[\frac{n!}{(n-i)!i!}\left(\frac{x}{n-x}\right)^{i}\right]i^{2}}x-1$$
$$\approx e^{-x}\left(1-\frac{1}{T_s^*}\right)+\frac{\sum_{i=1}^{N}\frac{1}{(i-1)!}x^{i}}{\sum_{i=1}^{N}\frac{i}{(i-1)!}x^{i}}x-1=0, \qquad (16)$$

where we use the fact that $\lim_{\frac{x}{n}\to 0}\left(1-\frac{x}{n}\right)^{-\frac{n}{x}}=e$. Eq. (16) implies the average number of transmitting STAs or the average transmission rate at MAC layer is almost constant under the condition of a large number of contending STAs in the network in the case of achieving maximum throughput and, consequently, the optimal transmission probability τ is just inversely proportional to the number of contending STAs. Substituting the result of Eq. (15) into the throughput equation (14), the maximum throughput is obtained as

$$\text{Throughput}_{\text{max}} = \frac{\mathbb{E}[P]}{T_s}\cdot\frac{1}{n\tau}\sum_{i=1}^{N}\binom{n}{i}i^{2}\tau^{i}(1-\tau)^{n-i}$$
$$\approx \frac{\mathbb{E}[P]}{T_s}\cdot\frac{1}{x}e^{-x}\sum_{i=1}^{N}\frac{i}{(i-1)!}x^{i}. \qquad (17)$$

We here define the normalized maximum throughput as follows:

$$\text{Normalized throughput}_{\text{max}} \approx \frac{1}{x}e^{-x}\sum_{i=1}^{N}\frac{i}{(i-1)!}x^{i}. \qquad (18)$$

Then, the normalized maximum throughput is almost constant because x is independent of n and the maximum throughput only depends on the data rate at PHY layer. If we calculate the ratio of the normalized maximum throughput between the MU-MIMO and the SU-MIMO schemes, we can obtain a MIMO multiplexing gain achieved by the MU-MIMO scheme at MAC layer. If the MIMO multiplexing gain of the SU-MIMO scheme at PHY layer is smaller than that of the MU-MIMO scheme at MAC layer, we should select the MU-MIMO scheme in the uplink WLANs when the system has the ability to achieve maximum throughput through controlling the transmission probability of each STA.

The maximum throughput can be obtained by adjusting the CWmin size of each STA in uplink WLANs. From the relationship between τ and the stage transition probability p shown by Eq. (5), we can adjust the CWmin size as follows:

$$W=\left(\frac{2n}{x}-1\right)\frac{(1-2p)(1+p^{R+1})}{p[1-(2p)^{L}]+(1-2p)(1-2^{L}p^{R+1})}, \qquad (19)$$

where the stage transition probability p can be approximated as

$$p = 1-\sum_{i=0}^{N-1}\binom{n-1}{i}\tau^{i}(1-\tau)^{n-1-i}$$
$$\approx 1-\frac{1}{x}e^{-x}\sum_{i=1}^{N}\frac{1}{(i-1)!}x^{i}. \qquad (20)$$

From Eq. (20), we can find the state transition probability p is almost constant regardless of the number of contending STAs. Thus, the value of CWmin is linearly increased with increasing the number of contending STAs, as shown in Eq. (19). With this calculated value of CWmin, each STA can eventually adjust the transmission probability.

TABLE II
SYSTEM PARAMETERS

PHY layer parameters	
Path loss at 1m	44.2 dB
Path-loss exponent	4
Transmit power of STA	200 mW
N_0	-199 dBW/Hz
Bandwidth	20 MHz
Target FER	0.01
MAC layer parameters	
DIFS	34 μs
SlotTime	9 μs
SIFS	16 μs
ACKtime	64 μs
ACKtimeout	80 μs
PHY overhead	40 μs
$CW_{\min}$	16
$CW_{\max}$	1024
Retry limit	7

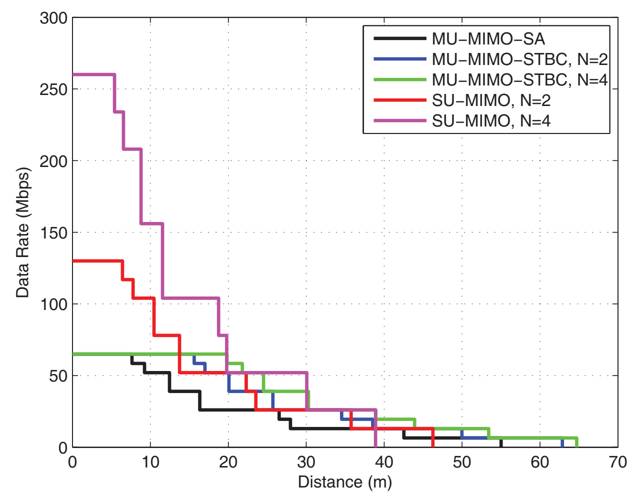

Fig. 2. Transmission data rates over distances.

V. NUMERICAL RESULTS

We consider a scenario in which an AP is located at the center and there exist only uplink traffic and their corresponding ACK frames transmitted from the AP. STAs are uniformly distributed in a BSS and adaptively select their data rates based on their average receive SNR values for a target FER of 0.01. The STAs are assumed to always have packets to transmit. The system parameters are listed in Table II in which the PHY layer parameters selected from [5] and the MAC layer parameters are selected from IEEE 802.11n standard [9]. The ACKtime and ACKtimeout are calculated by assuming the lowest MCS level for successful decoding. The selection of CWmin and CWmax values is based on the description part of MIMO-OFDM PHY characteristics in IEEE 802.11n standard [9].

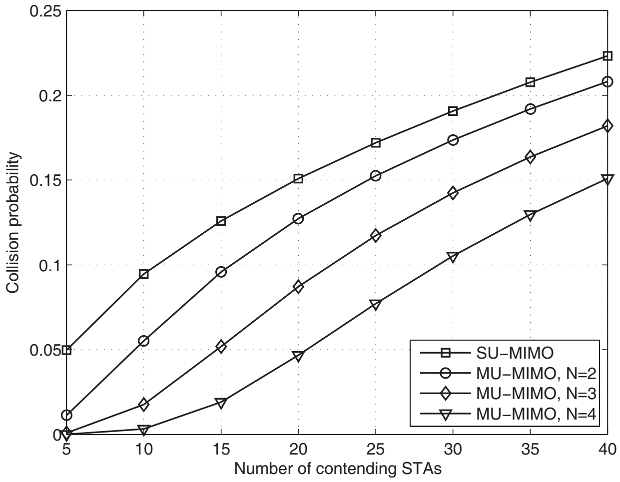

Fig. 3. Collision probability.

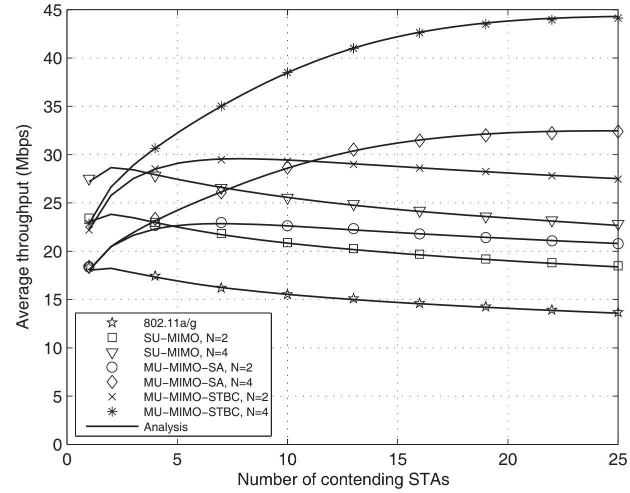

Fig. 4. Average throughput with payload size 1000 bytes and radius 20m.

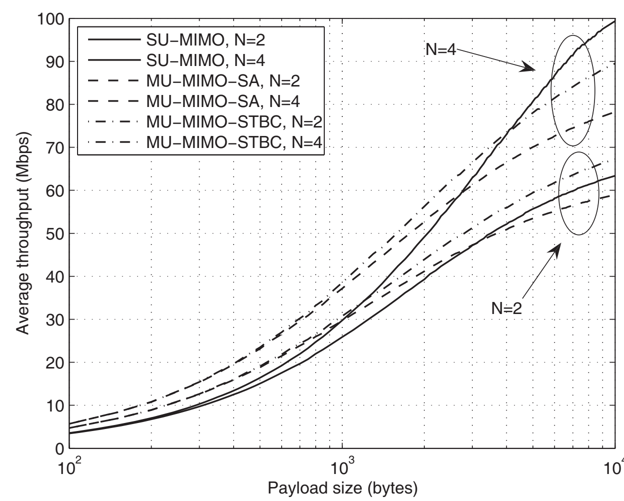

Fig. 5. Average throughput with 10 STAs and radius 10m.

A. Throughput Performance

The average SNR for the i-th STA with distance D_i from the AP is

$$\gamma(D_i)(dB) = \gamma_0(dB) - 44.2 - 40\log_{10} D_i, \tag{21}$$

where the path loss at 1m is set to 44.2dB and the path loss exponent is set to 4 as listed in Table II. The transmit power of each STA is set to 200mW. Fig. 2 shows the data rate for varying distances from the viewpoint of each STA when the payload size is set to 1000 bytes and the required FER constraint is set to 0.01. With the MU-MIMO-SA scheme, since each STA transmits its data steam with a single antenna, the data rate is lower than that of the SU-MIMO scheme. For the SU-MIMO scheme, with an increasing number of antennas at the STA and AP, the data rate can be increased. Since the MU-MIMO-STBC scheme achieves a diversity gain at PHY layer, it shows better data rates than that of the MU-MIMO-SA scheme.

Fig. 3 shows the collision probability for both the SU- and MU-MIMO systems. The collision probability is one of the most critical factors affecting the MAC layer throughput performance. With the SU-MIMO scheme, a collision occurs if more than two STAs simultaneously transmit their frames. With the MU-MIMO scheme, as noted in Section II-C, a collision occurs only when the number of simultaneous transmissions is larger than that of receive antennas at the AP. Hence, the collision probability of the MU-MIMO scheme decreases as the number of receive antennas at the AP increases, while the collision probability of the SU-MIMO scheme does not vary according to the number of antennas at the AP. The MU-MIMO scheme yields a lower collision probability than that of the SU-MIMO scheme.

Fig. 4 shows the average throughput for varying the number of STAs and the number of receive antennas at the AP when the payload size is set to 1000bytes and cell radius is set to 20m. For each simulation, we distribute STAs uniformly in a BSS and perform simulations 200 times and then obtain the average results. Although the throughput performance is analyzed for the lower bound, it agrees well with simulation results and we can use it as an approximation of the average throughput performance. Both the SU- and MU-MIMO schemes yield better average throughput than legacy IEEE 802.11a/g systems in which the AP has one receive antenna. Moreover, we can find there is an tradeoff between the SU- and MU-MIMO schemes. At the extreme case of one STA in a BSS, we can find both the MU-MIMO-SA and the MU-MIMO-STBC schemes show less throughput performance than that of the SU-MIMO scheme. With increasing the number of contending STAs, the MU-MIMO scheme shows better throughput performance than that of the SU-MIMO scheme.

Fig. 5 shows the average throughput performance for varying the payload size from 100bytes to 10000bytes when there are 10 contending STAs in a BSS and the cell radius is 10m. We can find that the MU-MIMO scheme shows better throughput performance than the SU-MIMO scheme when the payload size is small. In the case of $N = 4$, if we increase the payload size, the throughput performance of the SU-MIMO scheme increases faster than that of the MU-MIMO schemes, and eventually shows better performance with a large payload size. In the case of $N = 2$, there is a throughput performance tradeoff between the MU-MIMO-SA scheme and the SU-MIMO scheme while the MU-MIMO-STBC scheme always shows better throughput performance than that of the SU-

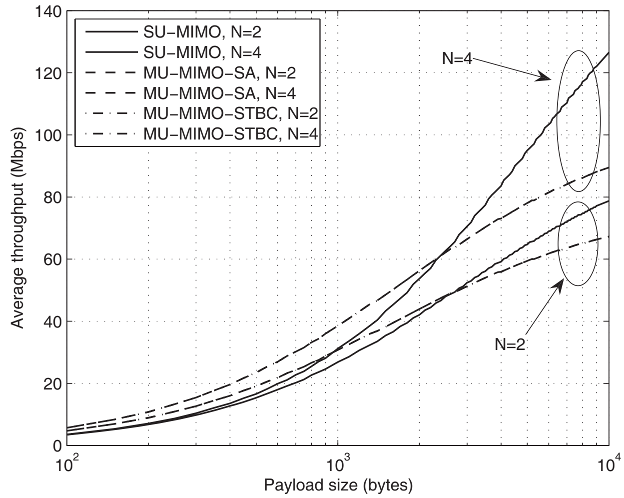

Fig. 6. Average throughput with 10 STAs and radius 5m.

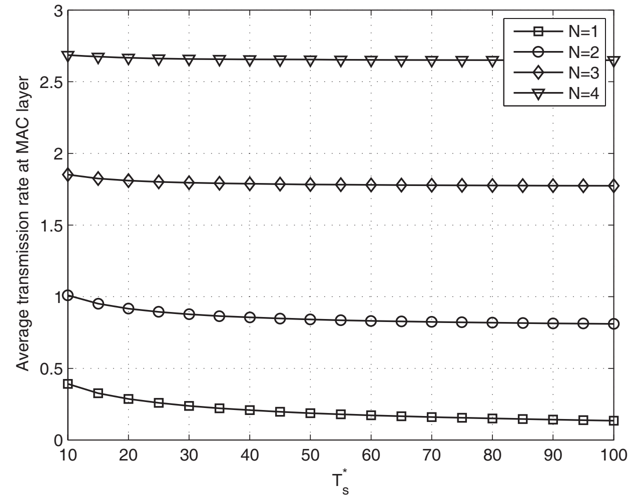

Fig. 7. Average transmission rate at MAC layer

MIMO scheme. The MU-MIMO-STBC obtains a diversity gain at PHY layer so that the STAs can select the higher data rate at low SNR region while still obtaining a multiplexing gain at MAC layer. Thus, with high probability, the MU-MIMO-STBC yields better throughput performance than that of the SU-MIMO scheme with a large cell size. Fig. 6 shows the average throughput performance with a small cell radius of 5m. If the cell size becomes small, an increase in the data rate with the diversity gain is limited. Thus, STAs with both the MU-MIMO-SA and MU-MIMO-STBC schemes select almost the same data rate, i.e., the highest MCS level and, consequently, the throughput performance is also identical and shows a tradeoff, compared to the SU-MIMO scheme.

In summary, if the payload size is small and there is a large number of contending STAs, the MU-MIMO scheme yields better throughput performance than the SU-MIMO scheme. In addition to the payload size and network population, the channel parameters such as the path loss exponent and channel distribution and the location distribution of STAs may also affect the throughput performance, it is hard to predict the accurate tradeoff boundary of the throughput performance between the SU- and MU-MIMO schemes. In order to find a more meaningful insight, we compare the SU- and MU-MIMO schemes in the case of the maximum throughput in the next subsection.

B. Maximum Throughput Comparison

For the conventional WLANs, the maximum throughput was first analyzed in [2]. The result in [2] shows that the transmission probability of each STA should be controlled as a function of the number of contending STAs and the average channel occupation time in order to obtain the maximum throughput. The effect of normalized channel occupancy time on the optimal transmission probability cannot be negligible as shown in by $\tau \approx 1/(n\sqrt{T_s^*/2})$, which is the result in [2].

Fig. 7 shows the average transmission rate at MAC layer for varying the normalized channel occupancy time T_s^* which is obtained from numerically solving Eq. (16). An increase in the value of T_s^* reduces the average transmission rate at MAC layer. Especially, the decrement is relatively large in the case of $N = 1$. However, as the number of receive antennas at the AP increases, the negative slope becomes nearly flatter and, thus, the effect of normalized channel occupancy time on the optimal average transmission rate at MAC layer becomes smaller. In the case of $N = 4$, the average transmission rate at MAC layer is almost constant. The detailed values are listed in Table III. Even though the channel occupancy time tends to infinity, the average number of transmitting STAs is almost not changed in the case of $N = 4$. The independence between the average number of transmitting STAs and channel occupancy time makes it easy to achieve maximum throughput because we do not need to consider the normalized channel occupancy time in tuning the MAC layer parameters. If the normalized channel occupancy time is hard to predict, we can simply use the value of the average transmission rate at MAC layer with a case of the infinite value of T_s^* to control the CWmin size described in Eq. (19).

TABLE III
AVERAGE TRANSMISSION RATE AT MAC LAYER VERSUS THE TRANSMISSION TIME AND THE NUMBER OF RECEIVE ANTENNAS AT THE AP

	Average transmission rate at MAC layer			
	N=1	N=2	N=3	N=4
$T_s^* = 10$	0.3917	1.0099	1.8519	2.6862
$T_s^* = 100$	0.1352	0.8101	1.7737	2.6497
$T_s^* = \infty$	0.0114	0.7736	1.7637	2.6454

Fig. 8 shows the normalized maximum throughput ratio between the MU-MIMO scheme and the SU-MIMO scheme. In the case of $T_s^* = 100$, the throughput ratios for $N = 2, 3$, and 4 are given by 1.33, 1.80 and 2.37, respectively. This result can be used to determine a MIMO mode selection criterion between the SU- and MU-MIMO schemes in uplink WLANs. For example, in the case of $T_s^* = 100$ and $N = 2$, it is better for each STA to choose the SU-MIMO scheme if the spectral efficiency of the SU-MIMO scheme is 1.33 times higher than that of the MU-MIMO scheme at PHY layer.

VI. CONCLUSIONS

In this paper, we first compared the performance of multi-rate uplink WLANs with the SU- and MU-MIMO schemes in

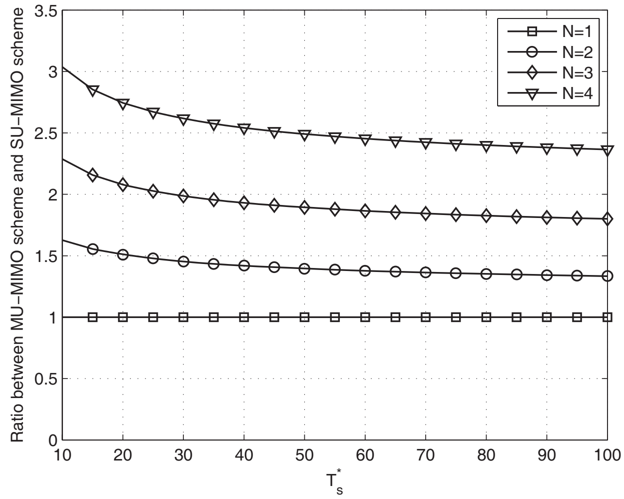

Fig. 8. Normalized throughput ratio between the MU-MIMO scheme and the SU-MIMO scheme.

terms of average throughput in a random STA distribution scenario. The transmission data rate at PHY layer was analyzed in a Rayleigh fading channel and the lower bound of the average throughput was also approximately analyzed at MAC layer. The SU-MIMO system yields better transmission data rate at PHY layer with short distances, while the MU-MIMO system shows a lower collision probability at MAC layer. The MU-MIMO system yields better throughput performance for small payload sizes and a large number of contending STAs. The inefficiency of the SU-MIMO scheme for small-sized packets was also indicated and the MU-MIMO scheme is a good candidate for improving the performance from this viewpoint. Moreover, we also analyzed and compared the normalized maximum throughput of uplink WLANs with the SU- and MU-MIMO schemes and found a decision criterion to select a proper MIMO mode through comparing the normalized maximum throughput and the spectral efficiency at PHY layer.

APPENDIX

In order to prove the convexity of the throughput formula shown by Eq. (8), we need to take the second derivative and prove that it is equal to or larger than 0. First, we define the denominator of the throughput formular as $f(p_1, p_2, \ldots, , p_K)$,

$$\begin{aligned} & f(p_1, p_2, \ldots, , p_K) \\ & = P_{tr}^0 \sigma + \sum_{m=1}^{N} P_{tr}^m T_{tr}^m + \sum_{m=N+1}^{n} P_{tr}^m T_c^m \\ & = P_{tr}^0 \sigma + \sum_{m=1}^{N} P_{tr}^m [SIFS + ACKtime + DIFS] \\ & \quad + \sum_{m=N+1}^{n} P_{tr}^m [ACKtimeout + DIFS] \\ & \quad + \sum_{m=1}^{N} P_{tr}^m \mathbb{E}[MaxDataTime_m] \\ & \quad + \sum_{m=N+1}^{n} P_{tr}^m \mathbb{E}[MaxDataTime_m] \\ & = C + \sum_{m=1}^{N} P_{tr}^m \mathbb{E}[MaxDataTime_m] \\ & \quad + \sum_{m=N+1}^{n} P_{tr}^m \mathbb{E}[MaxDataTime_m], \end{aligned} \tag{22}$$

where $C = P_{tr}^0 \sigma + \sum_{m=1}^{N} P_{tr}^m [SIFS + ACKtime + DIFS] + \sum_{m=N+1}^{n} P_{tr}^m [ACKtimeout + DIFS]$.

We substitute the result for $MaxDataTime$ in Eq. (11), then for any value $r (1 \leq r \leq K)$ we have

$$\begin{aligned} & f(p_1, p_2, \ldots, , p_K) \\ & = C + \sum_{m=1}^{n} P_{tr}^m \\ & \quad \cdot \left\{ \sum_{k=1}^{K} T_k \left[\left(\sum_{j=k}^{K} p_j \right)^m - \left(1 - \sum_{j=1}^{k} p_j \right)^m \right] \right\} \\ & = C + \sum_{m=1}^{n} P_{tr}^m \left\{ T_1 \left[1 - \left(\sum_{j=2}^{K} p_j \right)^m \right] \right. \\ & \quad + \sum_{k=2}^{K-1} T_k \left[\left(\sum_{j=k}^{K} p_j \right)^m - \left(\sum_{j=k+1}^{K} p_j \right)^m \right] \\ & \quad \left. + T_K \left(1 - \sum_{j=1}^{K-1} p_j \right)^m \right\} \\ & = C + \sum_{m=1}^{n} P_{tr}^m \left\{ T_1 \left[1 - \left(\sum_{j=2}^{K} p_j \right)^m \right] \right. \\ & \quad + \sum_{k=2}^{r-1} T_k \left[\left(\sum_{j=k}^{K} p_j \right)^m - \left(\sum_{j=k+1}^{K} p_j \right)^m \right] \\ & \quad + T_r \left(\sum_{j=r}^{K} p_j \right)^m - T_r \left(1 - \sum_{j=1}^{r} p_j \right)^m \\ & \quad + \sum_{k=r+1}^{K-1} T_k \left[\left(1 - \sum_{j=1}^{k-1} p_j \right)^m - \left(1 - \sum_{j=1}^{k} p_j \right)^m \right] \\ & \quad \left. + T_K \left(1 - \sum_{j=1}^{K-1} p_j \right)^m \right\}. \end{aligned} \tag{23}$$

$$\begin{aligned} & \frac{df(p_1, p_2, \ldots, , p_K)}{dp_r} \\ & = \sum_{m=1}^{n} m P_{tr}^m \left\{ -T_1 \left(\sum_{j=2}^{K} p_j \right)^{m-1} \right. \\ & \quad + \sum_{k=2}^{r-1} T_k \left[\left(\sum_{j=k}^{K} p_j \right)^{m-1} - \left(\sum_{j=k+1}^{K} p_j \right)^{m-1} \right] \\ & \quad + T_r \left(\sum_{j=r}^{K} p_j \right)^{m-1} + T_r \left(1 - \sum_{j=1}^{r} p_j \right)^{m-1} \\ & \quad + \sum_{k=r+1}^{K-1} T_k \left[- \left(1 - \sum_{j=1}^{k-1} p_j \right)^{m-1} \right. \\ & \quad \left. \left. + \left(1 - \sum_{j=1}^{k} p_j \right)^{m-1} \right] - T_K \left(1 - \sum_{j=1}^{K-1} p_j \right)^{m-1} \right\} \\ & = \sum_{m=2}^{n} m P_{tr}^m \left\{ -T_1 \left(\sum_{j=2}^{K} p_j \right)^{m-1} \right. \\ & \quad + \sum_{k=2}^{r-1} T_k \left[\left(\sum_{j=k}^{K} p_j \right)^{m-1} - \left(\sum_{j=k+1}^{K} p_j \right)^{m-1} \right] \\ & \quad + T_r \left(\sum_{j=r}^{K} p_j \right)^{m-1} + T_r \left(1 - \sum_{j=1}^{r} p_j \right)^{m-1} \\ & \quad + \sum_{k=r+1}^{K-1} T_k \left[- \left(1 - \sum_{j=1}^{k-1} p_j \right)^{m-1} \right. \\ & \quad \left. \left. + \left(1 - \sum_{j=1}^{k} p_j \right)^{m-1} \right] - T_K \left(1 - \sum_{j=1}^{K-1} p_j \right)^{m-1} \right\} \\ & \quad + P_{tr}^1 \left[-T_1 + 2T_r - T_K \right]. \end{aligned} \tag{24}$$

$$\begin{aligned} & \frac{df^2(p_1, p_2, \ldots, , p_K)}{dp_r^2} \\ & = \sum_{m=2}^{n} m(m-1) P_{tr}^m \left\{ -T_1 \left(\sum_{j=2}^{K} p_j \right)^{m-2} \right. \\ & \quad + \sum_{k=2}^{r-1} T_k \left[\left(\sum_{j=k}^{K} p_j \right)^{m-2} - \left(\sum_{j=k+1}^{K} p_j \right)^{m-2} \right] \\ & \quad + T_r \left(\sum_{j=r}^{K} p_j \right)^{m-2} - T_r \left(1 - \sum_{j=1}^{r} p_j \right)^{m-2} \\ & \quad + \sum_{k=r+1}^{K-1} T_k \left[\left(1 - \sum_{j=1}^{k-1} p_j \right)^{m-2} \right. \\ & \quad \left. \left. - \left(1 - \sum_{j=1}^{k} p_j \right)^{m-2} \right] + T_K \left(1 - \sum_{j=1}^{K-1} p_j \right)^{m-2} \right\} \\ & = \sum_{m=2}^{n} m(m-1) P_{tr}^m \\ & \quad \cdot \left\{ \sum_{k=1}^{r-1} (-T_k + T_{k+1}) \left(\sum_{j=k+1}^{K} p_j \right)^{m-2} \right. \\ & \quad \left. + \sum_{k=r}^{K-1} (-T_k + T_{k+1}) \left(1 - \sum_{j=1}^{k} p_j \right)^{m-2} \right\} \leq 0. \end{aligned} \tag{25}$$

The last inequality is based on the fact of $T_1 > T_2 > \ldots > T_K$. As a consequence, we have

$$\frac{d^2\text{Throughput}}{dp_r^2} = \mathbb{E}[\text{payload}] \cdot \left\{ \frac{-f''(p_1, p_2, ..., , p_K)}{f^2(p_1, p_2, ..., , p_K)} + \frac{2\left[f'(p_1, p_2, ..., , p_K)\right]^2}{f^3(p_1, p_2, ..., , p_K)} \right\} \geq 0, \tag{26}$$

which shows the throughput is a convex function of any p_r.

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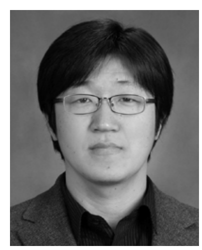

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