On the Multi-User Diversity with Fixed Power Transmission in Cognitive Radio Networks

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***Abstract*—In this letter, we investigate the multi-user diversity (MUD) in an underlay cognitive radio (CR) network where multiple secondary transmitters and primary receivers exist. Many studies on MUD in the underlay CR network have assumed that the secondary transmitters adaptively control their transmit power in order to achieve the optimal MUD gain, maintaining the interference at the primary receivers below a pre-determined level. We, however, prove the *optimal* MUD gain can be also achieved by the *fixed power* transmission strategy. In contrast to the adaptive power transmission strategy, the fixed power operation in the secondary transmitters relaxes the coordination constraint between the primary and secondary networks, and significantly reduces the signaling and feedback overhead from the secondary transmitters to the secondary receiver.**

***Index Terms*—Cognitive radio networks, spectrum sharing, multi-user diversity, power control, opportunistic scheduling.**

I. INTRODUCTION

SPECTRUM shortage is one of the most challenging issues in next-generation mobile communication systems because the demand for radio spectrum has been continuously increasing to support explosively growing mobile data traffic [1]. As a promising approach to solve the spectrum shortage problem, the federal communication commission (FCC) [2] proposed the concept of spectrum sharing, in which the spectrum band licensed to a primary network can be shared by another secondary network as long as the interference level perceived at primary receivers (PRs) is maintained below a pre-determined level. In general, the allowable interference level, called *interference temperature*, is imposed by the primary network. Although the spectrum sharing technique increases the spectral efficiency, the interference constraint dramatically aggravates the performance of the secondary network.

There have been many studies on mitigating the drawback of the spectrum sharing technique by adopting user scheduling algorithms [3]–[8]. Especially, these studies have investigated the effect of multi-user diversity (MUD) gain in spectrum sharing systems. [3]–[5] showed that the throughput in the uplink secondary network, i.e., multiple access channel (MAC), scales according to $\log\log N_s$, where N_s denotes the number of secondary transmitters (STs) when a single PR exists in the primary network. The scaling law achieved in the underlay cognitive radio (CR) networks is optimal in that it is the same as the throughput scaling without any interference constraint. [6] showed that the throughput of the secondary network can also achieve the same scaling in both broadcast channel (BC) and parallel access channel (PAC). In spite of these studies on MUD in the secondary network, most studies are based on the assumption that each ST has the perfect knowledge on the channel state information (CSI) from itself to the PRs. For the acquisition of the perfect CSI, a significant overhead is imposed to the primary network. Furthermore, the SR should have information on transmit power levels of the STs.

In this letter, therefore, we first propose a fixed power strategy for STs. The proposed scheme can operate based on *1-bit indicators* instead of the perfect CSI on interference channels from primary network even when the primary network adopts frequency division duplexing (FDD). In addition, the proposed scheme can significantly reduce the signaling and feedback overhead in the secondary network due to its inherent opportunistic feedback nature, compared with the conventional scheme with the adaptive power transmission at each ST. We investigate the characteristics of MUD in the secondary network when multiple PRs exist and prove that the proposed fixed power transmission strategy achieves the optimal throughput scaling. To the best of our knowledge, there has been no such study on the characteristics of MUD in the secondary network when each ST sends data with the fixed power and there exist multiple PRs.

II. SYSTEM MODEL

Fig. 1 depicts an underlay CR network where STs share a common spectrum which is primarily licensed to a primary network. A secondary network operates in frequency division duplex (FDD) mode. N_s STs have data to send to a SR, which corresponds to the uplink of cellular networks. On the other hand, in a primary network, a single transmitter has data to send to N_p PRs, which corresponds to the downlink of cellular networks. We assume that all transmitters and receivers are equipped with a single antenna. The interference temperature denoted by Q is pre-determined by the PRs. Although a single PR out of N_p may receive data from the primary transmitter (PT), the STs have no information on which PR is receiving the data. Hence, all STs need to satisfy N_p interference constraints imposed by the PRs regardless of the scheduling result of the primary network. $g_{i,j}$ and h_i denote the interference channel gain from the i-th ST to the j-th PR and signal channel gain from the i-th ST to the SR, respectively. Without loss of generality, all channel gains are assumed to be *i.i.d.* exponential random variables with unit mean. We consider a block-fading channel model where the

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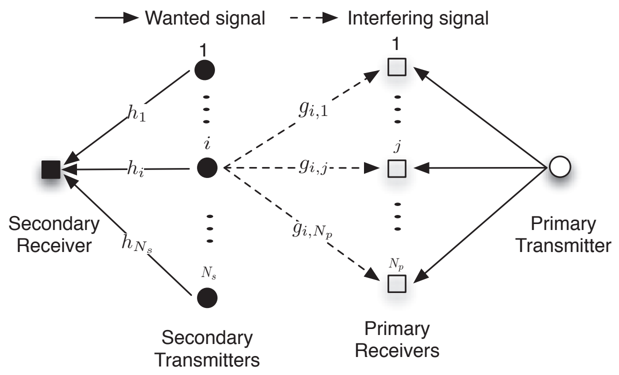

Fig. 1. An underlay cognitive radio network where there exist N_s STs and N_p PRs.

channel gains are constant during one transmission block and independently changes every transmission block. It is also assumed that there is no direct link between the PT and the SR. The received signal at the SR from the i-th ST is given as

$$y_i = \sqrt{h_i} x_i + z_i, \tag{1}$$

where $x_i (\mathbb{E}\left[|x_i|^2\right] \leq P)$ and $z_i \sim N(0,1)$ denote the transmit signal of the i-th ST and Gaussian noise at the SR, respectively.

III. FIXED POWER TRANSMISSION AND USER SCHEDULING

In this section, we first explain the adaptive power transmission strategy of STs as a reference and then the user scheduling algorithm with the fixed power transmission of STs is proposed for the secondary network. We also compare two schemes in terms of performance and complexity.

A. Adaptive Power Transmission

In the adaptive power transmission (APT) strategy, each ST adjusts the transmit power according to the interference channel from itself to PRs within available power constraint. The transmit power of the i-th ST is given by

$$p_i^A = \begin{cases} P, & P g_i \leq Q \\ \frac{Q}{g_i}, & P g_i > Q, \end{cases} \tag{2}$$

where $g_i \triangleq \max_{1 \leq j \leq N_p} g_{i,j}$, which implies the effective interference channel gain of the i-th ST. In the APT strategy, all STs should transmit sounding signals to enable the SR to estimate the gains of signal channels, h_i, as in the cellular uplink. In addition, the STs which can not use the peak transmit power should feed their allowable transmit power levels, p_i^A back to the SR. Then, the SR can calculate the received SNRs for all STs, $p_i^A h_i$, and can select one ST with the highest SNR among N_s STs.

B. Fixed Power Transmission

Contrary to the APT strategy, the transmit power of each ST is fixed to P in fixed power transmission (FPT) strategy. Thus, the STs satisfying the interference constraint at all PRs are only eligible for data transmission. The transmit power of the i-th ST in the FPT strategy is given by

$$p_i^F = \begin{cases} P, & P g_i \leq Q \\ 0, & P g_i > Q. \end{cases} \tag{3}$$

In the FPT strategy, the eligible STs only transmit their sounding signals to the SR for channel estimation. The SR does not require the feedback on transmit power levels of the eligible STs because their transmit power levels are fixed. Then, the SR opportunistically selects one ST with the highest SNR. The average transmission rate in FPT strategy is given by

$$C^F = \sum_{n=0}^{N_s} \mathsf{Pr}(n) \cdot C(n), \tag{4}$$

where $\mathsf{Pr}(n)$ denotes the probability that n STs are eligible for scheduling and $C(n)$ denotes the average transmission rate for a given n. $\mathsf{Pr}(n)$ and $C(n)$ are expressed as [3]:

$$\mathsf{Pr}(n) = \binom{N_s}{n} \left(1 - e^{-\frac{Q}{P}}\right)^{nN_p} \left(1 - \left(1 - e^{-\frac{Q}{P}}\right)^{N_p}\right)^{N_s - n} \tag{5}$$

$$C(n) = n \log_2(e) \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{(-1)^k}{k+1} e^{\frac{k+1}{P}} E_1\left(\frac{k+1}{P}\right). \tag{6}$$

C. Comparison and Discussions

The APT strategy requires all STs to obtain perfect CSI on the interference channels with all PRs as shown in (2). If the primary network operates in time division duplexing (TDD), the STs can obtain the CSI by overhearing the uplink sounding signals that the PRs transmit to the PT. However, if the primary network operates in frequency division duplexing (FDD) that most commercial wireless communication systems tend to adopt, then the APT strategy inevitably requires a large amount of feedback on the CSI from the primary network, which is possible only when a tight coordination is allowed between the primary and secondary networks. However, the tight cooperation requirement may not feasible in practical CR networks. In addition, in order to estimate received SNRs, the SR requires *signalings* such as sounding from all the STs and *feedback* information on the allowable transmit power levels of the STs which should adjust their transmit power. Thus, the signaling and feedback overhead in the secondary network becomes significant as the number of STs increases.

On the other hand, in the FPT strategy, each ST can know its eligibility for scheduling through 1-bit feedback from the PRs even when the primary network operates in FDD. Furthermore, eligible STs only transmit their sounding signals to the SR for channel estimation without the power level feedback.

To analyze the overhead caused by sounding signals and power level feedback in the secondary network, the overhead ratio of the FTP strategy to the APT strategy is defined as

$$\rho \triangleq \frac{\mathsf{Pr}\left(g_i \leq \frac{Q}{P}\right) \cdot O_s}{\mathsf{Pr}\left(g_i \leq \frac{Q}{P}\right) \cdot O_s + \mathsf{Pr}\left(g_i > \frac{Q}{P}\right) \cdot (O_s + O_f)}, \tag{7}$$

where O_s and O_f denote the overhead caused by sounding signals and power level feedback, respectively. For simplicity, we derive the upper bound by ignoring O_f as follows:

$$\begin{aligned}\rho &< \frac{\Pr\left(g_i \le \frac{Q}{P}\right)\cdot O_s}{\Pr\left(g_i \le \frac{Q}{P}\right)\cdot O_s + \Pr\left(g_i > \frac{Q}{P}\right)\cdot O_s}\\ &= \Pr\left(g_i \le \frac{Q}{P}\right) = \left(1-e^{-\frac{Q}{P}}\right)^{N_p}. \qquad (8)\end{aligned}$$

Note that Eq. (8) definitely underestimates the overhead reduction gain of the FPT scheme over the APT because it is upper-bounded. The CSI feedback overhead of the primary network operating in FDD mode is not considered to focus on the secondary network.

IV. OPTIMALITY OF FPT IN TERMS OF MUD GAIN

It is intractable to mathematically derive the scaling law of the transmission rate in the presence of multiple PRs [4]. In this section, however, we show that the optimal MUD gain can be achieved by the FPT strategy even in the case of multiple PRs. For mathematical analysis, we modify the FPT strategy proposed in Section III without changing the fundamental concept. In detail, STs first examine the following two criteria:

$$P g_{i,j} \le Q, \quad \forall\, j = 1, \cdots, N_p \qquad (9)$$

$$h_i \ge \eta, \qquad (10)$$

where η denotes pre-determined positive threshold. Then, STs satisfying both criteria in (9) and (10) send their sounding signals to the SR, while the original algorithm of FPT strategy in Section III only utilizes the first criterion in (9). Then, the SR *randomly* selects one ST and allows it to transmit data.

Lemma 1: If η is set to $\epsilon \ln N_s (0 < \epsilon < 1)$ and $N_s = \omega\left(P^{\frac{N_p}{1-\epsilon}}\right)^1$, there exists at least one ST satisfying both (9) and (10) when $P \gg Q$.

Proof: Since $g_{i,j}$ and h_i are exponentially distributed, the probability that a ST satisfies the criteria is given by

$$\psi = \left(1-e^{-P^{-1}Q}\right)^{N_p} e^{-\eta}. \qquad (11)$$

Then, the probability that there exists at least one ST who satisfies the criteria among N_s STs is given by

$$\begin{aligned}\Psi &= 1-(1-\psi)^{N_s}\\ &= 1-\left(1-\left(1-e^{-P^{-1}Q}\right)^{N_p} e^{-\eta}\right)^{N_s}. \qquad (12)\end{aligned}$$

By using Lemma 1 in [9], (12) converges to 1 as N_s goes to ∞ if and only if

$$\lim_{N_s\to\infty} N_s\left(1-e^{-P^{-1}Q}\right)^{N_p} e^{-\eta} \to \infty. \qquad (13)$$

When $P \gg Q$, the left term in (13) can be rewritten as

$$\begin{aligned}&\lim_{N_s\to\infty} N_s\left(1-e^{-P^{-1}Q}\right)^{N_p} e^{-\eta}\\ \approx\ &\lim_{N_s\to\infty} N_s\left(P^{-1}Q\right)^{N_p} e^{-\epsilon \ln N_s}\\ =\ &\lim_{N_s\to\infty} N_s\left(P^{-1}Q\right)^{N_p} N_s^{-\epsilon}\\ =\ &Q^{N_p}\lim_{N_s\to\infty}\frac{N_s^{1-\epsilon}}{P^{N_p}}, \qquad (14)\end{aligned}$$

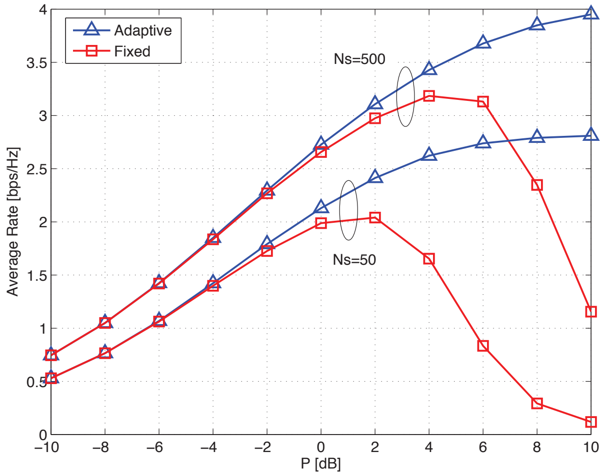

Fig. 2. Average transmission rate of secondary network according to P when $N_p = 3$ and $Q = 1$.

which converges to ∞ if N_s scales faster than $P^{\frac{N_p}{1-\epsilon}}$. ■

Theorem 1 (Scaling law in high transmit power regime): If η is set to $\epsilon \ln N_s$, the FPT strategy achieves $\log\left(P\log(N_s)\right)$ throughput scaling in high transmit power regime, i.e., $P \gg Q$, when $N_s = \omega\left(P^{\frac{N_p}{1-\epsilon}}\right)$.

Proof: Assuming that the $\hat{i}$-th ST is randomly selected, the average transmission rate of the FPT strategy is given by

$$R = \mathbb{E}_{h_{\hat{i}}}\left[\log\left(1+h_{\hat{i}}P\right)\right]. \qquad (15)$$

From *Lemma* 1, there exists at least one secondary transmitter who satisfies (9) and (10). Thus, transmission rate can be lower-bounded by

$$\begin{aligned}R &\ge \log(1+\eta P)\\ &= \log(1+\epsilon P \ln N_s)\\ &= \log(1+\epsilon P \ln(2)\log N_s). \qquad (16)\end{aligned}$$

This completes the proof the *Theorem 1*. ■

Remark 1: The original FPT strategy also achieves $\log\left(P\log(N_s)\right)$ throughput scaling because the modified FPT strategy in this section can be regarded as a lower-bound of the original FPT strategy.

V. PERFORMANCE EVALUATION

We evaluate the performance of both APT and FPT strategies in terms of average transmission rate in the secondary network through Monte-Carlo simulations. Fig. 2 shows the average transmission rate of the secondary network for varying P when $N_p = 3$ and $Q = 1$. When P is small, both strategies have a similar performance. However, the average transmission rate of the APT strategy outperforms the FPT strategy as P increases because the decreasing number of STs eligible for scheduling reduces the MUD gain.

Fig. 3 shows the average transmission rate of the secondary network for varying N_s when $N_p = 3$, $Q = 1$, and $P = 0, 5$dB. It is shown that transmission rates for both schemes increase as N_s increases due to the increasing MUD gain. The performance gap between two schemes is negligible when P is small, but the gap increases as P increases. Note that

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[1] $f(x) = \omega(g(x))$ if $\lim_{x\to\infty}\frac{g(x)}{f(x)} = 0$.

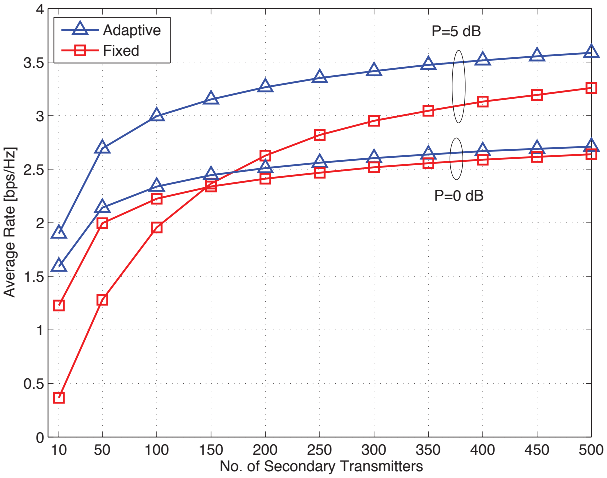

Fig. 3. Average transmission rate of secondary network for varying the number of STs (N_s) when $N_p = 3$, $Q = 1$, and $P = 0, 5$dB

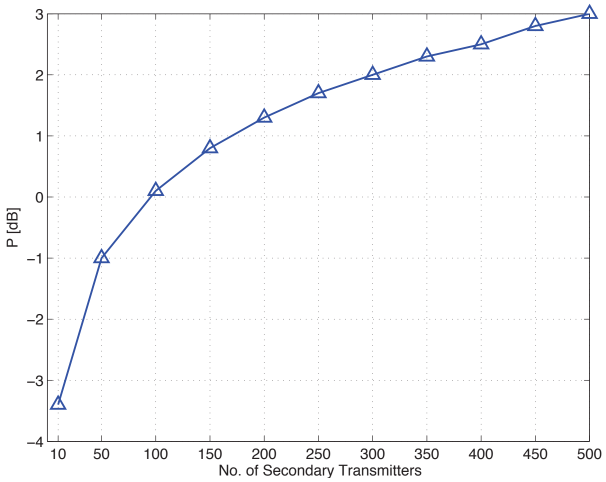

Fig. 4. Maximum P for guaranteeing that $\gamma \geq 0.95$ according to N_s when $N_p = 3$ and $Q = 1$

the performance gap decreases due to the MUD gain as N_s increases.

We define the average throughput ratio of the FPT scheme to the APT scheme as γ for clear performance comparison between them. When $N_s = 300$, for example, $\gamma = 0.96$ if $P = 0$dB, while $\gamma = 0.89$ if $P = 5$dB. Then, we need to investigate what is the maximum power of STs to guarantee a given γ. Fig. 4 shows the maximum P to guarantee that $\gamma \geq 0.95$ for N_s when $N_p = 3$ and $Q = 1$. When $N_s = 300$, $\gamma \geq 0.95$ if $P \leq 2$dB and thus the maximum P value is given by 2 dB . As N_s increases, the maximum power for achieving $\gamma \geq 0.95$ also increases. This indicates that the performance of the FPT strategy approaches that of the APT strategy even in high P regime.

Fig. 5 shows the upper bound of the overhead ratio defined as ρ in (8). The ρ decreases as either $\frac{P}{Q}$ or N_p increases because the number of eligible STs for scheduling decreases. Note that ρ is independent of N_s. Thus, the FPT scheme can achieve the comparable performance with that of the APT scheme and can significantly reduce the signaling and feedback overhead in the secondary network as N_s increases.

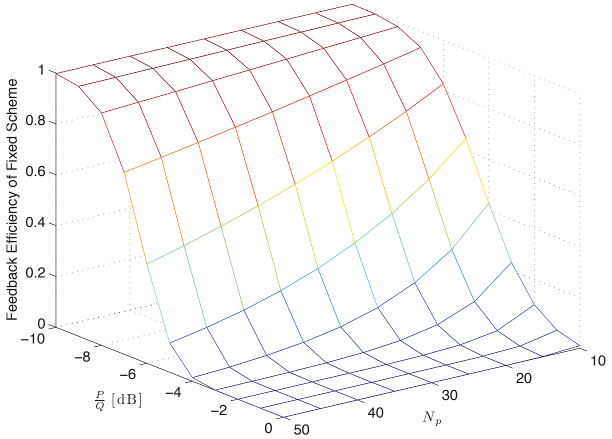

Fig. 5. The upper bound of overhead ratio of the FPT scheme to the APT scheme

VI. CONCLUSION

In this letter, we investigated the fixed power transmission (FPT) strategy at secondary transmitters (STs) and the user scheduling algorithm for underlay cognitive radio networks. The FPT strategy can significantly reduce the signaling and feedback overhead in the secondary network. We proved that the FPT strategy achieves the optimal MUD gain and also derived more elaborate condition on the number of STs, N_s, for the optimal throughput scaling. Furthermore, the FPT strategy can be operated based on 1-bit indicators from a primary network, which is another advantage of the FPT strategy, compared with the APT strategy which requires the perfect CSI on interference channels.

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