

# On the Multiuser Diversity in SIMO Interfering Multiple Access Channels: Distributed User Scheduling Framework

Won-Yong Shin, Dohyung Park, and Bang Chul Jung

**Abstract:** Due to the difficulty of coordination in the cellular uplink, it is a practical challenge how to achieve the optimal throughput scaling with distributed scheduling. In this paper, we propose a distributed and opportunistic user scheduling (DOUS) that achieves the optimal throughput scaling in a single-input multiple-output interfering multiple-access channel, i.e., a multi-cell uplink network, with  $M$  antennas at each base station (BS) and  $N$  users in a cell. In a distributed fashion, each BS adopts  $M$  random receive beamforming vectors and then selects  $M$  users such that both sufficiently large desired signal power and sufficiently small generating interference are guaranteed. As a main result, it is proved that full multiuser diversity gain can be achieved in each cell when a sufficiently large number of users exist. Numerical evaluation confirms that in a practical setting of the multi-cell network, the proposed DOUS outperforms the existing distributed user scheduling algorithms in terms of sum-rate.

**Index Terms:** Diversity, inter-cell interference (ICI), multi-cell network, multi-user, throughput scaling, uplink, user scheduling.

## I. INTRODUCTION

INTERFERENCE management is a critical issue which should be taken into account to provide high transmission rate in cellular networks. In multiuser environments, each mobile user (or each base station (BS)) suffers from intra-cell interference (ICI) as well as inter-cell interference in the cellular downlink (or in the cellular uplink). To resolve such interference issues of cellular networks, a simple infinite cellular multiple-access channel (MAC) model, referred to as the Wyner's model, was characterized and then its achievable throughput performance was analyzed in [1]–[3]. However, the Wyner's model is hardly realistic even though it gives a remarkable insight to the complex and analytically intractable practical cellular environ-

ments. While it has been elusive to find the optimal strategy with respect to the Shannon-theoretic capacity in multiuser interference networks, interference alignment (IA) was recently proposed by Cadambe and Jafar for fundamentally solving the interference problem along with an alternative approach [4]. It was shown that the IA scheme can achieve the optimal degrees-of-freedom in the  $K$ -user interference channel with time-varying channel coefficients. Subsequent work has shown that IA can be well applicable to other wireless network environments including multiple-input multiple-output (MIMO) interference channels [5]–[7] and cellular networks [8]–[12].

On the other hand, there have been some remarkable schemes that can exploit the usefulness of fading in single-cell broadcast channels, thus resulting in a *multiuser diversity (MUD)* gain: Opportunistic scheduling [13], opportunistic beamforming [14], and random beamforming [15]. Moreover, scenarios obtaining the MUD gain have been studied in cooperative networks by applying an opportunistic two-hop relying [16] and an opportunistic routing [17], and in cognitive radio networks with opportunistic scheduling [18], [19]. Such an opportunism have also been utilized in multi-cell broadcast channels (or equivalently, interfering broadcast channels) by using opportunistic IA [20]–[22]. In a decentralized/distributed manner, it however remains open how to design a constructive user scheduling method that can obtain the optimal MUD gain in multi-cell *uplink* networks, which are fundamentally different from downlink environments since for uplink, there exists a channel mismatch [9] between the amount of generating interference at each user and the amount of interference suffered by each BS from multiple users.

Recently, the authors of [23] considered the optimal MUD in multi-cell uplink networks in which there exists the channel mismatch problem among cells, and they showed that the optimal MUD gain can be achieved with a distributed user scheduling operation in each cell when each user and BS has a *single* antenna. However, multiple antennas at a BS make the problem complicated because the ICI from users in other cells forms vector channel at the receiver and the quantity of the ICI can be changed according to beamforming strategy adopted at the receiver. This motivates us to design a distributed user scheduling in the multi-cell uplink for the optimal MUD, where multiple antennas exist at BSs and a single antenna exists at users, respectively, which forms a single-input multiple-output (SIMO) channel for a single user.

In this paper, we introduce a *distributed and opportunistic user scheduling (DOUS)* which achieves the optimal MUD gain

Manuscript received December 27, 2013 approved for publication by Kim, Il-Min, Division II Editor, October 6, 2014.

This research was supported in part by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2013R1A1A2A10004905, 2014R1A1A2054577), and in part by the research fund of Dankook University(BK21 Plus) in 2013.

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Digital object identifier 10.1109/JCN.2015.000049

in time-division duplexing (TDD)  $K$ -cell uplink networks having  $N$  single-antenna users in each cell and one BS, each having  $M$  antennas, which is a practical cellular setting. In the proposed scheme, each BS adopts  $M$  random receive beamforming vectors and then selects  $M$  users based on two pre-determined thresholds (i.e., scheduling criteria). The first threshold is related to generating interference at each user to receivers (i.e., BSs) while the second one is related to the channel gain between a user and its home cell BS. Such a selection guarantees both sufficiently large desired signal power as well as sufficiently small generating interference to BSs, thus enabling to achieve full MUD gain. As our main result, it is shown that the achievable sum-rate in each cell scales as  $M \log(\text{SNR} \log N)$  in a high signal-to-noise ratio (SNR) regime, provided that the two thresholds are properly determined and the number of users in a cell scales as  $\text{SNR}^{(KM-1)/(1-\epsilon_0)}$  for a certain small  $\epsilon_0 > 0$ . Note that our scheme operates as a decentralized manner while requiring only local channel state information (CSI) at each user that can be acquired from all received channel links via pilot signaling. In addition, we evaluate the performance of the proposed scheduling for finite SNR regimes in order to investigate the feasibility of the proposed DOUS. Simulation results show that the proposed scheme significantly outperforms the conventional scheduling algorithms.

The rest of the paper is organized as follows. Section II describes the system and channel models. The DOUS protocol is specified in Section III. In Section IV, the achievable throughput of DOUS is analyzed. Numerical results of the DOUS protocol is shown in Section V. Finally, conclusions are drawn in Section VI.

## II. SYSTEM AND CHANNEL MODELS

Let us describe a practical cellular network by taking into account the interfering MAC (IMAC) model in [8], which is considered as one of the most useful channel models in these days. There are multiple cells, each of which has one BS and multiple mobile users. Specifically, suppose that there are  $K$  BSs, each having  $M$  antennas, and there are  $N$  users with a single-antenna in a cell. Each BS is interested only in traffic demands of users in the corresponding cell. Then, the baseband uplink channel in each cell can be modeled by a SIMO MAC. If  $N$  is much greater than  $M$ , then it is possible to exploit the channel randomness, thereby leading to the opportunistic gain.

We assume a block fading channel model, where the channel vectors are constant during a transmission block (e.g., frame), consisting of one scheduling period and one channel coding block, and independently change for every transmission block. Then, the received signal at the  $i$ th BS,  $\mathbf{y}_i \in \mathbb{C}^{M \times 1}$ , is given by

$$\mathbf{y}_i = \sum_{j=1}^S \mathbf{h}_{i,j}^{(i)} x_j^{(i)} + \sum_{k=1, k \neq i}^K \sum_{j=1}^S \beta_{ik} \mathbf{h}_{i,j}^{(k)} x_j^{(k)} + \mathbf{z}_i$$

where  $x_j^{(k)} \in \mathbb{C}$  denotes the signal transmitted from the  $j$ th user in the  $k$ th cell ( $j \in \{1, \dots, N\}$  and  $i, k \in \{1, \dots, K\}$ ),  $S$  denotes the number of simultaneously transmitting users in each cell, and  $\mathbf{z}_i \in \mathbb{C}^{M \times 1}$  represents the independent and identically

distributed (i.i.d.) and circularly symmetric complex additive white Gaussian noise vector with zero-mean and covariance matrix of  $N_0 \mathbf{I}_M$ . The term  $\beta_{ik} \mathbf{h}_{i,j}^{(k)} \in \mathbb{C}^{M \times 1}$  denotes the uplink channel vector from the  $j$ th user in the  $k$ th cell to the  $i$ th BS, consisting of the large-scale pathloss component  $0 < \beta_{ik} \leq 1$  and the small-scale complex fading component  $\mathbf{h}_{i,j}^{(k)}$ . For simplicity, we assume that transmitters (users) in the same cell experience the same degrees of pathloss attenuation. Especially, when  $i = k$ , the large-scale term  $\beta_{ik}$  is assumed to be 1. The fading term  $\mathbf{h}_{i,j}^{(k)}$  is assumed to be Rayleigh, having zero-mean and identity covariance matrix, and to be independent across different  $i, j$ , and  $k$ . To achieve full degrees-of-freedom, we assume that in each cell,  $M$  users are selected by the home-cell BS and they send their messages simultaneously at a time slot, i.e.,  $S = M$ . We also assume that each user has an average transmit power constraint  $\mathbb{E}[|x_j^{(i)}|^2] \leq P$ .

## III. DISTRIBUTED AND OPPORTUNISTIC USER SCHEDULING

In this section, we introduce a distributed scheduling protocol, named DOUS, in which  $M$  users in each cell are selected in the sense of obtaining the power gain as well as generating the amount of sufficiently small interference to BSs, thus enabling to achieve full MUD gain for the SIMO IMAC model. To this end, for each time slot, each BS first constructs  $M$  orthonormal random vectors, each of which is utilized for decoding the signal from the selected users at the BS.<sup>1</sup>

Now let us focus on the user selection strategy. Assuming that the channel reciprocity of TDD systems is used, it is possible for the  $j$ th user in the  $k$ th cell to obtain all the received channel vectors  $\{\beta_{1k} \mathbf{h}_{1,j}^{(k)}, \dots, \beta_{Kk} \mathbf{h}_{K,j}^{(k)}\}$  by utilizing orthogonal pilot signaling from BSs, where  $j \in \{1, \dots, N\}$  and  $i \in \{1, \dots, K\}$ . Each BS broadcasts its beamforming vectors to all users, constructed by an  $M \times M$  unitary matrix

$$\mathbf{U}_i = [\mathbf{u}_i^{[1]}, \mathbf{u}_i^{[2]}, \dots, \mathbf{u}_i^{[M]}]$$

where  $\mathbf{u}_i^{[l]} \in \mathbb{C}^{M \times 1}$  is the  $l$ th random vector of the  $i$ th BS ( $l \in \{1, \dots, M\}$ ), generated according to the isotropic distribution.<sup>2</sup> Here,  $\mathbf{u}_i^{[l]} \in \mathbb{C}^{M \times 1}$  is generated according to an isotropic distribution. After receiving all the matrices  $\{\mathbf{U}_1, \dots, \mathbf{U}_K\}$ , the  $j$ th user in the  $i$ th cell finds the index  $l^* \in \{1, \dots, M\}$  satisfying the following two criteria:

$$(C1) \quad \left| \mathbf{u}_i^{[l^*]T} \mathbf{h}_{i,j}^{(i)} \right|^2 \geq \eta_{\text{tr}} \quad (1)$$

$$(C2) \quad \underbrace{\sum_{l=1, l \neq l^*}^M \left| \mathbf{u}_i^{[l]T} \mathbf{h}_{i,j}^{(i)} \right|^2}_{\text{Intra-cell interference}} + \underbrace{\sum_{k=1, k \neq i}^K \sum_{l=1}^M \left| \beta_{ki} \mathbf{u}_k^{[l]T} \mathbf{h}_{k,j}^{(i)} \right|^2}_{\text{Inter-cell interference}} \leq \eta_I \quad (2)$$

where  $\eta_{\text{tr}}$  and  $\eta_I$  denote pre-determined positive thresholds. The criterion (C1) is satisfied if the desired signal strength is greater

<sup>1</sup>These random vectors can be regarded as the dual vectors of the transmit random beamforming vectors used for cellular downlink in [15].

<sup>2</sup>Alternatively, a set of vectors can be generated with prior knowledge in a pseudo-random manner, and thus can be acquired by all users before data transmission without any signaling overhead.

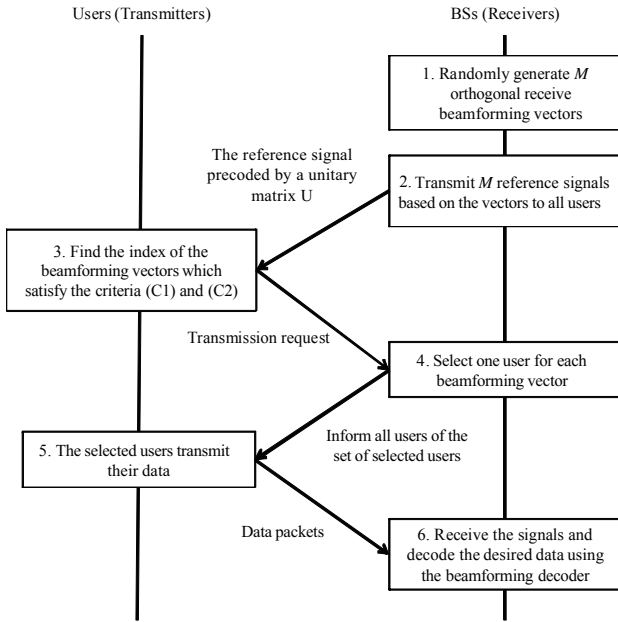


Fig. 1. A block diagram for the DOUS protocol.

than or equal to  $\eta_{tr} > 0$ , which is set in such a way that the users' desired signal power received at the corresponding BS is large enough to obtain the MUD gain. On the other hand, the criterion (C2) is satisfied if the sum power of  $MK - 1$  interfering signals to other BSs is less than or equal to  $\eta_I > 0$ , which is set to a sufficiently small constant to assure that generating interference of the target user is in deep-fading while not impeding the system obtaining full MUD gain. The left-hand side of (2) represents the total sum of intra-cell interference and inter-cell interference, which is generated by the  $j$ th user. After the random beamforming vectors other than  $l^*$  of the home cell BS are combined with the associated channels. The values of  $\eta_{tr}$  and  $\eta_I$  will be specified in Section IV.

When a user has at least one index  $l^*$  satisfying the above criteria, then it feeds back one of the indices to the corresponding BS. Otherwise, it feeds back nothing. This feedback strategy implies that the users such that the criteria are satisfied request transmission to their home cell BS. Thereafter, each BS randomly selects one out of the users that feed back the same receiver beamforming vector index  $l^*$ . If there exists a vector index that was not fed back from any users, then the beamforming vector is not used. This may cause some performance degradation, but we show that the probability that such an event occurs is arbitrarily small if we set the threshold values appropriately, which will be analyzed in Section IV. Finally, the selected users in each cell send their data packets. Each BS decodes the users' signal by using the beamforming vectors, while treating all the interference as noise. The whole procedure of the DOUS protocol is summarized in Fig. 1.

Note that it is sufficient to know the effective channels  $\mathbf{U}_i^T \mathbf{h}_{i,j}^{(k)}$ , but not the original channel vectors  $\mathbf{h}_{i,j}^{(k)}$ , if we assume the channel reciprocity of TDD systems. This precoded reference signal has similarly been taken into account in the literature [14], [24].<sup>3</sup>

<sup>3</sup>In the third-generation partnership project long term evolution (3GPP LTE)

#### IV. THROUGHPUT SCALING ANALYSIS

In this section, we analyze the throughput performance of the proposed DOUS in terms of scaling laws. We show that the DOUS asymptotically achieves the optimal MUD gain, where the throughput of each cell scales as  $M \log \text{SNR}(\log N)$  with increasing  $N$  at asymptotically high SNRs. The achievability result is conditioned by the scaling behavior between the number of per-cell users,  $N$ , and the received SNR. In other words, we analyze how  $N$  scales with SNR so as to achieve the logarithmic gain as well as the degrees-of-freedom gain. The total transmission rate of the  $K$ -cell uplink network,  $R(\text{SNR})$ , is given by

$$R(\text{SNR}) = \sum_{i=1}^K \sum_{l^*=1}^M R_i^{[l^*]}(\text{SNR}) \quad (3)$$

where  $R_i^{[l^*]}(\text{SNR})$  is the transmission rate of user  $\pi_{il^*}$  in the  $i$ th cell ( $i \in \{1, \dots, K\}$ ) whose signal is decoded by using the  $l^*$ th receive beamforming vector of the  $i$ th BS. Assuming all the interference to be Gaussian, the rate  $R_i^{[l^*]}(\text{SNR})$  is lower-bounded by

$$R_i^{[l^*]}(\text{SNR}) \geq \mathcal{P}_i^{[l^*]} \log \left( 1 + \frac{|\mathbf{u}_i^{[l^*]T} \mathbf{h}_{i,\pi_{il^*}}^{(i)}|^2 \text{SNR}}{1 + I_i^{[l^*]}} \right) \quad (4)$$

where  $\mathcal{P}_i^{[l^*]}$  denotes the probability that at least one user in the  $i$ th cell satisfies the two criteria (C1) and (C2) for the  $l^*$ th random beamforming vector of the  $i$ th BS,<sup>4</sup>

$$I_i^{[l^*]} = \left( \sum_{l=1, l \neq l^*}^M \left| \mathbf{u}_i^{[l]T} \mathbf{h}_{i,\pi_{il}}^{(i)} \right|^2 + \sum_{k=1, k \neq i}^K \sum_{l=1}^M \left| \beta_{ik} \mathbf{u}_i^{[l]T} \mathbf{h}_{i,\pi_{kl}}^{(k)} \right|^2 \right) \cdot \text{SNR}$$

and SNR is defined as  $P/N_0$ . The criterion (C2) represents the amount of generating interference at each user, which is fundamentally different from the amount of interference suffered by each receiver (BS) from multiple users. For this reason, let the total interference received at each BS be upper-bounded by  $MK\eta_I$ . We will show later that this simple upper bound on the total interference does not cause any performance loss in terms of scaling laws. In consequence, (4) is again lower-bounded by

$$R_i^{[l^*]}(\text{SNR}) \geq \mathcal{P}_i^{[l^*]} \log \left( 1 + \frac{\eta_{tr} \text{SNR}}{1 + MK\eta_I \text{SNR}} \right). \quad (5)$$

Now, let us focus on analyzing the probability  $\mathcal{P}_i^{[l^*]}$ . We start with the following lemma.

**Lemma 1:** Let  $f(x)$  denote a continuous function of  $x \in [0, \infty)$ , where  $0 < f(x) \leq 1$ . Then,  $\lim_{x \rightarrow \infty} (1 - f(x))^x$  converges to zero if and only if  $\lim_{x \rightarrow \infty} x f(x)$  tends to infinity.

standard [24], the user equipment (UE)-specific reference signal (antenna port 5) has been used.

<sup>4</sup>For analytical convenience, note that if no user in a cell satisfies the two given criteria, then the rate corresponding to each decoding vector at the receiver is treated as zero, which gives us a lower bound on the performance.

*Proof:* Refer to Appendix A.  $\square$

We then characterize the probability such that each criterion is satisfied for a certain user. First, the probability that the  $j$ th user in the  $i$ th cell satisfies (C1) for the  $l^*$ th beamforming vector of the  $i$ th BS is given by

$$\Pr(\text{C1}) = \Pr \left\{ \left| \mathbf{u}_i^{[l^*]T} \mathbf{h}_{i,j}^{(i)} \right|^2 \geq \eta_{\text{tr}} \right\} = e^{-\eta_{\text{tr}}} \quad (6)$$

since the beamforming vector  $\mathbf{u}_i^{[l^*]}$  is assumed to be isotropically distributed and thus  $\left| \mathbf{u}_i^{[l^*]T} \mathbf{h}_{i,j}^{(i)} \right|^2$  is exponentially distributed (refer to [15] for more details). The parameters  $i$ ,  $j$ , and  $l^*$  are omitted since the probability in (6) is identical for any  $i$ ,  $j$ , and  $l^*$  due to the fact that we assume the i.i.d. channel vectors. Second, we show the probability that the  $j$ th user in the  $i$ th cell satisfies (C2) for the  $l^*$ th beamforming vector of the corresponding BS, which is lower-bounded by

$$\begin{aligned} \Pr(\text{C2}) &\geq \Pr \left\{ \sum_{l=1, l \neq l^*}^M \left| \mathbf{u}_i^{[l]T} \mathbf{h}_{i,j}^{(i)} \right|^2 + \sum_{k=1, k \neq i}^K \sum_{l=1}^M \left| \mathbf{u}_k^{[l]T} \mathbf{h}_{k,j}^{(i)} \right|^2 \leq \eta_I \right\} \\ &\triangleq F(\eta_I) \end{aligned} \quad (7)$$

due to the fact that  $\beta_{ik} \leq 1$ . The parameters  $i$ ,  $j$ , and  $l^*$  are also omitted from (7) since  $F(\eta_I)$  is independent across different  $i$ ,  $j$ , and  $l^*$ . It is seen that  $\left| \mathbf{u}_i^{[l]T} \mathbf{h}_{i,j}^{(i)} \right|^2$  and  $\left| \mathbf{u}_k^{[l]T} \mathbf{h}_{k,j}^{(i)} \right|^2$  are exponentially distributed and are independent of each other for any parameters  $l$  and  $k$ . Hence, the total sum of  $MK-1$  squared values is distributed according to the chi-square distribution with  $2(MK-1)$  degrees of freedom [15], which thus results in

$$F(x) = \frac{\gamma(MK-1, x/2)}{\Gamma(MK-1)} \quad (8)$$

where  $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$  is the Gamma function and  $\gamma(z, x) = \int_0^x t^{z-1} e^{-t} dt$  is the lower incomplete Gamma function. Then, we provide the following lower bound on  $F(x)$ .

**Lemma 2:** For any  $0 \leq x < 2$ , the cumulative distribution function (CDF)  $F(x)$  is lower-bounded by

$$F(x) \geq c_1 x^{MK-1} \quad (9)$$

where

$$c_1 = \frac{e^{-1} 2^{-(MK-1)}}{(MK-1)\Gamma(MK-1)}$$

which is independent of  $N$ .

*Proof:* Refer to Appendix B.  $\square$

From Lemma 2, it follows that if  $0 \leq \eta_I < 2$ , then  $\Pr(\text{C2}) \geq c_1 \eta_I^{MK-1}$ . We now show the achievable sum-rate scaling for the SIMO IMAC model using the proposed DOUS protocol.

**Theorem 1:** Suppose that  $\eta_{\text{tr}} = \epsilon \log N$  and  $\eta_I = \text{SNR}^{-1}$  for an arbitrarily small  $\epsilon > 0$ .<sup>5</sup> Then, the DOUS achieves

$$\Theta(KM \log \text{SNR}(\log N)) \quad (10)$$

<sup>5</sup>Note that  $0 \leq \text{SNR}^{-1} < 2$  holds in the high SNR regime.

sum-rate scaling with high probability in the high SNR regime when  $N = \Theta(\text{SNR}^{(MK-1)/(1-\epsilon_0)})$  for a constant  $\epsilon_0 \in (\epsilon, 1)$ .<sup>6</sup>

*Proof:* By using the probabilities (6) and (7), the probability  $\mathcal{P}_i^{[l^*]}$  is lower-bounded by

$$\begin{aligned} \mathcal{P}_i^{[l^*]} &\geq 1 - (1 - \Pr(\text{C1})\Pr(\text{C2}))^N \\ &\geq 1 - (1 - F(\eta_I)e^{-\eta_{\text{tr}}})^N. \end{aligned}$$

From Lemma 1 and the fact that  $0 < F(\eta_I)e^{-\eta_{\text{tr}}} \leq 1$ , it follows that  $\mathcal{P}_i^{[l^*]}$  converges to 1 as  $N$  tends to infinity if

$$\lim_{N \rightarrow \infty} NF(\eta_I)e^{-\eta_{\text{tr}}} \rightarrow \infty. \quad (11)$$

Using Lemma 2, the term in (11) can be lower-bounded by

$$\begin{aligned} \lim_{N \rightarrow \infty} c_1 N \eta_I^{MK-1} e^{-\eta_{\text{tr}}} &= \lim_{N \rightarrow \infty} \frac{N}{\text{SNR}^{MK-1}} e^{-\epsilon \log N} \\ &= \lim_{N \rightarrow \infty} \frac{N^{1-\epsilon}}{\text{SNR}^{MK-1}} \end{aligned}$$

which increases with  $N$  as  $N$  scales as  $\text{SNR}^{(MK-1)/(1-\epsilon_0)}$ , where the first equality comes from  $\eta_{\text{tr}} = \epsilon \log N$  and  $\eta_I = \text{SNR}^{-1}$  for a constant  $\epsilon \in (0, 1)$ . Hence, the probability  $\mathcal{P}_i^{[l^*]}$  converges to 1 with high probability for any parameters  $i$  and  $l^*$  as  $N$  tends to infinity. From (3) and (5), a lower bound on the sum-rate  $R(\text{SNR})$  is finally given by

$$\begin{aligned} R(\text{SNR}) &\geq KM \log \left( 1 + \frac{\eta_{\text{tr}} \text{SNR}}{1 + MK \eta_I \text{SNR}} \right) \\ &= KM \log \left( 1 + \frac{\epsilon (\log N) \text{SNR}}{1 + MK} \right) \end{aligned}$$

which scales as (10) under the condition

$N = \Theta(\text{SNR}^{(MK-1)/(1-\epsilon_0)})$ . This completes the proof of this theorem.  $\square$

To verify the optimality of the proposed DOUS scheme, it would be worthy to compare our achievability result with an upper bound on the sum-rate scaling. From a genie-aided interference removal, we obtain  $K$  parallel SIMO MAC systems, each of which has  $N$  transmitters and one receiver consisting of  $M$  antennas. Under the basic assumption that (at most)  $M$  users in a cell transmit their data at the same time slot, the sum-rate is upper-bounded by  $KM \log \text{SNR}(\log N)$ . Hence, it is clear that this simple upper bound on the sum-rate scaling matches our lower bound in (10) that is achieved using the proposed DOUS based only on local CSI at each user.<sup>7</sup>

In the following, our achievability result is extended to a MIMO scenario.

**Remark 1:** Let us assume a MIMO IMAC model where each user is equipped with multiple transmit antennas. Each selected user is assumed to transmit a single data stream since multi-stream transmission does not increase the throughput scaling

<sup>6</sup>We use the following notation: i)  $f(x) = O(g(x))$  means that there exist constants  $C$  and  $c$  such that  $f(x) \leq Cg(x)$  for all  $x > c$ . ii)  $f(x) = \Theta(g(x))$  if  $f(x) = O(g(x))$  and  $g(x) = O(f(x))$  [25].

<sup>7</sup>We also remark that the same sum-rate scaling can be achieved by assuming that each receive antenna is treated as one single-antenna BS and the proposed DOUS process is performed with a slight modification.

Table 1. Optimal value of  $\eta_I$  for various system parameters.

	$N = 50$	$N = 100$	$N = 200$	$N = 300$
$K = 2, M = 2$	1.4	1.0	0.8	0.7
$K = 2, M = 3$	3.4	2.9	2.5	2.3
$K = 2, M = 4$	5.6	5.0	4.4	4.1
$K = 3, M = 2$	3.4	2.9	2.5	2.3
$K = 3, M = 3$	8.1	6.2	5.6	5.3
$K = 3, M = 4$	10.7	10.0	8.8	8.6

in our model. Then, as in [11], we consider the case where each transmitter (user) performs the singular-value decomposition (SVD)-type precoding. Using the precoder, each user designs the weight vector in terms of minimizing the total sum of interfering signals to all BSs, consisting of intra-cell and inter-cell interference. Due to the fact that a large portion of interfering links to the BSs can be completely canceled through the SVD-based beamforming, the total rank of the effective interfering channels from each user to all the BSs can be reduced accordingly. Since the SNR exponent in the user scaling law corresponds to the total rank of the interfering channels, it is expected that the resulting user scaling law for the MIMO system using the SVD-type precoding may be significantly reduced, while the same sum-rate scaling is achieved as the SIMO case in Theorem 1.

It is also worth noting that our DOUS protocol is very robust to imperfect knowledge of the channel phase information at the transmitters. Specifically, even in the presence of random phase error due to the channel uncertainty between a transmitter (user) and a receiver (BS), there is no fundamental change of the scheduling criteria (see (1) and (2)) and the set of selected users, thereby resulting in no performance degradation on the throughput scaling, compared to the perfect channel knowledge case. The effect of general channel uncertainty, including both phase and amplitude errors, on the sum-rate will be numerically verified using computer simulations in the next section.

## V. NUMERICAL EVALUATION

We perform computer simulations to evaluate the throughput performance of the proposed DOUS in a practical network setting for parameters  $N$  and SNR. For simplicity, when we show numerical results, we assume no large-scale pathloss component, i.e.,  $\beta_{ik} = 1$  for  $i, k \in \{1, \dots, K\}$ . In our simulation, each channel coefficient is generated  $1 \times 10^5$  times for each system parameter.

We modify our DOUS so that it is suitable for numerical evaluation. Specifically, among the users satisfying the criterion (C2) for given threshold  $\eta_I$ , the one whose desired signal strength (given in (1)) is the largest is selected for each beamforming vector. To do this, when a user has at least one beamforming vector index satisfying the criterion (C2), it feeds back the desired signal power levels in (1) along with the corresponding indices to the home cell BS. Assuming less  $\eta_I$  reduces the total generating interference but corresponds to a smaller MUD gain. On the other hand, greater  $\eta_I$  leads to a larger MUD gain at the cost of increased interference. Hence, an issue that needs to be taken into account for the practical setting is how to determine the threshold  $\eta_I$  for given parameters  $K$ ,  $M$ , and  $N$ . When

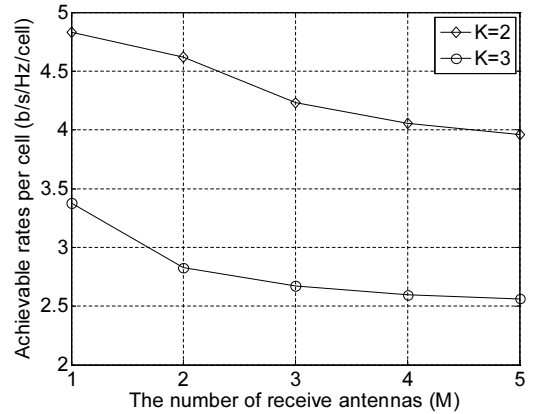


Fig. 2. The sum-rates of the SIMO IMAC model with respect to  $M$ . The system with  $N = 100$  and SNR = 20 dB is considered.

we use the optimal  $\eta_I^*$  that leads to the maximum average sum-rate performance for given parameters  $K$ ,  $M$ , and  $N$ , the value  $\eta_I^*$  can be numerically determined once off-line regardless of instantaneous channel realizations before data transmission. More specifically, the optimization problem is given by

$$\eta_I^* = \arg \max_{\eta_I} E \left[ \sum_{i=1}^K \sum_{l^*=1}^M \tilde{R}_i^{[l^*]}(\text{SNR}) \right] \quad (12a)$$

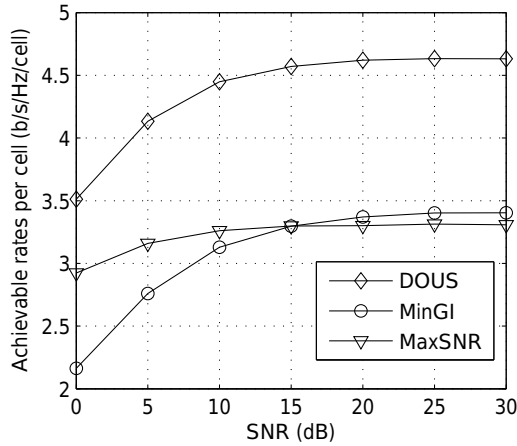
$$\text{subject to} \quad \sum_{l=1, l \neq l^*}^M \left| \mathbf{u}_i^{[l]T} \mathbf{h}_{i,j}^{(i)} \right|^2 + \sum_{k=1, k \neq i}^K \sum_{l=1}^M \left| \mathbf{u}_k^{[l^*]T} \mathbf{h}_{k,j}^{(i)} \right|^2 \leq \eta_I \quad (12b)$$

where the expectation is taken over the channel realizations, and  $\tilde{R}_i^{[l^*]}(\text{SNR})$  denotes the transmission rate of user  $\tilde{\pi}_{il^*}$  in the  $i$ th cell such that

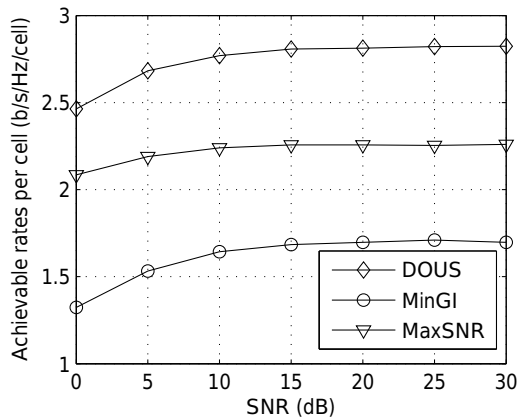
$$\tilde{\pi}_{il^*} = \arg \max_{j \in \tilde{\Pi}_{il^*}} \left| \mathbf{u}_i^{[l^*]T} \mathbf{h}_{i,j}^{(i)} \right|^2.$$

Here,  $\tilde{\Pi}_{il^*}$  denotes the set of users satisfying the constraint in (12b) when the  $l^*$ th receive beamforming vector of the  $i$ th BS is used. The optimal  $\eta_I^*$  is optimized via exhaustive search up to the first decimal place, which is summarized in Table 1 for various system parameters. Interestingly, it turns out that the optimal value of  $\eta_I$  does not depend on the received SNR values. Moreover, the following observations are made based on Table 1. As  $N$  increases, the optimal  $\eta_I^*$  gets decreased since, for large  $N$ , there exist a sufficiently large number of users satisfying the constraint constraint in (12b). On the other hand, as another parameter  $K$  or  $M$  increases, the optimal  $\eta_I^*$  also needs to rise accordingly since, otherwise, the number of users satisfying (12b) may not be sufficient due to increased interferers.

In Fig. 2, the average sum-rate in each cell is evaluated according to  $M$  for the SIMO IMAC model, where  $N = 100$  and SNR = 20 dB are assumed. In the simulation environments, the optimal  $\eta_I$  is given by (0.4, 1.0, 2.9, 5.0, 7.6) and (0.5, 2.9, 6.2, 10.0, 13.1) for  $K = 2$  and  $K = 3$ , respectively, where  $M = 1, \dots, 5$ . It is seen that the sum-rate performance gets degraded with increasing  $M$ , due to the fact that each re-



(a)



(b)

Fig. 3. The sum-rates of the SIMO IMAC model for various protocols: (a)  $K = 2$ ,  $M = 2$  and  $N = 100$  ( $\eta_I = 1.0$ ) and (b)  $K = 3$ ,  $M = 2$ , and  $N = 100$  ( $\eta_I = 3.0$ ).

ceiver suffers from increased interferers in a *finite*  $N$  regime where full multiuser diversity gain may not be obtained.

Figs. 3(a) and 3(b) show the average sum-rate in each cell of three different schemes according to received SNRs (in dB scale) for the SIMO IMAC model with  $(K, M, N) = (2, 2, 100)$  and  $(K, M, N) = (3, 2, 100)$ , respectively. In the simulation environments, the optimal  $\eta_I$  is given by 1.0 and 3.0 for  $K = 2$  and 3, respectively. The proposed DOUS scheme is compared with two other scheduling methods for data transmission: 1) Selecting the users having the maximum desired signal strength and 2) selecting the users having the minimum amount of generating interference (we represent them with *MaxSNR* and *MinGI*, respectively, in the figure). More specifically, the MinGI scheme operates in the sense that each BS selects  $M$  users such that the total generating interference, shown in (C2), is minimized. It is shown that our DOUS outperforms the existing methods for all practical SNR regimes.

In addition, to verify the robustness of our DOUS protocol against the presence of (effective) channel uncertainty at the transmitters (users), the average sum-rate in each cell is evaluated according to the normalized mean-square-error (MSE) of the channel vector  $\mathbf{h}_{k,j}^{(i)}$  (for all  $i, k \in \{1, \dots, K\}$  and  $j \in$

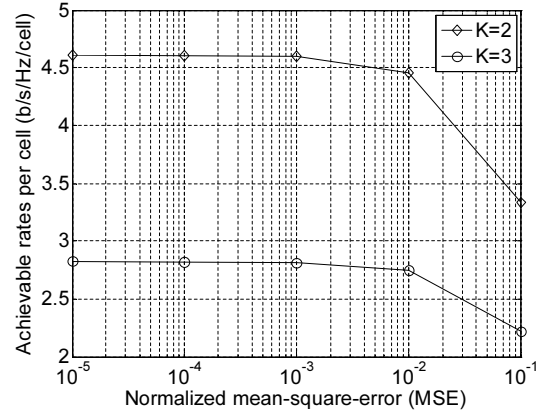


Fig. 4. The sum-rates of the SIMO IMAC model with respect to the normalized MSE. The system with  $N = 100$ ,  $M = 2$ , and  $\text{SNR} = 20$  dB is considered.

Table 2. Comparisons among three scheduling methods.

(a) Computations		
Method	The total amount	Ratio
DOUS	Desired signal power + generating interference	$MK$
MaxSNR	Desired signal power	1
MinGI	Generating interference	$MK - 1$

(b) The amount of feedback		
Method	The total amount	Ratio
DOUS	(Indices + power levels) $\times$ number of requesting users	$F(\eta_I)$
MaxSNR	(Indices + power levels) $\times$ number of all users	1
MinGI	(Indices + power levels) $\times$ number of all users	1

$\{1, \dots, N\}$ , where  $N = 100$ ,  $M = 2$ , and  $\text{SNR} = 20$  dB are assumed. That is, we are interested in the case where each user computes two criteria (C1) and (C2) based on the imperfect channel knowledge. In the simulation environments, the optimal  $\eta_I$  is given by 1.0 and 3.0 for  $K = 2$  and 3, respectively. As illustrated in Fig. 4, it is shown that there is almost no performance degradation on the sum-rate up to  $\text{MSE} = 10^{-2}$ . This is because the user scheduling process requires not accurate channel characterization including its phase information but a scalar expression of the effective channel gain, shown in (C1) and (C2).

Finally, the computational load and the amount of necessary feedback of our DOUS are compared with those of the two conventional methods, which is summarized in Table 2. In the table, a quantitative comparison is made among the three methods, where the ratio is normalized to the minimum amount among the three. Here,  $F(\eta_I)$  is the CDF defined in (7) and represents the average ratio of the number of requesting users to the number of all users. It is seen that our scheme has a significantly smaller amount of feedback at the cost of slightly larger computation, because the users only satisfying the criterion (C2) request transmission. We also remark that the three scheduling methods need the same amount of pilot symbols (i.e., CSI overheads at the transmitters) since, at the downlink phase, each user can acquire all the received channel links utilizing orthogonal pilot signaling from BSs.

## VI. CONCLUSION

In this paper, we proposed the DOUS method that efficiently exploits the MUD gain in a distributed manner with no coordination among cells for multi-cell uplink networks. The achievable sum-rate scaling was then analyzed—the DOUS scheme asymptotically achieves  $KM \log \text{SNR}(\log N)$  sum-rate scaling under the condition that  $N$  scales as  $\text{SNR}^{(MK-1)/(1-\epsilon_0)}$  for a constant  $\epsilon_0 \in (\epsilon, 1)$ . It thus turned out that full MUD gain, which provides a logarithmic throughput boost, is obtained even under multiple interfering MAC environments. The performance of DOUS in the practical network setting was also evaluated via computer simulation. It was shown that the proposed scheme outperforms the two conventional schemes in terms of sum-rate.

## APPENDIX

## A. Proof of Lemma 1

If  $\lim_{x \rightarrow \infty} xf(x) \rightarrow \infty$ , then it follows that  $f(x) = \omega(1/x)$ , thus resulting in

$$\begin{aligned} \lim_{x \rightarrow \infty} (1 - f(x))^x &= o\left(\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x\right) \\ &= o(1) \end{aligned}$$

for  $0 < f(x) \leq 1$ . It is hence seen that  $\lim_{x \rightarrow \infty} (1 - f(x))^x$  converges to zero. If  $\lim_{x \rightarrow \infty} xf(x)$  is finite, then there exists a constant  $c_2 > 0$  such that  $xf(x) < c_2$  for any  $x \geq 0$ . We then have

$$\begin{aligned} \lim_{x \rightarrow \infty} (1 - f(x))^x &> \lim_{x \rightarrow \infty} \left(1 - \frac{c_2}{x}\right)^x \\ &= e^{-c_2} > 0, \end{aligned}$$

which complete the proof.

## B. Proof of Lemma 2

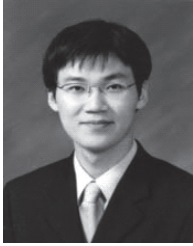
The lower incomplete Gamma function satisfies the inequality  $\gamma(z, x) \geq (1/z)x^z e^{-x}$  for  $z > 0$  and  $0 \leq x < 1$  since

$$\begin{aligned} \gamma(z, x) &= \frac{1}{z}x^z e^{-x} + \frac{1}{z}\gamma(z+1, x) \\ &= \frac{1}{z}x^z e^{-x} + \frac{1}{z(z+1)}x^{z+1}e^{-x} + \dots \\ &\geq \frac{1}{z}x^z e^{-1}. \end{aligned}$$

Applying the above bound to (8), we finally obtain (9), which completes the proof.

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