Opportunistic Downlink Interference Alignment for Multi-Cell MIMO Networks

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Abstract—In this paper, we propose an opportunistic downlink interference alignment (ODIA) for interference-limited cellular downlink, which intelligently combines user scheduling and downlink IA techniques. The proposed ODIA not only efficiently reduces the effect of inter-cell interference from other-cell base stations (BSs) but also eliminates intra-cell interference among spatial streams in the same cell. We show that the minimum number of users required to achieve a target degrees-of-freedom can be fundamentally reduced, i.e., the fundamental user scaling law can be improved by using the ODIA, compared with the existing downlink IA schemes. In addition, we adopt a limited feedback strategy in the ODIA framework, and then analyze the number of feedback bits required for the system with limited feedback to achieve the same user scaling law of the ODIA as the system with perfect channel state information. We also modify the original ODIA in order to further improve the sum-rate, which achieves the optimal multiuser diversity gain, i.e., $\log \log N$, per spatial stream even in the presence of downlink inter-cell interference, where N denotes the number of users in a cell. Simulation results show that the ODIA significantly outperforms existing interference management techniques in terms of sum rate in realistic cellular environments. Note that the ODIA operates in a non-collaborative and decoupled manner, i.e., it requires no information exchange among BSs and no iterative beamformer optimization between BSs and users, thus leading to an easier implementation.

Index Terms—Inter-cell interference, interference alignment, degrees-of-freedom (DoF), transmit & receive beamforming, limited feedback, multiuser diversity, user scheduling.

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I. INTRODUCTION

INTERFERENCE management has been taken into account as one of the most challenging issues to increase the throughput of cellular networks serving multiple users. In multiuser cellular environments, each receiver may suffer from intra-cell and inter-cell interference. Interference alignment (IA) was proposed by fundamentally solving the interference problem when there are multiple communication pairs [1]. It was shown that the IA scheme can achieve the optimal degrees-of-freedom (DoF)[1] in the multiuser interference channel with time-varying channel coefficients. Subsequent studies have shown that the IA is also useful and indeed achieves the optimal DoF in various wireless multiuser network setups: multiple-input multiple-output (MIMO) interference channels [2], [3] and cellular networks [4], [5]. In particular, IA techniques [4], [5] for cellular uplink and downlink networks, also known as the interfering multiple-access channel (IMAC) or interfering broadcast channel (IBC), respectively, have received much attention. The existing IA framework for cellular networks, however, still has several practical challenges: the scheme proposed in [5] requires arbitrarily large frequency/time-domain dimension extension, and the scheme proposed in [4] is based on iterative optimization of processing matrices and cannot be optimally extended to an arbitrary downlink cellular network in terms of achievable DoF.

In the literature, there are some results on the usefulness of fading in single-cell downlink broadcast channels, where one can obtain multiuser diversity gain along with user scheduling as the number of users is sufficiently large: opportunistic scheduling [6], opportunistic beamforming [7], and random beamforming [8]. Scenarios exploiting multiuser diversity gain have been studied also in ad hoc networks [9], cognitive radio networks [10], and cellular networks [11].

Recently, the concept of opportunistic IA (OIA) was introduced in [12]–[14] for the K-cell uplink network (i,e., IMAC model), where there are one M-antenna base station (BS) and N users in each cell. The OIA scheme incorporates user scheduling into the classical IA framework by opportunistically selecting S ($S \leq M$) users amongst the N users in each cell in the sense that inter-cell interference is aligned at a pre-defined interference space. It was shown in [13] and [14] that one can asymptotically achieve the optimal DoF if the number of users in a cell scales as a certain function of the

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[1]It is referred that 'optimal' DoF is achievable if the outer-bound on DoF for given network configuration is achievable.

signal-to-noise-ratio (SNR). For the K-cell downlink network (i.e., IBC model) assuming one M-antenna base station (BS) and N per-cell users, studies on the OIA have been conducted in [15]–[20]. More specifically, the user scaling condition for obtaining the optimal DoF was characterized for the K-cell multiple-input single-output (MISO) IBC [15], and then such an analysis of the DoF achievability was extended to the K-cell MIMO IBC with L receive antennas at each user [16]–[20]— full DoF can be achieved asymptotically, provided that N scales faster than SNR^{KM-L}, for the K-cell MIMO IBC using OIA [19], [20].

In this paper, we propose an *opportunistic downlink IA (ODIA)* framework as a promising interference management technique for K-cell downlink networks, where each cell consists of one BS with M antennas and N users having L antennas each. The proposed ODIA jointly takes into account user scheduling and downlink IA issues. In particular, inspired by the precoder design in [4], we use two cascaded beamforming matrices to construct our precoder at each BS. To design the first transmit beamforming matrix, we use a user-specific beamforming, which conducts a linear zero-forcing (ZF) filtering and thus eliminates intra-cell interference among spatial streams in the same cell. To design the second transmit beamforming matrix, we use a predetermined reference beamforming matrix, which plays the same role of random beamforming for cellular downlink [15], [19], [20] and thus efficiently reduces the effect of inter-cell interference from other-cell BSs. On the other hand, the receive beamforming vector is designed at each user in the sense of minimizing the total amount of received inter-cell interference using *local* channel state information (CSI) in a decentralized manner. Each user feeds back both the effective channel vector and the quantity of received inter-cell interference to its home-cell BS. The user selection and transmit beamforming at the BSs and the design of receive beamforming at the users are completely decoupled. Hence, the ODIA operates in a non-collaborative manner while requiring no information exchange among BSs and no iterative optimization between transmitters and receivers, thereby resulting in an easier implementation.

The main contribution of this paper is four-fold as follows.

- We first show that the minimum number of users required to achieve S DoF ($S \le M$) can be fundamentally reduced to $\mathsf{SNR}^{(K-1)S-L+1}$ by using the ODIA at the expense of acquiring perfect CSI at the BSs from users, compared to the existing downlink IA schemes requiring the user scaling law $N = \omega(\mathsf{SNR}^{KS-L})$ [19], [20],[2] where S denotes the number of spatial streams per cell. The interference decaying rate with respect to N for given SNR is also characterized in regards to the derived user scaling law.
- We introduce a limited feedback strategy in the ODIA framework, and then analyze the required number of feedback bits leading to the same DoF performance as that of the ODIA assuming perfect feedback, which is given by $\omega\left(\log_2 \mathsf{SNR}\right)$.

- We present a user scheduling method for the ODIA to achieve optimal multiuser diversity gain, i.e., $\log \log N$ per stream even in the presence of downlink inter-cell interference.
- To verify the ODIA schemes, we perform numerical evaluation via computer simulations. Simulation results show that the proposed ODIA significantly outperforms existing interference management and user scheduling techniques in terms of sum-rate in realistic cellular environments.

The remainder of this paper is organized as follows. Section II describes the system and channel models. Section III presents the overall procedure of the proposed ODIA. In Section IV, the DoF achievablility result is shown. Section V presents the ODIA scheme with limited feedback. In Section VI, the achievability of the spectrally efficient ODIA leading to a better sum-rate performance is characterized. Numerical results are shown in Section VII. Section VIII summarizes the paper with some concluding remarks.

II. SYSTEM AND CHANNEL MODELS

We consider a K-cell MIMO IBC where each cell consists of a BS with M antennas and N users with L antennas each. The number of selected users in each cell is denoted by $S(\le M)$. It is assumed that each selected user receives a single spatial stream. To consider nontrivial cases, we assume that $L < (K-1)S + 1$, because all inter-cell interference can be completely canceled at the receivers (i.e., users) otherwise. Moreover, the number of antennas at the users is in general limited due to the size of the form factor, and hence it is more safe to assume that L is relatively small compared to $(K-1)S + 1$. The channel matrix from the k-th BS to the j-th user in the i-th cell is denoted by $\mathbf{H}_k^{[i,j]} \in \mathbb{C}^{L \times M}$, where $i, k \in \mathcal{K} \triangleq \{1, \ldots, K\}$ and $j \in \mathcal{N} \triangleq \{1, \ldots, N\}$. Each element of $\mathbf{H}_k^{[i,j]}$ is assumed to be independent and identically distributed (i.i.d.) according to $\mathcal{CN}(0, 1)$. In addition, quasi-static frequency-flat fading is assumed, i.e., channel coefficients are constant during one transmission block and change to new independent values for every transmission block. The j-th user in the i-th cell can estimate the channels $\mathbf{H}_k^{[i,j]}$, $k = 1, \ldots, K$, using pilot signals sent from all the BSs.

The received signal vector at the j-th user in the i-th cell is expressed as:

$$\mathbf{y}^{[i,j]} = \sum_{k=1}^{K} \mathbf{H}_k^{[i,j]} \mathbf{s}_k + \mathbf{z}^{[i,j]}, \tag{1}$$

where $\mathbf{s}_k \in \mathbb{C}^{M \times 1}$ is the transmit signal vector at the k-th BS with unit average power, i.e., $E \|\mathbf{s}_k\|^2 = 1$, and $\mathbf{z}^{[i,j]} \in \mathbb{C}^{L \times 1}$ denotes additive noise, each element of which is independent and identically distributed complex Gaussian with zero mean and the variance of N_0. The average SNR is given by $\mathsf{SNR} = \mathbb{E}\left[\left\|\mathbf{H}_i^{[i,j]} \mathbf{s}_i\right\|^2\right] / \mathbb{E}\left[\left\|\mathbf{z}^{[i,j]}\right\|^2\right] = 1/N_0$. Thus, in what follows we shall use the notation $N_0 = \mathsf{SNR}^{-1}$ for notational simplicity.

Figure 1 shows an example of the MIMO IBC model, where $K = 3$, $M = 3$, $S = 2$, $L = 3$, and $N = 2$.

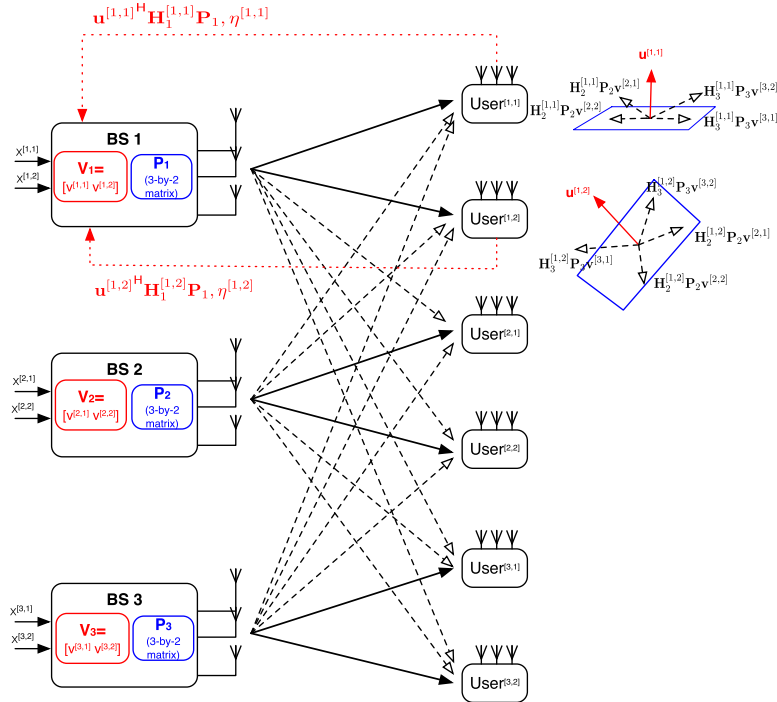[2] $f(x) = \omega(g(x))$ implies that $\lim_{x \to \infty} \frac{g(x)}{f(x)} = 0$.

Fig. 1. MIMO IBC model, where $K = 3$, $M = 3$, $S = 2$, $L = 3$, and $N = 2$.

The details in the figure will be described in the subsequent section.

III. PROPOSED ODIA

We first describe the overall procedure of our proposed ODIA scheme for the MIMO IBC, and then define its achievable sum-rate and DoF.

A. Overall Procedure

The ODIA scheme is described according to the following four steps.

1) Initialization (Broadcast of Reference Beamforming Matrices): The reference beamforming matrix at the BS in the k-th cell is given by $\mathbf{P}_k = [\mathbf{p}_{1,k}, \ldots, \mathbf{p}_{S,k}]$, where $\mathbf{p}_{s,k} \in \mathbb{C}^{M \times 1}$ is an orthonormal vector for $k \in \mathcal{K}$ and $s = 1, \ldots, S$. That is, $\mathbf{P}_k$ is an orthonormal basis for an S-dimensional subspace of $\mathbb{C}^{M \times M}$. Each BS randomly generates $\mathbf{P}_k$ independently of the other BSs. If the reference beamforming matrix is generated in a pseudo-random fashion, i.e., it changes based on a certain pattern as if it changes randomly and the pattern is known by the BSs as well as the users, BSs do not need to broadcast them to users. Then, the j-th user in the i-th cell obtains $\mathbf{H}_k^{[i,j]}$ and $\mathbf{P}_k$, $k = 1, \ldots, K$.

2) Receive Beamforming and Scheduling Metric Feedback: In the second step, we explain how to decide a user scheduling metric at each user along with given receive beamforming, where the design of receive beamforming will be explained in Section IV. Let $\mathbf{u}^{[i,j]} \in \mathbb{C}^{L \times 1}$ denote the unit-norm weight vector at the j-th user in the i-th cell, i.e., $\|\mathbf{u}^{[i,j]}\|^2 = 1$. Note that the user-specific beamforming $\mathbf{V}_k$ will be utilized only to cancel intra-cell interference out, and the inter-cell interference will be suppressed from user scheduling, which

will be specified later. Thus, from the notion of $\mathbf{P}_k$ and $\mathbf{H}_k^{[i,j]}$, the j-th user in the i-th cell can compute the following quantity while using its receive beamforming vector $\mathbf{u}^{[i,j]}$, which is given by

$$\tilde{\eta}_k^{[i,j]} = \left\| \mathbf{u}^{[i,j]\mathsf{H}} \mathbf{H}_k^{[i,j]} \mathbf{P}_k \right\|^2, \tag{2}$$

where $i \in \mathcal{K}$, $j \in \mathcal{N}$, and $k \in \mathcal{K} \setminus i = \{1, \ldots, i-1, i+1, \ldots, K\}$. Using (2), the scheduling metric at the j-th user in the i-th cell, denoted by $\eta^{[i,j]}$, is defined as the sum of $\tilde{\eta}_k^{[i,j]}$. That is,

$$\eta^{[i,j]} = \sum_{k=1, k \neq i}^{K} \tilde{\eta}_k^{[i,j]}. \tag{3}$$

As illustrated in Fig. 1, each user feeds the metric in (3) back to its home-cell BS. In addition to the scheduling metric in (3), for each BS to design the user-specific beamforming $\mathbf{V}_k$, each user needs to feed back the information of the following vector

$$\mathbf{f}_i^{[i,j]} \triangleq \left(\mathbf{u}^{[i,j]\mathsf{H}} \mathbf{H}_i^{[i,j]} \mathbf{P}_i \right)^{\mathsf{H}}. \tag{4}$$

3) User Scheduling: Upon receiving N users' scheduling metrics in the serving cell, each BS selects S users having the metrics up to the S-th smallest one. Without loss of generality, the indices of selected users in every cell are assumed to be $(1, \ldots, S)$. Although $\tilde{\eta}_k^{[i,j]}$ is not exactly the amount of the generating interference from the k-th BS to the j-th user in the i-th cell due to the absence of $\mathbf{V}_k$, it decouples the design of the user-specific precoding matrix $\mathbf{V}_k$ from the user scheduling metric calculation, i.e., $\eta^{[i,j]}$ includes no information of $\mathbf{V}_k$. In addition, we shall show in the sequel that the inter-cell

interference can be successfully suppressed by using the metric $\eta_k^{[i,j]}$ even with $\mathbf{V}_k$ excluded and that the optimal DoF can be achieved.

At this point, it is worthwhile to note that the role of $\mathbf{P}_k$ is two-fold. First, it determines the dimension of the effective received channel according to given parameter S. By multiplying $\mathbf{P}_k$ to the channel matrix, the dimension of the effective channel is reduced to S rather than M, which results in reduced number of inter-cell interference terms as well as reduced average interference level for each interference term. We shall show in the sequel that $\mathbf{P}_k$ plays a role in the end of rendering the user scaling law dependent on the parameter S.

Second, $\mathbf{P}_k$ separates the user scheduling procedure from the user-specific precoding matrix design of $\mathbf{V}_k$ and also from the receiver beamforming vector design of $\mathbf{u}_k$. By employing the cascaded precoding matrix design, the scheduling metric in (1) becomes independent of $\mathbf{V}_k$ or $\mathbf{u}_k$, and $\mathbf{u}_k$ can be obtained as a function of only $\mathbf{H}_k^{[i,j]}$ and $\mathbf{P}_k$ as shown in (18).

The reason why $\mathbf{P}_k$ is designed to change in a pseudo-random fashion is to increase the fairness of the users scheduling by randomizing the scheduling metric of each user, but can also be fixed if the fairness is not a matter or the channel changes fast enough. In addition, if one wants to further improve the achievable rate, $\mathbf{P}_k$ may be channel-specifically designed combined with the user scheduling, which however results in a collaborative and iterative user scheduling and precoding matrix design.

In this and subsequent sections, we focus on how to simply design a user scheduling method to guarantee the optimal DoF. An enhanced scheduling algorithm jointly taking into account the vector to be fed back in (4) and the scheduling metric in (3) may provide a better performance in terms of sum-rate, which shall be discussed in Section VI.

4) Transmit Beamforming and Downlink Data Transmission: As illustrated in Fig. 1, the precoding matrix at each BS is composed of the product of the predetermined reference beamforming matrix $\mathbf{P}_k$ and the user-specific precoding matrix $\mathbf{V}_i = \left[\mathbf{v}^{[i,1]}, \ldots, \mathbf{v}^{[i,S]} \right]$, where $\mathbf{v}^{[i,s]} \in \mathbb{C}^{S \times 1}$, $i \in \mathcal{K}$. Let us denote the transmit symbol at the i-th BS transmitted to the j-th user by $x^{[i,j]}$, where $E\left| x^{[i,s]} \right|^2 = 1/S$ for $s = 1, \ldots, S$. Denoting the transmit symbol vector by $\mathbf{x}_i = \left[x^{[i,1]}, \ldots, x^{[i,S]} \right]^T$, the transmit signal vector at the i-th BS is given by $\mathbf{s}_i = \mathbf{P}_i \mathbf{V}_i \mathbf{x}_i$, and the received signal vector at the j-th user in the i-th cell is written as

$$
\begin{aligned}
\mathbf{y}^{[i,j]} &= \mathbf{H}_i^{[i,j]} \mathbf{P}_i \mathbf{V}_i \mathbf{x}_i + \sum_{k=1,k\neq i}^{K} \mathbf{H}_k^{[i,j]} \mathbf{P}_k \mathbf{V}_k \mathbf{x}_k + \mathbf{z}^{[i,j]} \\
&= \underbrace{\mathbf{H}_i^{[i,j]} \mathbf{P}_i \mathbf{v}^{[i,j]} x^{[i,j]}}_{\text{desired signal}} + \underbrace{\sum_{s=1,s\neq j}^{S} \mathbf{H}_i^{[i,j]} \mathbf{P}_i \mathbf{v}^{[i,s]} x^{[i,s]}}_{\text{intra-cell interference}} \\
&\quad + \underbrace{\sum_{k=1,k\neq i}^{K} \mathbf{H}_k^{[i,j]} \mathbf{P}_k \mathbf{V}_k \mathbf{x}_k}_{\text{inter-cell interference}} + \mathbf{z}^{[i,j]}.
\end{aligned} \tag{5}
$$

The received signal vector after receive beamforming, denoted by $\tilde{y}^{[i,j]} = \mathbf{u}^{[i,j]\mathsf{H}} \mathbf{y}^{[i,j]}$, can be rewritten as:

$$
\begin{aligned}
\tilde{y}^{[i,j]} &= \mathbf{f}_i^{[i,j]\mathsf{H}} \mathbf{v}^{[i,j]} x^{[i,j]} + \mathbf{f}_i^{[i,j]\mathsf{H}} \sum_{s=1,s\neq j}^{S} \mathbf{v}^{[i,s]} x^{[i,s]} \\
&\quad + \sum_{k=1,k\neq i}^{K} \mathbf{f}_k^{[i,j]\mathsf{H}} \mathbf{V}_k \mathbf{x}_k + \mathbf{u}^{[i,j]\mathsf{H}} \mathbf{z}^{[i,j]},
\end{aligned} \tag{6}
$$

where $\mathbf{f}_k^{[i,j]\mathsf{H}} = \mathbf{u}^{[i,j]\mathsf{H}} \mathbf{H}_k^{[i,j]} \mathbf{P}_k$. By selecting users with small $\eta^{[i,j]}$ in (3), $\mathbf{H}_k^{[i,j]} \mathbf{P}_k$ tends to be orthogonal to the receive beamforming vector $\mathbf{u}^{[i,j]}$; thus, inter-cell interference channel matrices $\mathbf{H}_k^{[i,j]} \mathbf{P}_k \mathbf{V}_k$ in (6) also tend to be orthogonal to $\mathbf{u}^{[i,j]}$ as illustrated in Fig. 1.

To cancel out intra-cell interference, the user-specific beamforming matrix $\mathbf{V}_i \in \mathbb{C}^{S\times S}$ is given by

$$
\begin{aligned}
\mathbf{V}_i &= [\mathbf{v}^{[i,1]}, \mathbf{v}^{[i,2]}, \ldots, \mathbf{v}^{[i,S]}] \\
&= \begin{bmatrix} \mathbf{u}^{[i,1]\mathsf{H}} \mathbf{H}_i^{[i,1]} \mathbf{P}_i \\ \mathbf{u}^{[i,2]\mathsf{H}} \mathbf{H}_i^{[i,2]} \mathbf{P}_i \\ \vdots \\ \mathbf{u}^{[i,S]\mathsf{H}} \mathbf{H}_i^{[i,S]} \mathbf{P}_i \end{bmatrix}^{-1} \\
&\quad \cdot \begin{bmatrix} \sqrt{\gamma^{[i,1]}} & 0 & \cdots & 0 \\ 0 & \sqrt{\gamma^{[i,2]}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{\gamma^{[i,S]}} \end{bmatrix},
\end{aligned} \tag{7}
$$

where $\sqrt{\gamma^{[i,j]}}$ denotes a normalization factor for satisfying the unit-transmit power constraint for each spatial stream, i.e., $\gamma^{[i,j]} = 1/ \left\| \mathbf{P}_i \mathbf{v}^{[i,j]} \right\|$. In consequence, the received signal can be simplified to

$$
\tilde{y}^{[i,j]} = \sqrt{\gamma^{[i,j]}} x^{[i,j]} + \underbrace{\sum_{k=1,k\neq i}^{K} \mathbf{f}_k^{[i,j]\mathsf{H}} \mathbf{V}_k \mathbf{x}_k}_{\text{inter-cell interference}} + \mathbf{u}^{[i,j]\mathsf{H}} \mathbf{z}^{[i,j]}, \tag{8}
$$

which thus does not contain the intra-cell interference term.

As in [13] and [21]–[25], we assume no loss in exchanging signaling messages such as information of effective channels, scheduling metrics, and receive beamforming vectors.

B. Achievable Sum-Rate and DoF

From (8), the achievable rate of the j-th user in the i-th cell is given by

$$
\begin{aligned}
R^{[i,j]} &= \log_2 \left(1 + \mathsf{SINR}^{[i,j]} \right) \\
&= \log_2 \left(1 + \frac{\gamma^{[i,j]} \cdot |x^{[i,j]}|^2}{\left| \mathbf{u}^{[i,j]\mathsf{H}} \mathbf{z}^{[i,j]} \right|^2 + \tilde{I}^{[i,j]}} \right) \\
&= \log_2 \left(1 + \frac{\gamma^{[i,j]}}{\frac{S}{\mathsf{SNR}} + \sum_{k=1,k\neq i}^{K} \sum_{s=1}^{S} \left| \mathbf{f}_k^{[i,j]\mathsf{H}} \mathbf{v}^{[k,s]} \right|^2} \right),
\end{aligned} \tag{9}
$$

where $\tilde{I}^{[i,j]} \triangleq \sum_{k=1,k\neq i}^{K} \left| \mathbf{f}_k^{[i,j]\mathsf{H}} \mathbf{V}_k \mathbf{x}_k \right|^2$.

Using (9), the achievable total DoF can be defined as [26]

$$\text{DoF} = \lim_{\text{SNR} \to \infty} \frac{\sum_{i=1}^{K} \sum_{j=1}^{S} R^{[i,j]}}{\log \text{SNR}}. \qquad (10)$$

IV. DoF ACHIEVABILITY

In this section, we characterize the DoF achievability in terms of the user scaling law with the optimal receive beamforming technique. To this end, we start with the receive beamforming design that maximizes the achievable DoF. For given channel instance, from (9), each user can attain the maximum DoF of 1 if and only if the interference $\sum_{k=1,k\neq i}^{K} \sum_{s=1}^{S} \left| \mathbf{f}_k^{[i,j]\mathsf{H}} \mathbf{v}^{[k,s]} \right|^2 \cdot \text{SNR}$ remains constant for increasing SNR. Note that $R^{[i,j]}$ can be bounded as

$$R^{[i,j]} \geq \log_2 \left(1 + \frac{\gamma^{[i,j]}}{\frac{S}{\text{SNR}} + \sum_{k=1,k\neq i}^{K} \sum_{s=1}^{S} \left\| \mathbf{f}_k^{[i,j]} \right\|^2 \left\| \mathbf{v}^{[k,s]} \right\|^2} \right) \qquad (11)$$

$$\geq \log_2 \left(1 + \frac{\gamma^{[i,j]}}{\frac{S}{\text{SNR}} + \sum_{k\neq i}^{K} \sum_{s=1}^{S} \left\| \mathbf{f}_k^{[i,j]} \right\|^2 \left\| \mathbf{v}_i^{(\max)} \right\|^2} \right) \qquad (12)$$

$$= \log_2 (\text{SNR}) + \log_2 \left(\frac{1}{\text{SNR}} + \frac{\frac{\gamma^{[i,j]}}{\left\| \mathbf{v}_i^{(\max)} \right\|^2}}{\frac{S}{\left\| \mathbf{v}_i^{(\max)} \right\|^2} + I^{[i,j]}} \right), \qquad (13)$$

where $\mathbf{v}_i^{(\max)}$ in (12) is defined by

$$\mathbf{v}_i^{(\max)} = \arg\max \left\{ \left\| \mathbf{v}^{[i',j']} \right\|^2 : i' \in \mathcal{K} \setminus i, \, j' \in \mathcal{S} \right\}, \quad (14)$$

$\mathcal{S} \triangleq \{1, \ldots, S\}$, and $I^{[i,j]}$ in (13) is defined by

$$I^{[i,j]} \triangleq \sum_{k=1,k\neq i}^{K} \sum_{s=1}^{S} \left\| \mathbf{f}_k^{[i,j]} \right\|^2 \cdot \text{SNR}. \qquad (15)$$

Here, $\mathbf{v}_i^{(\max)}$ is fixed for given channel instance, because $\mathbf{v}^{[i,j]}$ is determined by $\mathbf{H}_i^{[i,j]}$, $j = 1, \ldots, S$. Recalling that the indices of the selected users are $(1, \ldots, S)$ for all cells, we can expect the DoF of 1 for each user if and only if for some $0 \leq \epsilon < \infty$,

$$I^{[i,j]} < \epsilon, \quad \forall j \in \mathcal{S}, \, i \in \mathcal{K}. \qquad (16)$$

To maximize the achievable DoF, we aim to minimize the sum-interference $\sum_{i=1}^{K} \sum_{j=1}^{S} I^{[i,j]}$ through receive beamforming at the users. Since $I^{[i,j]} = \sum_{s=1}^{S} \eta^{[i,j]} \text{SNR}$, we have

$$\sum_{i=1}^{K} \sum_{j=1}^{S} I^{[i,j]} = \sum_{i=1}^{K} \sum_{j=1}^{S} \sum_{s=1}^{S} \eta^{[i,j]} \text{SNR}$$

$$= S \sum_{i=1}^{K} \sum_{j=1}^{S} \eta^{[i,j]} \text{SNR}. \qquad (17)$$

The equation (17) implies that the collection of distributed effort to minimize $\eta^{[i,j]}$ at the users can reduce the sum of received interference. Therefore, each user finds the beamforming vector that minimizes $\eta^{[i,j]}$ from

$$\mathbf{u}^{[i,j]} = \arg\min_{\mathbf{u}} \eta^{[i,j]} = \arg\min_{\mathbf{u}} \sum_{k=1,k\neq i}^{K} \left\| \mathbf{u}^{\mathsf{H}} \mathbf{H}_k^{[i,j]} \mathbf{P}_k \right\|^2. \quad (18)$$

Let us denote the augmented interference matrix by

$$\mathbf{G}^{[i,j]} \triangleq \left[\left(\mathbf{H}_1^{[i,j]} \mathbf{P}_1 \right), \ldots, \left(\mathbf{H}_{i-1}^{[i,j]} \mathbf{P}_{i-1} \right), \left(\mathbf{H}_{i+1}^{[i,j]} \mathbf{P}_{i+1} \right), \right.$$

$$\left. \ldots, \left(\mathbf{H}_K^{[i,j]} \mathbf{P}_K \right) \right]^{\mathsf{H}} \in \mathbb{C}^{(K-1)S \times L}, \qquad (19)$$

and the singular value decomposition of $\mathbf{G}^{[i,j]}$ by

$$\mathbf{G}^{[i,j]} = \mathbf{\Omega}^{[i,j]} \mathbf{\Sigma}^{[i,j]} \mathbf{Q}^{[i,j]\mathsf{H}}, \qquad (20)$$

where $\mathbf{\Omega}^{[i,j]} \in \mathbb{C}^{(K-1)S \times L}$ and $\mathbf{Q}^{[i,j]} \in \mathbb{C}^{L \times L}$ consist of L orthonormal columns, and $\mathbf{\Sigma}^{[i,j]} = \text{diag}\left(\sigma_1^{[i,j]}, \ldots, \sigma_L^{[i,j]} \right)$, where $\sigma_1^{[i,j]} \geq \cdots \geq \sigma_L^{[i,j]}$. Then, the optimal $\mathbf{u}^{[i,j]}$ is determined as

$$\mathbf{u}^{[i,j]} = \mathbf{q}_L^{[i,j]}, \qquad (21)$$

where $\mathbf{q}_L^{[i,j]}$ is the L-th column of $\mathbf{Q}^{[i,j]}$. With this choice the scheduling metric is simplified to

$$\eta^{[i,j]} = \sigma_L^{[i,j]2}. \qquad (22)$$

Since each column of $\mathbf{P}_k$ is isotropically and independently distributed, each element of the effective interference channel matrix $\mathbf{G}^{[i,j]}$ is i.i.d. complex Gaussian with zero mean and unit variance.

Remark 1: In general, the conventional scheduling metric such as SNR or SINR in the IBC is dependent on the precoding matrices at the transmitters, which makes the joint optimization of the precoder design and user scheduling difficult to be separated from each other and implemented with feasible signaling overhead and low complexity. The previous schemes [2], [27] for the IBC only consider the design of the precoding matrices and receive filters without any consideration of user scheduling.

With the cascaded precoding matrix design, however, the proposed scheme decouples the user scheduling metric calculation and the user-specific precoding matrix $\mathbf{V}_i$, as shown in (2). In addition, the receive beamforming vector design can also be decoupled from $\mathbf{V}_i$ as shown in (18). A similar cascaded precoding matrix design was used in [4] for some particular cases of the antenna configuration without the consideration of user scheduling. However, the proposed scheme applies to an arbitrary antenna and channel configuration, where the inter-cell interference is suppressed with the aid of opportunistic user scheduling. In addition, we shall show in the sequel that the optimal DoF can be achievable under a certain user scaling condition for an arbitrary antenna configuration without any iterative optimization procedure between the users and BSs.

Remark 2: Note that although it is assumed in the proposed scheme that each user feeds back the $(1 \times S)$-dimensional vector $\mathbf{f}_i^{[i,j]}$ to its home cell, the amount of CSI feedback is equivalent to that in the conventional single-cell MU-MIMO scheme such as ZF or minimum mean-squared error (MMSE) precoding. On the other hand, the previous iterative transceiver design schemes [2], [27] based on local CSI for the IBC require all the selected users to feed back the information of the receive beamformer to all the BSs in the network, which results in K times more feedback compared to the single-cell MU-MIMO scheme even for one iteration where the users feed back their receive beamformers and the BSs update their transmit precoders once. Furthermore, the information of weight coefficients also needs to be fed back to all the BSs in [27]. We shall show via numerical simulations in the sequel that even with K times less feedback the proposed scheme exhibits superior sum-rate compared to the iterative scheme [27].

We start with the following lemma to derive the achievable DoF.

Lemma 1 [14, Lemma 1]: The CDF of $\eta^{[i,j]}$, denoted by $F_\eta(x)$, can be written as

$$F_\eta(x) = c_0 x^{(K-1)S-L+1} + o\left(x^{(K-1)S-L+1}\right), \qquad (23)$$

for $0 \leq x < 1$, where $f(x) = o(g(x))$ means $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$, and $\tilde{c}_0$ is a constant determined by K, S, and L.

We further present the following lemma for the probabilistic interference level of the ODIA.

Lemma 2: The sum-interference remains constant with high probability for increasing SNR, that is,

$$\mathcal{P} \triangleq \lim_{SNR\to\infty} Pr\left\{ \sum_{i=1}^{K} \sum_{j=1}^{S} I^{[i,j]} \leq \epsilon \right\} = 1 \qquad (24)$$

for any $0 < \epsilon < \infty$, if

$$N = \omega\left(\text{SNR}^{(K-1)S-L+1}\right). \qquad (25)$$

Proof: See appendix A.   □

Now, the following theorem establishes the DoF achievability of the proposed ODIA.

Theorem 1 (User Scaling Law): The proposed ODIA scheme with the scheduling metric (22) achieves the optimal KS DoF for given S with high probability if

$$N = \omega\left(SNR^{(K-1)S-L+1}\right). \qquad (26)$$

Proof: If the sum-interference remains constant for increasing SNR with probability $\mathcal{P}$, the achievable rate in (13) can be further bounded by

$$
R^{[i,j]}
$$
$$
\geq \mathcal{P} \cdot \left[\log_2(\text{SNR}) + \log_2 \left(\frac{1}{\text{SNR}} + \frac{\gamma^{[i,j]} / \left(S \left\| \mathbf{v}_i^{(\max)} \right\|^2 \right)}{1 / \left\| \mathbf{v}_i^{(\max)} \right\|^2 + \epsilon} \right) \right],
$$
$$(27)$$

for any $0 \leq \epsilon < \infty$. Thus, the achievable DoF in (10) can be bounded by

$$\text{DoF} \geq KS \cdot \mathcal{P}. \qquad (28)$$

From Lemma 2, it is immediate to show that $\mathcal{P}$ tends to 1, and hence KS DoF is achievable if $N = \omega\left(\text{SNR}^{(K-1)S-L+1}\right)$, which proves the theorem.   □

From Theorem 1, it is shown that there exist a fundamental trade-off between the achievable DoF KS and required user scaling of $N = \omega\left(\text{SNR}^{(K-1)S-L+1}\right)$. This trade-off can also be observed in terms of the sum-rate even under a practical system setup, as we shall show in Section VII. Therefore, a higher S value can be chosen to achieve higher DoF or sum-rate if there exist more users in the network.

The following remark discusses the uplink and downlink duality on the DoF achievability within the OIA framework.

Remark 3 (Uplink-Downlink Duality on the DoF Achievability): The same scaling condition of $N = \omega\left(\text{SNR}^{K(S-1)-L+1}\right)$ was achieved to obtain KS DoF in the K-cell uplink interference channel [14], each cell of which is composed of a BS with M antennas and N users each with L antennas. Similarly as in the proposed scheme, the uplink scheme [14] also selects S users that generate the minimal interference to the receivers (BSs). In the uplink scheme, the transmitters (users) perform SVD-based beamforming and the receivers (BSs) employ a ZF equalization, while in the proposed downlink case the transmitters (BSs) perform ZF precoding and the receivers (users) employ SVD-based beamforming. In addition, each transmitter sends the information on effective channel vectors to the corresponding receiver in the uplink case, and vise versa in the downlink case. The transmit power per spatial stream is the same for both the cases. Therefore, Theorem 1 implies that the same DoF is achievable with the same user scaling law for the downlink and uplink cases.

The user scaling law characterizes the trade-off between the asymptotic DoF and number of users, i.e., the more number of users, the more achievable DoF. In addition, we relate the derived user scaling law to the interference decaying rate with respect to N for given SNR in the following theorem.

Theorem 2 (Interference Decaying Rate): If the user scaling condition to achieve a target DoF is given by $N = \omega\left(SNR^{\tau'}\right)$ for some $\tau' > 0$, then the interference decaying rate is given by

$$E\left\{ \frac{1}{\eta^{[i,j]}} \right\} \geq \Theta\left(N^{1/\tau'}\right), \qquad (29)$$

where $f(x) = \Theta(g(x))$ if $f(x) = O(g(x))$ and $g(x) = O(f(x))$.

Proof: From the proof of Theorem 1, the user scaling condition to achieve a target DoF is given by $N = \omega\left(\text{SNR}^{\tau'}\right)$ if and only if the CDF of $\eta^{[i,j]}$ is given by $a_0 x^{-\tau'} + o(x^{-\tau'})$ for $\tau' > 0$. The theorem can be shown by following the footsteps of the proof of [28, Lemma 4], and the detailed proof is omitted.   □

From Theorem 2, the interference decaying rate of the proposed ODIA for the jth selected user in the i-th cell with

respect to N is given by

$$E\left\{\frac{1}{\eta^{[i,j]}}\right\} \geq \Theta\left(N^{1/((K-1)S-L+1)}\right), \quad (30)$$

which is also the same as the result in the uplink channel [28]. The user scaling law also provides an insight on the interference decaying rate with respect to N for given SNR; that is, the smaller SNR exponent of the user scaling law, the faster interference decreasing rate with respect to N.

A. Comparison to the Previous Results

In this subsection, the DoF achievability is compared with the previous results in [15], [17], and [19]. From [19, Lemma 4.2], choosing $M_i = S$ ($S \leq M$) therein, where M_i denotes the number of spatial streams in the i-th cell, S DoF is achievable per cell, i.e., KS DoF in total, if $N = \Theta(\text{SNR}^\rho)$ for $\rho > KS - L$; or equivalently,

$$N = \omega\left(\text{SNR}^{KS-L}\right). \quad (31)$$

In addition, from [17, Th. 6], choosing $d = S$ ($S \leq M$) therein, which is the target DoF for each cell, KS DoF is achievable, under the same scaling condition given in (31). The same conclusion was obtained in [15]. Intuitively, the exponent of SNR in the user scaling condition represents the number of interference spatial streams after suppression and nulling. Note that the number of total interference spatial streams received at each user is $KS - 1$ excluding one desired spatial stream, and that the receive diversity for nulling received interference is $L - 1$ leaving one spatial domain for receiving a desired stream. Thus, the exponent of SNR becomes $(KS - 1) - (L - 1) = KS - L$ as shown in (31).

On the other hand, the proposed ODIA pre-nulls $S - 1$ intra-cell interference signals at the transmitter, and hence the exponent becomes $(KS - 1) - (S - 1) - (L - 1) = (K - 1)S - L + 1$ as shown in Theorem 1. This improvement in the user scaling condition is attributed to the additional CSI feedback of $\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_i^{[i,j]}\mathbf{P}_i$, which are used to design the precoding matrix $\mathbf{V}_i$ in (7). This feedback procedure corresponds to the feedforward of the effective channel vectors in the uplink OIA case [14].

Note that even with this feedback procedure, a straightforward dual transceiver and user scheduling scheme inspired by the uplink OIA would result in an infinitely-iterative optimization between the user scheduling and transceiver design, because the received interference changes according to the precoding matrix and receive beamforming vector. Furthermore, only with the cascaded precoding matrix, the iterative optimization is still needed, since the coupled optimization issue is still there, as shown in [4]. It is indeed the proposed ODIA that can achieve the same user scaling condition of the uplink case, i.e., $N = \omega\left(\text{SNR}^{(K-1)S-L+1}\right)$, without any iterative design. In addition, the proposed ODIA applies to an arbitrary M, L, and K, whereas the optimal DoF is achievable only in a few special cases in the scheme proposed in [4].

V. ODIA WITH LIMITED FEEDBACK

In the proposed ODIA scheme, the vectors $(\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_i^{[i,j]}\mathbf{P}_i)$ in (4) can be fed back to the corresponding BS using pilots rotated by the effective channels [29]. However, this analog feedback requires two consecutive pilot phases for each user: regular pilot for uplink channel estimation and analog feedback for effective channel estimation. Hence, pilot overhead grows with respect to the number of users in the network. As a result, in practical systems with massive users, it is more preferable to follow the widely-used limited feedback approach [30], in which the information of $\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_i^{[i,j]}\mathbf{P}_i$ is fed back using codebooks.

For limited feedback, we define the codebook by

$$\mathcal{C}_f = \{\mathbf{c}_1, \ldots, \mathbf{c}_{N_f}\}, \quad (32)$$

where N_f is the codebook size and $\mathbf{c}_k \in \mathbb{C}^{S \times 1}$ is a unit-norm codeword, i.e., $\|\mathbf{c}_i\|^2 = 1$. Hence, the number of feedback bits used is given by

$$n_f = \lceil \log_2 N_f \rceil \, (\text{bits}) \quad (33)$$

For $\mathbf{f}_i^{[i,j]\mathsf{H}} = \mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_i^{[i,j]}\mathbf{P}_i$, each user quantizes the normalized vector for given $\mathcal{C}_f$ from

$$\tilde{\mathbf{f}}_i^{[i,j]} = \arg\max_{\{\mathbf{w}=\mathbf{c}_k : 1 \leq k \leq N_f\}} \frac{\left|\mathbf{f}_i^{[i,j]\mathsf{H}}\mathbf{w}\right|^2}{\left\|\mathbf{f}_i^{[i,j]}\right\|^2}. \quad (34)$$

Now, the user feeds back three types of information: 1) index of $\tilde{\mathbf{f}}_i^{[i,j]}$, 2) channel gain of $\left\|\mathbf{f}_i^{[i,j]}\right\|^2$, and 3) scheduling metric $\eta^{[i,j]}$. Note that the feedback of scalar information such as channel gains and scheduling metrics can be fed back relatively accurately with a few bits of uplink data, and the main challenge is on the feedback of the angle of vectors [30]. Thus, in what follows, the aim is to analyze the impact of the quantized feedback of the index of $\tilde{\mathbf{f}}_i^{[i,j]}$. Then, BS i constructs the quantized vectors $\hat{\mathbf{f}}^{[i,j]}$ from

$$\hat{\mathbf{f}}^{[i,j]} \triangleq \left\|\mathbf{f}_i^{[i,j]}\right\|^2 \cdot \tilde{\mathbf{f}}_i^{[i,j]}, \quad i = 1, \ldots, S, \quad (35)$$

and the precoding matrix $\hat{\mathbf{V}}_i$ from

$$\hat{\mathbf{V}}_i = \hat{\mathbf{F}}_i^{-1}\mathbf{\Gamma}_i, \quad (36)$$

where $\mathbf{\Gamma}_i = \text{diag}\left(\sqrt{\gamma^{[i,1]}}, \ldots, \sqrt{\gamma^{[i,S]}}\right)$ and $\hat{\mathbf{F}}_i = \left[\hat{\mathbf{f}}^{[i,1]}, \ldots, \hat{\mathbf{f}}^{[i,S]}\right]^{\mathsf{H}}$.

With limited feedback, the received signal vector after receive beamforming is written by

$$\tilde{y}^{[i,j]} = \mathbf{f}_i^{[i,j]\mathsf{H}}\hat{\mathbf{V}}_i\mathbf{x}_i + \cdot \sum_{k=1, k\neq i}^{K} \mathbf{f}_k^{[i,j]\mathsf{H}}\hat{\mathbf{V}}_k\mathbf{x}_k + \mathbf{u}^{[i,j]\mathsf{H}}\mathbf{z}^{[i,j]}$$

$$(37)$$

$$= \sqrt{\gamma^{[i,j]}}x^{[i,j]} + \underbrace{\left(\mathbf{f}_i^{[i,j]\mathsf{H}}\hat{\mathbf{V}}_i\mathbf{x}_i - \sqrt{\gamma^{[i,j]}}x^{[i,j]}\right)}_{\text{residual intra-cell interference}}$$

$$+ \sum_{k=1, k\neq i}^{K} \mathbf{f}_k^{[i,j]\mathsf{H}}\hat{\mathbf{V}}_k\mathbf{x}_k + \mathbf{u}^{[i,j]\mathsf{H}}\mathbf{z}^{[i,j]}, \quad (38)$$

where the residual intra-cell interference is non-zero due to the quantization error in $\hat{\mathbf{V}}_i$.

It is important to note that the residual intra-cell interference is a function of $\hat{\mathbf{V}}_i$, which includes other users' channel information, and thus each user treats this term as unpredictable noise and calculates only the inter-cell interference for the scheduling metric as in (3); that is, the scheduling metric is not changed for the ODIA with limited feedback.

The following theorem establishes the user scaling law for the ODIA with limited feedback.

Theorem 3: The ODIA with a Grassmannian[3] or random codebook achieves the same user scaling law of the ODIA with perfect CSI described in Theorem 1, if

$$n_f = \omega \left(\log_2 \mathsf{SNR} \right). \tag{39}$$

That is, KS DoF is achievable with high probability if $N = \omega \left(\mathsf{SNR}^{(K-1)S-L+1} \right)$ and (39) holds true.

Proof: Without loss of generality, the quantized vector $\hat{\mathbf{f}}^{[i,j]}$ can be decomposed as

$$\hat{\mathbf{f}}^{[i,j]} = \left\| \mathbf{f}_i^{[i,j]} \right\|^2 \cdot \tilde{\mathbf{f}}_i^{[i,j]}$$
$$= \sqrt{1 - d^{[i,j]2}} \cdot \mathbf{f}_i^{[i,j]} + d^{[i,j]} \left\| \mathbf{f}_i^{[i,j]} \right\|^2 \left(\mathbf{t}^{[i,j]} \right), \tag{40}$$

where $\mathbf{t}^{[i,j]}$ is a unit-norm vector i.i.d. over $\text{null} \left(\mathbf{f}_i^{[i,j]} \right)$ [21], [31]. At this point, we consider the worse performance case where each user finds $\hat{\mathbf{f}}^{[i,j]}$ such that with a slight abuse of notation

$$\hat{\mathbf{f}}^{[i,j]} = \sqrt{1 - d_i^{\max 2}} \cdot \mathbf{f}_i^{[i,j]} + d_i^{\max} \nu_i \cdot \mathbf{t}^{[i,j]}, \tag{41}$$

where

$$d_i^{\max} = \max \left\{ d^{[i,1]}, \ldots, d^{[i,S]} \right\},$$
$$\nu_i = \max \left\{ \left\| \mathbf{f}_i^{[i,j]} \right\|^2, j = 1, \ldots, S \right\}. \tag{42}$$

Note that more quantization error only degrades the achievable rate, and hence the quantization via (41) yields a performance lower-bound. Inserting (41) to (36) gives us

$$\hat{\mathbf{V}}_i = \left(\sqrt{1 - d_i^{\max 2}} \mathbf{F}_i + d_i^{\max} \nu_i \mathbf{T}_i \right)^{-1} \mathbf{\Gamma}_i, \tag{43}$$

where $\mathbf{F}_i = \left[\mathbf{f}_i^{[i,1]}, \ldots, \mathbf{f}_i^{[i,S]} \right]^{\mathsf{H}}$ and $\mathbf{T}_i = \left[\mathbf{t}^{[i,1]}, \ldots, \mathbf{t}^{[i,S]} \right]^{\mathsf{H}}$.

The Taylor expansion of $\left(\sqrt{1 - d_i^{\max 2}} \mathbf{F}_i + d_i^{\max} \nu_i \mathbf{T} \right)^{-1}$ in (36) gives us

$$\left(\sqrt{1 - d_i^{\max 2}} \mathbf{F}_i + d_i^{\max} \nu_i \mathbf{T}_i \right)^{-1}$$
$$= \mathbf{F}_i^{-1} - \mathbf{F}_i^{-1} \mathbf{T}_i \mathbf{F}_i^{-1} \nu_i d_i^{\max} + \sum_{k=2}^{\infty} \mathbf{A}_k \left(d_i^{\max} \right)^k, \tag{44}$$

where $\mathbf{A}_k$ is a function of $\mathbf{F}_i$ and $\mathbf{T}_i$. Thus, $\hat{\mathbf{V}}_i$ can be written by

$$\hat{\mathbf{V}}_i = \mathbf{F}_i^{-1} \mathbf{\Gamma}_i - d_i^{\max} \nu_i \mathbf{F}_i^{-1} \mathbf{T}_i \mathbf{F}_i^{-1} \mathbf{\Gamma}_i + \sum_{k=2}^{\infty} \left(d_i^{\max} \right)^k \mathbf{A}_k \mathbf{\Gamma}_i \tag{45}$$

Inserting (45) to (37) yields

$$\tilde{y}^{[i,j]} = \sqrt{\gamma^{[i,j]}} x^{[i,j]}$$
$$- \underbrace{d_i^{\max} \nu_i \mathbf{t}^{[i,j]\mathsf{H}} \mathbf{F}_i^{-1} \mathbf{\Gamma}_i \mathbf{x}_i + \sum_{k=2}^{\infty} \left(d_i^{\max} \right)^k \mathbf{f}_i^{[i,j]\mathsf{H}} \mathbf{A}_k \mathbf{\Gamma}_i \mathbf{x}_i}_{\text{residual intra-cell interference}}$$
$$+ \sum_{k=1, k \neq i}^{K} \mathbf{f}_k^{[i,j]\mathsf{H}} \hat{\mathbf{V}}_k \mathbf{x}_k + \mathbf{u}^{[i,j]\mathsf{H}} \mathbf{z}^{[i,j]}. \tag{46}$$

Consequently, the rate $R^{[i,j]}$ in (9) is given by

$$R^{[i,j]} = \log_2 \left(1 + \frac{\gamma^{[i,j]}}{\frac{S + \Delta^{[i,j]}}{\mathsf{SNR}} + \sum_{k \neq i}^{K} \sum_{s=1}^{S} \left| \mathbf{f}_k^{[i,j]\mathsf{H}} \mathbf{v}^{[k,s]} \right|^2} \right), \tag{47}$$

where

$$\Delta^{[i,j]} = \left(d_i^{\max} \right)^2 \delta_1 \cdot \mathsf{SNR} + \sum_{k=2}^{\infty} \left(d_i^{\max} \right)^{2k} \delta_k \cdot \mathsf{SNR}, \tag{48}$$

$$\delta_1 = \left(\nu_i^2 \mathbf{t}^{[i,j]\mathsf{H}} \mathbf{F}_i^{-1} \mathbf{\Gamma}_i^2 \mathbf{F}_i^{-\mathsf{H}} \mathbf{t}^{[i,j]} \right),$$
$$\delta_k = \left(\mathbf{f}_i^{[i,j]\mathsf{H}} \mathbf{A}_k \mathbf{\Gamma}_i^2 \mathbf{A}_k^{\mathsf{H}} \mathbf{f}_i^{[i,j]} \right). \tag{49}$$

As in (11) to (13), the achievable rate can be bounded by

$$R^{[i,j]} \geq \mathcal{P}' \cdot \left[\log_2 \mathsf{SNR} + \log_2 \left(\frac{1}{\mathsf{SNR}} + \frac{\frac{\gamma^{[i,j]}}{\left\| \mathbf{v}_i^{(\max)} \right\|^2}}{\frac{1}{\left\| \mathbf{v}_i^{(\max)} \right\|^2} + 2\epsilon} \right) \right], \tag{50}$$

where

$$\mathcal{P}' \triangleq \Pr \left\{ \left(\sum_{i=1}^{K} \sum_{j=1}^{S} I^{[i,j]} \leq \epsilon \right) \& \left(\Delta^{[i,j]} / \left\| \mathbf{v}_i^{(\max)} \right\|^2 \leq \epsilon \right), \right.$$
$$\left. \forall i \in \mathcal{K}, j \in \mathcal{S} \right\} \tag{51}$$

$$= \Pr \left\{ \sum_{i=1}^{K} \sum_{j=1}^{S} I^{[i,j]} \leq \epsilon, \forall i \in \mathcal{K}, j \in \mathcal{S} \right\}$$
$$\cdot \Pr \left\{ \Delta^{[i,j]} \leq \epsilon', \forall i \in \mathcal{K}, j \in \mathcal{S} \right\}, \tag{52}$$

where $\epsilon' \triangleq \epsilon \cdot \left\| \mathbf{v}_i^{(\max)} \right\|^2$. Here, (52) follows from the fact that the inter-cell interference $I^{[i,j]}$ and residual intra-cell interference $\Delta^{[i,j]}$ are independent each other. Note also that the level of residual intra-cell interference does not affect the user selection and is determined only by the codebook

---

[3]The Grassmannian codebook refers to a vector codebook having a maximized minimum chordal distance of any two codewords, which can be obtained by solving the Grassmannian line packing problem [30].

size N_f. Hence, the user selection result does not change for different N_f.

The achievable DoF is given by

$$\text{DoF} \geq \lim_{\text{SNR}\to\infty} KS \cdot \mathcal{P}'. \tag{53}$$

If $N = \omega\left(\text{SNR}^{(K-1)S-L+1}\right)$, the first term of (52) tends to 1 according to Theorem 1. Thus, the maximum DoF can be obtained if and only if $\Delta^{[i,j]} \leq \epsilon'$ for all selected users for increasing SNR.

In Appendix B, it is shown that $\Delta^{[i,j]} \leq \epsilon'$ for all selected users if $n_f = \omega\left(\log_2 \text{SNR}\right)$ for both Grassmannian and random codebooks. Therefore, if $N = \omega\left(\text{SNR}^{(K-1)S-L+1}\right)$ and $n_f = \omega\left(\log_2 \text{SNR}\right)$, $\mathcal{P}'$ in (52) tends to 1, which proves the theorem. $\qquad\square$

From Theorem 3, the minimum number of feedback bits n_f is characterized to achieve the optimal KS DoF, which increases with respect to $\log_2(\text{SNR})$. It is worthwhile to note that the results are the same for the Grassmannian and random codebooks.

We conclude this section by providing the following comparison to the well-known conventional results on limited feedback systems.

Remark 4: In the previous works on limited feedback systems, the performance analysis was focused on the average SNR or the average rate loss [32]. In an average sense, the Grassmannian codebook is in general outperforms the random codebook. However, our scheme focuses on the asymptotic codebook performance for given channel instance for increasing SNR, and it turned out that this asymptotic behaviour is the same for the two codebooks. In fact, this result agrees with the previous works e.g., [33], in which the performance gap between the two codebooks was shown to be negligible as n_f increases through computer simulations.

Remark 5: For the MIMO broadcast channel with limited feedback, where the transmitter has L antennas and employs the random codebook, it was shown [21] that the achievable rate loss for each user, denoted by ΔR, due to the finite size of the codebook is upper-bounded by

$$\Delta < \log_2\left(1 + \text{SNR} \cdot 2^{-n_f/(L-1)}\right). \tag{54}$$

Thus, to achieve the maximum 1 DoF for each user, or to make the rate loss negligible as the SNR increases, the term $\text{SNR} \cdot 2^{-n_f/(L-1)}$ should remain constant for increasing SNR. That is, n_f should scale faster than $(L-1)\log_2(\text{SNR})$. Note however that the proof of Theorem 3 is different from that in [21], since the residual interference due to the limited feedback, $\Delta^{[i,j]}$, needs to vanish for any given channel instance with respect to SNR to achieve a non-zero DoF per spatial stream. Though the system and proof are different, our results of Theorem 3 are consistent with this previous result.

VI. SPECTRALLY EFFICIENT ODIA (SE-ODIA)

In this section, we propose a spectrally efficient OIA (SE-ODIA) scheme and show that the proposed SE-ODIA achieves the optimal multiuser diversity gain $\log\log N$. For the DoF achievability, it was enough to design the user scheduling

in the sense to minimize inter-cell interference. However, to achieve optimal multiuser diversity gain, the gain of desired channels also needs to be considered in user scheduling. The overall procedure of the SE-ODIA follows that of the ODIA described in Section III except the the third stage 'User Scheduling'. In addition, we assume the perfect feedback of the effective desired channels $\mathbf{u}^{[i,j]\mathsf{H}}\mathbf{H}_i^{[i,j]}\mathbf{P}_i$ for the SE-ODIA. We incorporate the semiorthogonal user selection algorithm proposed in [34] to the ODIA framework taking into consideration inter-cell interference. Specifically, the algorithm for the user scheduling at the BS side is as follows:

- Step 1: Initialization:

$$\mathcal{N}_1 = \{1, \ldots, N\}, \quad s = 1 \tag{55}$$

- Step 2: For each user $j \in \mathcal{N}_s$ in the i-th cell, the s-th orthogonal projection vector, denoted by $\tilde{\mathbf{b}}_s^{[i,j]}$, for given $\left\{\mathbf{b}_1^{[i]}, \ldots, \mathbf{b}_{s-1}^{[i]}\right\}$ is calculated from:

$$\tilde{\mathbf{b}}_s^{[i,j]} = \mathbf{f}_i^{[i,j]} - \sum_{s'=0}^{s-1} \frac{\mathbf{b}_{s'}^{[i]\mathsf{H}}\mathbf{f}_i^{[i,j]}}{\|\mathbf{b}_{s'}^{[i]}\|^2}\mathbf{b}_{s'}^{[i]} \tag{56}$$

Note that if $s = 1$, $\tilde{\mathbf{b}}_1^{[i,j]} = \mathbf{f}_i^{[i,j]}$.

- Step 3: For the s-th user selection, a user is selected at random from the user pool $\mathcal{N}_s$ that satisfies the following two conditions:

$$\mathsf{C}_1 : \eta^{[i,j]} \leq \eta_I, \quad \mathsf{C}_2 : \|\tilde{\mathbf{b}}_s^{[i,j]}\|^2 \geq \eta_D \tag{57}$$

Denote the index of the selected user by $\pi(s)$ and define

$$\mathbf{b}_s^{[i]} = \tilde{\mathbf{b}}_s^{[i,\pi(s)]}. \tag{58}$$

- Step 4: If $s < S$, then find the $(s+1)$-th user pool $\mathcal{N}_{s+1}$ from:

$$\mathcal{N}_{s+1} = \left\{j : j \in \mathcal{N}_s, j \neq \pi(s), \frac{\left|\mathbf{f}_i^{[i,j]\mathsf{H}}\mathbf{b}_s^{[i]}\right|}{\|\mathbf{f}_i^{[i,j]}\|\|\mathbf{b}_s^{[i]}\|} < \alpha\right\},$$
$$s = s + 1, \tag{59}$$

where $\alpha > 0$ is a positive constant. Repeat Step 2 to Step 4 until $s = S$.

To show the SE-ODIA achieves the optimal multiuser diversity gain, we start with the following lemma for the bound on $|\mathcal{N}_s|$.

Lemma 3: The cardinality of $\mathcal{N}_s$ can be bounded by

$$|\mathcal{N}_s| \gtrsim N \cdot \alpha^{2(S-1)}. \tag{60}$$

The approximated inequality becomes tight as N increases.

Proof: See Appendix C. $\qquad\square$

We also introduce the following useful lemma.

Lemma 4: If $x \in \mathbb{C}^{M\times 1}$ has its element i.i.d. according to $\mathcal{CN}(0, \sigma^2)$ and $\mathbf{A}$ is an idempotent matrix of rank r (i.e., $\mathbf{A}^2 = \mathbf{A}$), then $\mathbf{x}^{\mathsf{H}}\mathbf{A}\mathbf{x}/\sigma^2$ has a Chi-squared distribution with $2r$ degrees-of-freedom.

Proof: See [35]. $\qquad\square$

In addition, the following lemma on the achievable rate of the SE-ODIA will be used to show the achievability of optimal multiuser diversity gain.

Lemma 5: For the j-th selected user in the i-th cell, the achievable rate is bounded by

$$R^{[i,j]} \geq \log_2 \left(1 + \frac{\frac{\left\| \mathbf{b}_j^{[i]} \right\|^2}{1+\frac{(S-1)^4 \alpha^2}{1-(S-1)\alpha^2}}}{\frac{S}{\mathsf{SNR}} + \sum_{k \neq i}^{K} \sum_{s=1}^{S} \left| \mathbf{f}_k^{[i,j]\mathsf{H}} \mathbf{v}^{[k,s]} \right|^2} \right). \quad (61)$$

Proof: Since the chosen channel vectors are not perfectly orthogonal, there is degradation in the effective channel gain $\gamma^{[i,j]}$. Specifically, for the j-th selected user in the i-th cell, we have

$$\gamma^{[i,j]} = \frac{1}{\left[\left(\mathbf{F}_i \mathbf{F}_i^{\mathsf{H}} \right)^{-1} \right]_{j,j}} > \frac{\left\| \mathbf{b}_j^{[i]} \right\|^2}{1 + \frac{(S-1)^4 \alpha^2}{1-(S-1)\alpha^2}}, \quad (62)$$

which follows from [34, Lemma 2]. Inserting (62) to the sum-rate lower bound in (9) proves the lemma.   □

Now the following theorem establishes the achievability of the optimal multiuser diversity gain.

Theorem 4: The proposed SE-ODIA scheme with

$$\eta_D = \epsilon_D \log \mathsf{SNR} \quad (63)$$

$$\eta_I = \epsilon_I \mathsf{SNR}^{-1} \quad (64)$$

for any $\epsilon_D, \epsilon_I > 0$ achieves the optimal multiuser diversity gain given by

$$R^{[i,j]} = \Theta \left(\log \left(\mathsf{SNR} \cdot \log N \right) \right), \quad (65)$$

with high probability for all selected users in the high SNR regime if

$$N = \omega \left(\mathsf{SNR}^{\frac{(K-1)S-L+1}{1-(\epsilon_D/2)}} \right). \quad (66)$$

Proof: Amongst $|\mathcal{N}_s|$ users, there should exist at least one user satisfying the conditions C_1 and C_2 to make the proposed user scheduling for the SE-ODIA valid. Thus, we first show the probability that there exist at least one valid user, denoted by p_s, converges to 1, for the s-th user selection, if N scales according to (66) with the choices (63) and (64).

The probability that each user satisfies the two conditions is given by $\Pr\{\mathsf{C}_1\} \cdot \Pr\{\mathsf{C}_2\}$, because the two conditions are independent of each other. Consequently, p_s is given by

$$\mathsf{p}_s = 1 - (1 - \Pr\{\mathsf{C}_1\} \cdot \Pr\{\mathsf{C}_2\})^{|\mathcal{N}_s|} \quad (67)$$

$$\gtrsim 1 - (1 - \Pr\{\mathsf{C}_1\} \cdot \Pr\{\mathsf{C}_2\})^{N \cdot \alpha^{2(S-1)}}. \quad (68)$$

Note that each element of $\mathbf{f}_i^{[i,j]\mathsf{H}} = \mathbf{u}^{[i,j]\mathsf{H}} \mathbf{H}_i^{[i,j]} \mathbf{P}_i$ is i.i.d. according to $\mathcal{CN}(0, 1)$, because $\mathbf{P}_k$ is independently and randomly chosen orthonormal basis for an S-dimensional subspace of $\mathbb{C}^{M \times M}$ and because $\mathbf{u}^{[i,j]\mathsf{H}}$ is designed independently of $\mathbf{H}_i^{[i,j]}$ and isotropically distributed over a unit sphere. Thus, $\mathbf{f}_i^{[i,j]\mathsf{H}} = \mathbf{u}^{[i,j]\mathsf{H}} \mathbf{H}_i^{[i,j]} \mathbf{P}_i$ has its element i.i.d. according to $\mathcal{CN}(0, 1)$.

Let us define $\mathbf{P}$ by

$$\mathbf{P} \triangleq \left(\mathbf{I} - \sum_{s'=0}^{s-1} \frac{\mathbf{b}_{s'}^{[i]} \mathbf{b}_{s'}^{[i]\mathsf{H}}}{\|\mathbf{b}_{s'}^{[i]}\|^2} \right), \quad (69)$$

which is a symmetric idempotent matrix with rank $(S-s+1)$. Since $\mathbf{b}_s^{[i]} = \mathbf{P} \mathbf{f}_i^{[i,j]}$, from Lemma 4, $\left\| \mathbf{b}_s^{[i]} \right\|^2$ is a Chi-squared random variable with $2(S-s+1)$ degrees-of-freedom.

In Appendix D, for $\eta_D > 2$, we show that

$$\lim_{\mathsf{SNR} \to \infty} \mathsf{p}_s = 1, \quad \text{if } N = \omega \left(\mathsf{SNR}^{\frac{(K-1)S-L+1}{1-(\epsilon_D/2)}} \right). \quad (70)$$

Now, given that there always exist at least one user that satisfies the conditions C_1 and C_2, the achievable sum-rate can be bounded from Lemma 5 by

$$R^{[i,j]} \geq \log_2 \left(1 + \frac{\frac{\left\| \mathbf{b}_j^{[i]} \right\|^2}{1+\frac{(S-1)^4 \alpha^2}{1-(S-1)\alpha^2}} \cdot \frac{1}{\left\| \mathbf{v}_i^{\max} \right\|^2}}{\frac{S}{\mathsf{SNR}\|\mathbf{v}_i^{\max}\|^2} + \sum_{k \neq i}^{K} \sum_{s=1}^{S} \left\| \mathbf{f}_k^{[i,j]} \right\|^2} \right) \quad (71)$$

$$\geq \log_2 \left(1 + \frac{\frac{\left\| \mathbf{b}_j^{[i]} \right\|^2}{1+\frac{(S-1)^4 \alpha^2}{1-(S-1)\alpha^2}} \cdot \mathsf{SNR}/ \left\| \mathbf{v}_i^{\max} \right\|^2}{S/ \left\| \mathbf{v}_i^{\max} \right\|^2 + K S \epsilon_I} \right) \quad (72)$$

$$= \log_2 \left(1 + \left\| \mathbf{b}_j^{[i]} \right\|^2 \mathsf{SNR} \cdot \xi \right) \quad (73)$$

$$\geq \log_2 \left(1 + \epsilon_D (\log N) \cdot \mathsf{SNR} \right), \quad (74)$$

where (72) follows from the fact that the sum-interference for all selected users, given by $\sum_{j=1}^{S} \sum_{i=1}^{K} \eta^{[i,j]} \mathsf{SNR}$ (See (17)), does not exceed $K S \epsilon_I$ by choosing $\eta_I = \epsilon_I \mathsf{SNR}^{-1}$. Furthermore, ξ is a constant given by

$$\xi = \frac{1}{\left\| \mathbf{v}_i^{\max} \right\|^2 \left(1 + \frac{(S-1)^4 \alpha^2}{1-(S-1)\alpha^2} \right) \left(S/ \left\| \mathbf{v}_i^{\max} \right\|^2 + K S \epsilon_I \right)}, \quad (75)$$

and (74) follows from $\|\mathbf{b}_j^{[i]}\|^2 \geq \eta_D = \epsilon_D \log N$. Therefore, the proposed SE-ODIA achieves the optimal multiuser diversity gain $\log \log N$ in the high SNR regime, if $N = \omega \left(\mathsf{SNR}^{\frac{(K-1)S-L+1}{1-(\epsilon_D/2)}} \right)$.   □

Therefore, the optimal multiuser gain of $\log \log N$ is achieved using the proposed SE-ODIA with the choices of (63) and (64). Note that since small ϵ_D suffices to obtain the optimal multiuser gain, the condition on N does not dramatically change compared with that required to achieve KS DoF (See Theorem 1). Thus, surprisingly, this means a slight increase in user scaling results in optimal multiuser diversity by using the proposed SE-ODIA. Combining the results in Theorem 1 and 4, we can conclude the achievability of the optimal DoF and multiuser gain as follows.

Remark 6: In fact, the ODIA described in Section III can be implemented using the SE-ODIA approach by choosing $\eta_D = 0$, $\alpha = 1$, and $\eta_I^{[i]} = \min \left\{ \eta^{[i,1]}, \ldots, \eta^{[i,N]} \right\}$, where $\eta_I^{[i]}$ denotes η_I at the i-th cell. In summary, the optimal KM DoF and optimal multiuser gain of $\log \log N$ can be achieved using the proposed ODIA framework, if the number of users per cell increases according to $N = \omega \left(\mathsf{SNR}^{\frac{(K-1)M-L+1}{1-(\epsilon_D/2)}} \right)$ for any $\epsilon_D > 0$.
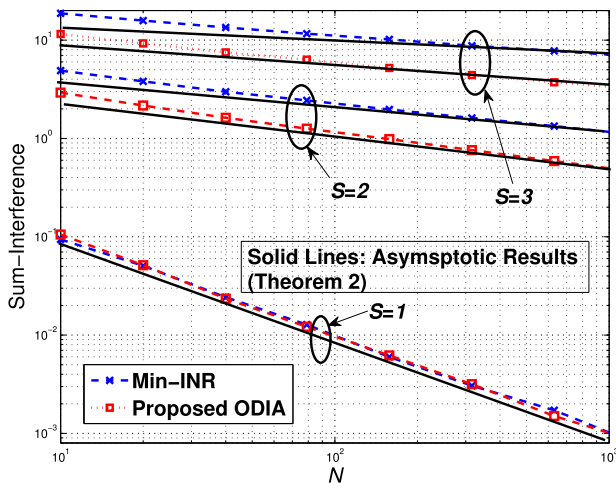
Fig. 2.   Normalized sum-interference versus N when $K = 3$, $M = 4$, $L = 2$.

Fig. 3.   Sum-rates versus SNR when $K = 2$, $M = 3$, $L = 2$, $S = 2$. The total achievable DoF for all cells is $KS = 4$. N and n_f change according to the SNR as $N = \text{SNR}^{(K-1)S-L+1} = \text{SNR}^1$ and $n_f = \log_2 \text{SNR}$, respectively.

VII. Numerical Results

In this section, we compare the performance of the proposed ODIA with two conventional schemes which also utilize the multi-cell random beamforming technique at BSs. First, we consider "max-SNR" technique, in which each user designs the receive beamforming vector in the sense to maximize the desired signal power, and feeds back the maximized signal power to the corresponding BS. Each BS selects S users who have higher received signal power. Second, "min-INR" technique is considered, in which each user performs receive beamforming in order to minimize the sum of inter-cell interference and intra-cell interference [19], [20]. Hence, intra-cell interference does not vanish at users, while the proposed ODIA perfectly eliminates it via transmit beamforming. Specifically, from (6), the j-th user in the i-th cell should calculate the following S scheduling metrics

$$\eta_{\text{min-INR},m}^{[i,j]} = \underbrace{\left\| \mathbf{u}^{[i,j],m\mathsf{H}} \mathbf{H}_i^{[i,j]} \tilde{\mathbf{P}}_{i,m} \right\|^2}_{\text{intra-cell interference}}$$
$$+ \underbrace{\sum_{k=1,k\neq i}^{K} \left\| \mathbf{u}^{[i,j],m\mathsf{H}} \mathbf{H}_k^{[i,j]} \mathbf{P}_k \right\|^2}_{\text{inter-cell interference}}, \quad m = 1, \dots, S, \tag{76}$$

where $\tilde{\mathbf{P}}_{i,m} \triangleq [\mathbf{p}_{1,i}, \dots, \mathbf{p}_{m-1,i}, \mathbf{p}_{m+1,i}, \dots, \mathbf{p}_{S,i}]$. For each m, the receive beamforming vector $\mathbf{u}^{[i,j],m}$ is assumed to be designed such that $\eta_{\text{min-INR},m}^{[i,j]}$ is minimized. Each user feedbacks S scheduling metrics to the corresponding BS, and the BS selects the user having the minimum scheduling metric for the m-th spatial stream, $m = 1, \dots, S$. For more details about the min-INR scheme, refer to [19] and [20].

Fig. 2 shows the sum-interference at all users for varying number of users per cell, N, when $K = 3$, $M = 4$, $L = 2$, and SNR=20dB. The solid lines are obtained from Theorem 2 with proper biases, and thus only the slopes of the solid lines are relevant. The decaying rates of sum-interference of the proposed ODIA are higher than those of the min-INR scheme since intra-cell interference is perfectly eliminated in
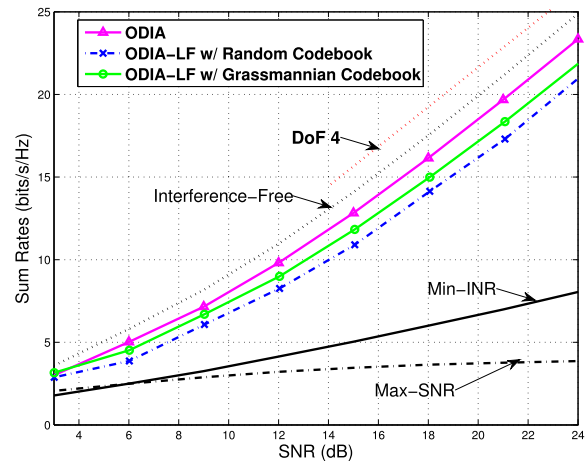
the proposed ODIA. In addition, the interference decaying rates of the proposed ODIA are consistent with the theoretical results of Theorem 2, which proves that the user scaling condition derived in Theorem 1 and the interference bound in Theorem 2 are in fact accurate and tight.

Fig. 3 shows the sum-rate vs. SNR when $K = 2$, $M = 3$, $L = 2$, and $S = 2$. Thus, the total achievable DoF is $KS = 4$. Here, to comply with Theorems 1 and 3, N and n_f are assumed to scale with respect to SNR as $N = \text{SNR}^{(K-1)S-L+1} = \text{SNR}^1$ and $n_f = \log_2 \text{SNR}$, respectively. For an upper bound, the genie-aided interference-free ODIA scheme is plotted as 'Interference-Free' in which both the intra- and inter-cell interference was removed in the achievable rate calculation of the ODIA scheme. It is seen that the proposed ODIA achieves the target DoF of 4 with $N = \text{SNR}^{(K-1)-L+1}$, which again proves Theorem 1. In addition, the ODIA with limited feedback (ODIA-LF) also achieves the target DoF of 4 for both random and Grassmannian codebooks with $n_f = \log_2(\text{SNR})$, which verifies Theorem 3. The Max-SNR scheme achieves zero DoF, since the interference is not suppressed at all for increasing SNR. The Min-INR scheme cannot achieve the target DoF, since the user scaling is not fast enough to satisfy $N = \text{SNR}^{KS-L} = \text{SNR}^2$ (See Section IV-A).

To evaluate the sum-rates of the SE-ODIA, the parameters η_I, η_D, and α need to be optimized for the SE-ODIA. Fig. 4 shows the sum-rate performance of the proposed SE-ODIA for varying η_I or η_D with two different α values when $K = 3$, $M = 4$, $L = 2$, $S = 2$, and $N = 20$. To obtain the sum-rate according to η_I, η_D was fixed to 1. Similarly, for the sum-rate according to η_D, η_I was fixed to 1. If η_I is too small, then there may not be eligible users that satisfy the conditions C_1 and C_2 in (57). Thus, *scheduling outage*[4] can occur frequently and the achievable sum-rate becomes low. On the other hand, if η_I is too large, then the received interference at users may not be sufficiently

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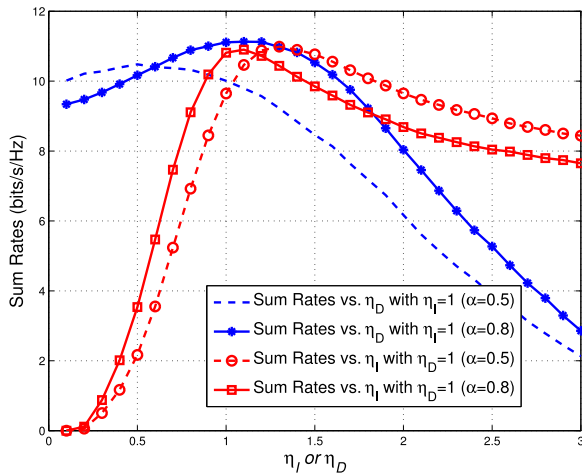[4]It indicates the situation that there are no users who are eligible for scheduling.

Fig. 4. Sum-rates of SE-ODIA versus η_D or η_I when $K = 3$, $M = 4$, $L = 2$, $S = 2$, and $N = 20$.

TABLE I
OPTIMIZED PARAMETERS (η_I, η_D, α) FOR DIFFERENT
SNRS AND N VALUES

	N=20	N=50
SNR=3dB	(2.5, 2.5, 0.8)	(2, 2.5, 0.8)
SNR=21dB	(1.5, 2, 0.8)	(1, 2, 0.8)

suppressed. Thus, the achievable sum-rate converges to that of the system without interference suppression. Similarly, if η_D is too large, then the scheduling outage occurs; and if η_D is too small, then desired channel gains cannot be improved. The orthogonality parameter α plays a similar role; if α is too small, the cardinality of the user pool $|\mathcal{N}_S|$ often becomes smaller than S, and scheduling outage happens frequently. If α is too large, then the orthogonality of the effective channel vectors of the selected users is not taken into account for scheduling. In short, the parameters η_I, η_D, and α need to be carefully chosen to improve the performance of the proposed SE-ODIA. In subsequent sum-rate simulations, proper sets of η_I, η_D, and α were numerically found for various N and SNR values and applied to the SE-ODIA. For instance, optimal (η_I, η_D, α) values that maximize the sum-rate for a few cases are provided in Table I. It is seen that in the noise-limited low SNR regime, large η_D helps, whereas in the interference-limited high SNR regime, small η_I improves the sum-rate. On the other hand, as N increases, interference can be suppressed by choosing smaller η_I values.

Fig. 5 shows the sum-rates for varying SNR values when $K = 3$, $M = 4$, $L = 2$, $S = 2$, and $N = 20$. In the noise-limited low SNR regime, the sum-rate of the min-INR scheme is even lower than that of the max-SNR scheme, because N is not large enough to suppress both intra- and inter-cell interference. For comparison, the sum-rate maximizing iterative transceiver design of [27] is also evaluated allowing one iteration between the BSs and users, i.e., the users feed back their receive beamforming vectors and BSs update their precoding matrices once. Even with one iteration, since each user needs to feed back the information of the receive beamformer to all the BSs in the network, the amount of
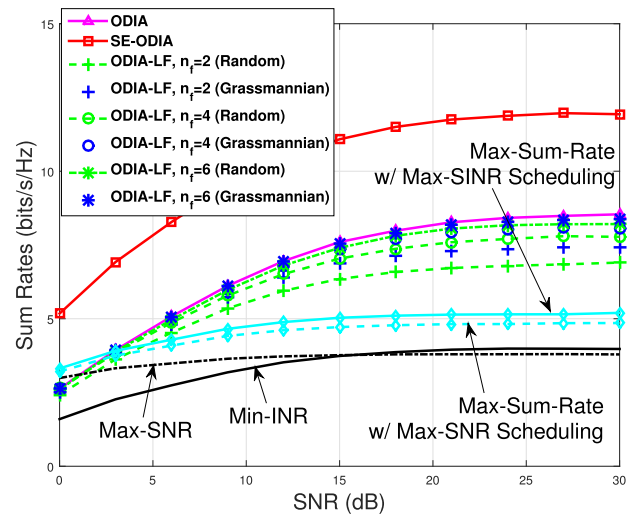

Fig. 5. Sum-rates versus SNR when $K = 3$, $M = 4$, $L = 2$, $S = 2$, and $N = 20$.

the feedback is K times more than in the proposed scheme. In addition, because [27] does not include any consideration of user scheduling, which is in general difficult to be separated from the precoding matrix design, we applied the conventional max-SNR and max-SINR scheduling schemes for the scheme of [27], which are labeled by 'Max-Sum-Rate w/ Max-SNR Scheduling' and 'Max-Sum-Rate w/ Max-SINR Scheduling,' respectively. The precoding matrix was fixed to be the one achieving the max-SNR in the scheduling metric calculation of [27], e.g., the scheduling metric for the max-SNR scheme is given by $\mathsf{SNR} \cdot \lambda_i^{[i,j]2}$, where $\lambda_i^{[i,j]}$ is the largest singular value of $\mathbf{H}_i^{[i,j]}$.

It is seen from the figure that the proposed ODIA outperforms the conventional schemes for SNRs larger than 3dB due to the combined effort of 1) transmit beamforming perfectly eliminating intra-cell interference and 2) receive beamforming effectively reducing inter-cell interference. In particular, the proposed ODIA shows higher sum-rate than the iterative transceiver design even with K times less feedback due to the separate joint optimization of the precoding matrix design and user scheduling.

The sum-rate performance of the ODIA-LF improves as n_f increases as expected. In practice, $n_f = 6$ exhibits a good compromise between the number of feedback bits and sum-rate performance for the codebook dimension of 2 (i.e., $S = 2$). On the other hand, the proposed SE-ODIA achieves higher sum-rates than the others including the ODIA for all SNR regime, because the SE-ODIA improves desired channel gains and suppresses interference simultaneously. Note however that the SE-ODIA includes the optimization on the parameters for given SNR and N and requires the user scheduling method based on perfect CSI feedback, which demands higher computational complexity than the user scheduling of the ODIA.

Fig. 6 shows the sum-rate performance of the proposed ODIA schemes for varying number of users per cell, N, when $K = 3$, $M = 4$, $L = 2$, $S = 2$, and SNR=20dB. For limited feedback, the Grassmannian codebook was employed.
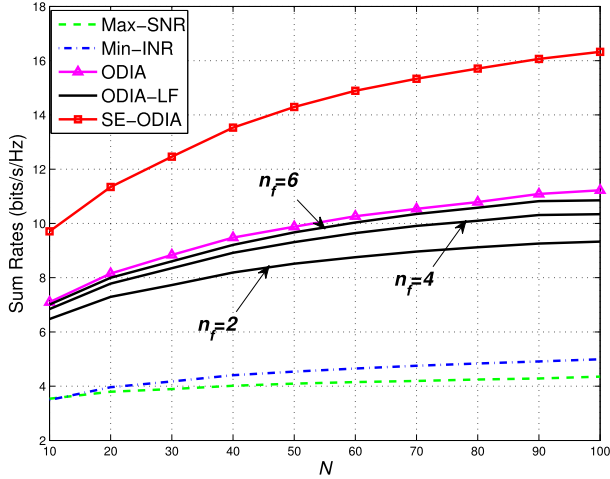
Fig. 6. Sum-rates versus N when $K = 3$, $M = 4$, $L = 2$, $S = 2$, and SNR=20dB.

The sum-rates of the proposed ODIA schemes increase faster than the two conventional schemes, which implies that the user scaling conditions of the proposed ODIA schemes required for a given DoF or MUD gain are lowered than the conventional schemes, as shown in Theorems 1 and 4.

VIII. CONCLUSION

In this paper, we proposed an opportunistic downlink interference alignment (ODIA) which intelligently combines user scheduling, transmit beamforming, and receive beamforming for multi-cell downlink networks. In the ODIA, the optimal DoF can be achieved with more relaxed user scaling condition $N = \left(\mathsf{SNR}^{(K-1)S-L+1}\right)$. To the best of our knowledge, this user scaling condition is the best known to date. We also considered a limited feedback approach for the ODIA, and analyzed the minimum number of feedback bits required to achieve the same user scaling condition of the ODIA with perfect feedback. We found that both Grassmannian and random codebooks yield the same condition on the number of required feedback bits. Finally, a spectrally efficient ODIA (SE-ODIA) was proposed to further improve the sum-rate of the ODIA, in which optimal multiuser diversity can be achieved even in the presence of inter-cell interference. Through numerical results, it was shown that the proposed ODIA schemes significantly outperform the conventional interference management schemes in practical environments.

APPENDIX A
PROOF OF LEMMA 2

Using (17), $\mathcal{P}$ can be bounded by

$$\mathcal{P} = \lim_{\mathsf{SNR}\to\infty} \Pr\left\{ \sum_{i=1}^{K} \sum_{j=1}^{S} \eta^{[i,j]}\mathsf{SNR} \le \epsilon \right\} \tag{77}$$

$$\ge \lim_{\mathsf{SNR}\to\infty} \Pr\left\{ \eta^{[i,j]} \le \frac{\mathsf{SNR}^{-1}\epsilon}{K S^2}, \forall i \in \mathcal{K}, \forall j \in \mathcal{S} \right\}. \tag{78}$$

Note that the selected users' $\eta^{[i,j]}$ are the minimum S values out of N i.i.d. random variables. Since the CDF of $\eta^{[i,j]}$ is

given by (23), (78) can be written by

$$\mathcal{P} \ge \lim_{\mathsf{SNR}\to\infty} \left[1 - \sum_{i=1}^{S-1} \binom{N}{i} \underbrace{\left(F_\eta\left(\frac{\epsilon\mathsf{SNR}^{-1}}{K S^2} \right) \right)^i}_{\triangleq A} \right.$$
$$\left. \cdot \left(1 - F_\eta\left(\frac{\epsilon\mathsf{SNR}^{-1}}{K S^2} \right) \right)^{N-i} \right] \tag{79}$$

$$\ge \lim_{\mathsf{SNR}\to\infty} \left[1 - \sum_{i=1}^{S-1} N^i A^i (1-A)^{-i} (1-A)^N \right], \tag{80}$$

where

$$(1-A)^N = \left(1 - c_0\left(\frac{\epsilon}{K S^2} \right)^{(K-1)S-L+1} \cdot \mathsf{SNR}^{-((K-1)S-L+1)} \right.$$
$$\left. - \Omega_{\mathsf{SNR}}\left(\mathsf{SNR}^{-((K-1)S-L)} \right) \right)^N. \tag{81}$$

Here, $f(x) = \Omega_x(g(x))$ means $\lim_{x\to\infty} \left| \frac{f(x)}{g(x)} \right| > 0$. Thus, $(1-A)^N$ tends to 0 (exponentially) if and only if N scales faster than $\mathsf{SNR}^{(K-1)S-L+1}$. Now, inserting $N = \omega\left(\mathsf{SNR}^{(K-1)S-L+1} \right)$ to (80) yields $\mathcal{P}$ tending to 1 for increasing SNR, because for given i, $(1-A)^N$ vanishes exponentially.

APPENDIX B
PROOF OF THEOREM 3

A. Grassmannian Codebook

For the Grassmannian codebook, the chordal distance between any two codewords is the same, i.e., $\sqrt{1 - |\mathbf{c}_i^{\mathsf{H}}\mathbf{c}_j|^2} = d$, $\forall i \neq j$. The Rankin, Gilbert-Varshamov, and Hamming bounds on the chordal distance give us [36]–[38]

$$d^{[i,j]2} \le \min\left\{ \frac{1}{2}, \frac{(S-1)N_f}{2S(N_f-1)}, \left(\frac{1}{N_f} \right)^{1/(S-1)} \right\}. \tag{82}$$

The bound in (82) is reduced to the third bound as N_f increases, thus providing arbitrarily tight upper-bound on $d^{[i,j]2}$. Thus, the first term of (48) remains constant if

$$\left(d_i^{\max} \right)^2 \delta_1 \cdot \mathsf{SNR} \le \left(\frac{1}{N_f} \right)^{1/(S-1)} \delta_1 \cdot \mathsf{SNR} \le \epsilon'. \tag{83}$$

This is reduced to $N_f^{-1/(S-1)} \le \epsilon'\delta_1^{-1}\mathsf{SNR}^{-1}$, or equivalently (39). Now, if (39) holds true, $d_i^{\max}$ tends to be arbitrarily small as SNR increases, and thus the second term of (48) is dominated by the first term. Therefore, if n_f scales with respect to $\log_2(\mathsf{SNR})$ as (39), the residual intra-cell interference $\Delta^{[i,j]}$ remains constant.

B. Random Codebook

In a random codebook, each codeword $\mathbf{c}_k$ is chosen isotropically and independently from the L-dimensional hyper sphere, and thus the maximum chordal distance of a random codebook is unbounded. Since $d^{[i,j]2}$ is the minimum of N_f chordal

distances resulting from N_f independent codewords, the CDF of $d^{[i,j]2}$ is given by [21] and [39]

$$F_d(z) \triangleq \Pr\left\{d^{[i,j]2} \le z\right\} = 1 - \left(1 - z^{S-1}\right)^{N_f}. \quad (84)$$

From (48), the second term of (52) can be bounded by

$$\Pr\left\{\Delta^{[i,j]} \le \epsilon', \forall i \in \mathcal{K}, j \in \mathcal{S}\right\}$$
$$\ge \Pr\left\{\left(d_i^{\max}\right)^2 \delta_1 \cdot \mathsf{SNR} \le \epsilon', \forall i \in \mathcal{K}\right\}$$
$$\cdot \Pr\left\{\sum_{k=2}^{\infty} \left(d_i^{\max}\right)^{2k} \delta_k \cdot \mathsf{SNR} \le \epsilon', \forall i \in \mathcal{K}\right\}. \quad (85)$$

Subsequently, we have

$$\Pr\left\{\left(d_i^{\max}\right)^2 \delta_1 \cdot \mathsf{SNR} \le \epsilon'\right\}$$
$$= \prod_{k=1}^{S} \Pr\left\{\left(d^{[k,i]}\right)^2 \delta_1 \cdot \mathsf{SNR} \le \epsilon'\right\}, \quad (86)$$

which follows from the fact that $d^{[k,i]}$ and $d^{[m,i]}$ are independent for $k \ne m$. From (84) we have

$$\Pr\left\{\left(d^{[k,i]}\right)^2 \delta_1 \cdot \mathsf{SNR} \le \epsilon'\right\}$$
$$= 1 - \left(1 - \epsilon'^{S-1} \delta_1^{-S+1} (\mathsf{SNR})^{-(S-1)}\right)^{N_f}. \quad (87)$$

Therefore, $\lim_{\mathsf{SNR}\to\infty} \Pr\left\{\left(d_i^{\max}\right)^2 \delta_1 \cdot \mathsf{SNR} \le \epsilon'\right\} = 1$ if and only if $N_f = \omega\left(\mathsf{SNR}^{S-1}\right)$, or equivalently (39). Now, if (39) holds true, $d_i^{\max}$ tends to arbitrarily small with high probability as SNR increases. Therefore, the second term of (48) is dominated by the first term, and hence $\Pr\left\{\Delta^{[i,j]} \le \epsilon', \forall i \in \mathcal{K}, j \in \mathcal{S}\right\}$ in (85) tends to 1.

APPENDIX C
PROOF OF LEMMA 3

Let us define the set Π_s by

$$\Pi_s$$
$$\triangleq \left\{\mathbf{h} \in \mathbb{C}^{S \times 1} : \frac{\mathbf{h}^{\mathsf{H}}\mathbf{v}}{\|\mathbf{h}\|\|\mathbf{v}\|} < \alpha, \forall \mathbf{v} \in \mathrm{span}\left(\mathbf{b}_1^{[i]}, \dots, \mathbf{b}_{s-1}^{[i]}\right)\right\}. \quad (88)$$

Since the s-th user pool is determined only by checking the orthogonality to the chosen users' channel vectors, for arbitrarily large N, we have the followings by the law of large numbers:

$$|\mathcal{N}_\mathcal{G}| \approx N \cdot \Pr\left\{\mathbf{h} \in \mathbb{C}^{S \times 1} : \frac{\mathbf{h}^{\mathsf{H}}\mathbf{b}_s^{[i]'}}{\|\mathbf{h}\|\|\mathbf{b}_{s'}^{[i]}\|} < \alpha, s' = 1, \dots, s-1\right\} \quad (89)$$
$$\ge N \cdot \Pr\left\{\mathbf{h} \in \mathbb{C}^{S \times 1} : \mathbf{h} \in \Pi_s\right\} \quad (90)$$
$$= N \cdot I_{\alpha^2}(s-1, S-s+1) \quad (91)$$
$$\ge N \cdot \alpha^{2(S-1)}, \quad (92)$$

where I_{α^2} is the regularized incomplete beta function (See [34, Lemma 3]), and (92) follows from $I_{\alpha^2}(s-1, S-s+1) \ge I_{\alpha^2}(S-1, 1) = \alpha^{2(S-1)}$.

APPENDIX D
PROOF OF (70)

Since $\left\|\mathbf{b}_s^{[i]}\right\|^2$ is a Chi-squared random variable with $2(S-s+1)$ degrees-of-freedom, for $\eta_D > 2$, we have

$$\Pr\{\mathsf{C}_2\} = 1 - \frac{\gamma\left((S-s+1), \eta_D/2\right)}{\Gamma(S-s+1)} \quad (93)$$
$$= \frac{\Gamma\left((S-s+1), \eta_D/2\right)}{\Gamma(S-s+1)} \quad (94)$$
$$= \sum_{m=0}^{S-s} e^{-(\eta_D/2)} \frac{(\eta_D/2)^m}{m!} \quad (95)$$
$$= \frac{e^{-(\eta_D/2)} \cdot (\eta_D/2)^{S-s}}{(S-s)!}\left(1 + O\left((\eta_D/2)^{-1}\right)\right) \quad (96)$$
$$\ge \frac{e^{-(\eta_D/2)}}{(S-s)!}, \quad (97)$$

where $\Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} dt$ is the upper incomplete gamma function and $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ is the lower incomplete gamma function.

Note that from the CDF of $\eta^{[i,j]}$ (See [14, Lemma 1]), $\Pr\left\{\eta^{[i,j]} \le \eta_I\right\} = c_0 \eta_I^\tau + o(\eta_I^\tau)$, where $\tau = (K-1)S-L+1$. Thus, from (63), (64), and (97), (68) can be bounded by

$$\mathsf{p}_s \gtrsim 1 - \left(1 - \left(c_0(\epsilon_I)^\tau \mathsf{SNR}^{-\tau} + \Omega\left(\mathsf{SNR}^{-(\tau-1)}\right)\right)\right.$$
$$\left. \times \frac{N^{-(\epsilon_D/2)}}{(S-s)!}\right)^{N \cdot \alpha^{2(S-1)}}. \quad (98)$$

The right-hand side of (98) converges to 1 for increasing SNR if and only if

$$\lim_{\mathsf{SNR}\to\infty} \left(N \cdot \alpha^{2(S-1)}\right) \cdot \left(c_0(\epsilon_I)^\tau \mathsf{SNR}^{-\tau} + \Omega\left(\mathsf{SNR}^{-(\tau-1)}\right)\right)$$
$$\cdot \frac{N^{-(\epsilon_D/2)}}{(S-s)!} = \infty. \quad (99)$$

Since the left-hand side of (99) can be written by $\tilde{c}_0 \frac{N^{1-(\epsilon_D/2)}}{\mathsf{SNR}^\tau} + \tilde{c}_1 \frac{N^{1-(\epsilon_D/2)}}{o(\mathsf{SNR}^\tau)}$, where $\tilde{c}_0$ and $\tilde{c}_1$ are positive constants independent of SNR and N, it tends to infinity for increasing SNR, and thereby p_s tends to 1 if and only if $N = \omega\left(\mathsf{SNR}^{\frac{(K-1)S-L+1}{1-(\epsilon_D/2)}}\right)$.

REFERENCES

[1] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[2] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3309–3322, Jun. 2011.

[3] T. Gou and S. A. Jafar, "Degrees of freedom of the K user M×N MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 12, pp. 6040–6057, Dec. 2010.

[4] C. Suh, M. Ho, and D. N. C. Tse, "Downlink interference alignment," *IEEE Trans. Commun.*, vol. 59, no. 9, pp. 2616–2626, Sep. 2011.

[5] C. Suh and D. Tse, "Interference alignment for cellular networks," in *Proc. 46th Annu. Allerton Conf. Commun., Control, Comput.*, Champaign, IL, USA, Sep. 2008, pp. 1037–1044.

[6] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. Int. Conf. Commun. (ICC)*, Seattle, WA, USA, Jun. 1995, pp. 331–335.

[7] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, Jun. 2002.

[8] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial side information," *IEEE Trans. Inf. Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.

[9] W.-Y. Shin, S.-Y. Chung, and Y. H. Lee, "Parallel opportunistic routing in wireless networks," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6290–6300, Oct. 2013.

[10] T. W. Ban, W. Choi, B. C. Jung, and D. K. Sung, "Multi-user diversity in a spectrum sharing system," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 102–106, Jan. 2009.

[11] W.-Y. Shin, D. Park, and B. C. Jung, "Can one achieve multiuser diversity in uplink multi-cell networks?" *IEEE Trans. Commun.*, vol. 60, no. 12, pp. 3535–3540, Dec. 2012.

[12] B. C. Jung and W.-Y. Shin, "Opportunistic interference alignment for interference-limited cellular TDD uplink," *IEEE Commun. Lett.*, vol. 15, no. 2, pp. 148–150, Feb. 2011.

[13] B. C. Jung, D. Park, and W. Y. Shin, "Opportunistic interference mitigation achieves optimal degrees-of-freedom in wireless multi-cell uplink networks," *IEEE Trans. Commun.*, vol. 60, no. 7, pp. 1935–1944, Jul. 2012.

[14] H. J. Yang, W.-Y. Shin, B. C. Jung, and A. Paulraj, "Opportunistic interference alignment for MIMO interfering multiple-access channels," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2180–2192, May 2013.

[15] W.-Y. Shin and B. C. Jung, "Network coordinated opportunistic beamforming in downlink cellular networks," *IEICE Trans. Commun.*, vol. E95-B, no. 4, pp. 1393–1396, Apr. 2012.

[16] J. Jose, S. Subramanian, X. Wu, and J. Li, "Opportunistic interference alignment in cellular downlink," in *Proc. 50th Annu. Allerton Conf. Commun., Control, Comput.*, Oct. 2012, pp. 1529–1545.

[17] J. H. Lee and W. Choi, "On the achievable DoF and user scaling law of opportunistic interference alignment in 3-transmitter MIMO interference channels," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2743–2753, Jun. 2013.

[18] H. D. Nguyen, R. Zhang, and H. T. Hui, "Multi-cell random beamforming: Achievable rate and degrees of freedom region," *IEEE Trans. Signal Process.*, vol. 61, no. 14, pp. 3532–3544, Jul. 2013.

[19] H. D. Nguyen, R. Zhang, and H. T. Hui. "Effect of receive spatial diversity on the degrees of freedom region in multi-cell random beamforming." [Online]. Available: http://arxiv.org/abs/1303.5947

[20] J. H. Lee, W. Choi, and B. D. Rao, "Multiuser diversity in interfering broadcast channels: Achievable degrees of freedom and user scaling law," *IEEE Trans. Wireless Commun.*, vol. 12, no. 11, pp. 5837–5849, Nov. 2013.

[21] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 5045–5060, Nov. 2006.

[22] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-antenna downlink channels with limited feedback and user selection," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, Sep. 2007.

[23] J. Thukral and H. Bölcskei, "Interference alignment with limited feedback," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Seoul, South Korea, Jul. 2009, pp. 1759–1763.

[24] R. T. Krishnamachari and M. K. Varanasi, "Interference alignment under limited feedback for MIMO interference channels," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Austin, TX, USA, Jun. 2010, pp. 619–623.

[25] S. Pereira, A. Paulraj, and G. Papanicolaou, "Opportunistic scheduling for multiantenna cellular: Interference limited regime," in *Proc. Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, USA, Nov. 2007, pp. 1171–1175.

[26] S. A. Jafar and S. Shamai, "Degrees of freedom region of the MIMO X channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 151–170, Jan. 2008.

[27] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331–4340, Sep. 2011.

[28] H. J. Yang, B. C. Jung, W.-Y. Shin, and A. Paulraj, "Codebook-based opportunistic interference alignment," *IEEE Trans. Signal Process.*, vol. 62, no. 11, pp. 2922–2937, Jun. 2014.

[29] L.-U. Choi and R. D. Murch, "A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 20–24, Jan. 2004.

[30] D. J. Love, R. W. Heath, and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2735–2747, Oct. 2003.

[31] Z. Peng, W. Xu, J. Zhu, H. Zhang, and C. Zhao, "On performance and feedback strategy of secure multiuser communications with MMSE channel estimate," *IEEE Trans. Wireless Commun.*, vol. 15, no. 2, pp. 1602–1616, Feb. 2016.

[32] C. K. Au-Yeung and D. J. Love, "Optimization and tradeoff analysis of two-way limited feedback beamforming systems," *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2570–2579, May 2009.

[33] B. Khoshnevis, "Multiple-antenna communications with limited channel state information," Ph.D. dissertation, Dept. Elect. Comput. Eng., Univ. Toronto, Toronto, ON, Canada, 2011.

[34] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 528–541, Mar. 2006.

[35] G. A. F. Seber and J. L. Alan, *Linear Regression Analysis*, 2nd ed. Hoboken, NJ, USA: Wiley, 2003.

[36] A. Barg and D. Y. Nogin, "Bounds on packings of spheres in the Grassmann manifold," *IEEE Trans. Inf. Theory*, vol. 48, no. 9, pp. 2450–2454, Sep. 2002.

[37] J. H. Conway, R. H. Hardin, and N. J. A. Sloane, "Packing lines, planes, etc.: Packings in Grassmannian spaces," *Experim. Math.*, vol. 5, no. 2, pp. 139–159, 1996.

[38] W. Dai, Y. Liu, and B. Rider, "Quantization bounds on Grassmann manifolds and applications to MIMO communications," *IEEE Trans. Inf. Theory*, vol. 54, no. 3, pp. 1108–1123, Mar. 2008.

[39] C. K. Au-yeung and D. J. Love, "On the performance of random vector quantization limited feedback beamforming in a MISO system," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 458–462, Feb. 2007.

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