IET Communications

Research Article

ISSN 1751-8628
Received on 26th February 2016
Revised 23rd September 2016
Accepted on 28th November 2016
E-First on 7th March 2017
doi: 10.1049/iet-com.2016.0199
www.ietdl.org

Pricing-based distributed spectrum access for cognitive radio networks with geolocation database

Woongsup Lee[1], *Bang Chul Jung*[2] ✉
[1]*Department of Information and Communication Engineering, Gyeongsang National University, Tongyeong, Republic of Korea*
[2]*Department of Electronics Engineering, Chungnam National University, Daejeon, Republic of Korea*
✉ *E-mail: bcjung@cnu.ac.kr*

Abstract: A pricing-based distributed spectrum access technique for cognitive radio (CR) networks which adopt the geolocation database (GD) is proposed. The GD contains which frequency bands are occupied by the primary system in a particular location. Given that multiple CR systems may attempt to transmit data over the same frequency band when the GD is used, the achievable rate of the CR systems becomes deteriorated due to interference. In the proposed technique, each (secondary) CR system determines whether it utilises vacant frequency bands by considering the cost of using them, which is calculated by taking into account the interference. The authors analyse the behaviour of the CR systems based on game theory. In particular, it is shown that the sum capacity of CR systems is maximised when the number of utilised bands is proportional to the relative channel gain with respect to the average channel gain at each CR system. In addition, the authors obtain the optimal cost for the vacant bands, which achieves the maximum sum capacity of CR systems. Finally, it is shown that the sum capacity of the (secondary) CR systems is significantly improved via proper pricing policy on the vacant frequency bands through extensive computer simulations.

1 Introduction

Cognitive radio (CR) technology has been proposed to efficiently utilise unused spectrum [1]. In CR technology, the wireless system uses a vacant band, that is not in use by a primary user (PU) who has the right to it, for data transmission, provided that its use does not interrupt the PU [2]. One of the most important characteristics of CR technology is that the wireless system finds a vacant band adaptively, so the technology used to detect vacant bands is important [3].

In general, two approaches have been considered for detecting vacant bands: (i) geolocation database (GD) and (ii) spectrum sensing (SS) [3–6]. In GD techniques, a CR system finds vacant bands by using the GD that contains location and the spectrum usage information of primary users (PUs) [4, 6]. In SS techniques, on the other hand, a CR system finds the status of the bands through sensing the spectrum [2, 7]. The spectrum allocation or access technique has not been much investigated in the GD-based CR networks, while the spectrum allocation has been extensively studied for the SS-based CR networks [8, 9]. Proper spectrum management including spectrum access and allocation is also crucial for achieving better performance in the GD-based CR networks. Especially, decentralised spectrum management techniques will be more required for practical applications.

In this paper, we propose a pricing-based *distributed* spectrum access technique for GD-based CR networks. Unlike existing pricing schemes where the amount of payment by a CR system is determined based on its channel and interference conditions [10], the price of the vacant bands is determined according to the number of bands being used by the CR system. A single price is applied for all CR systems and thus the proposed pricing technique is easy to implement in practice. In particular, the behaviour of CR systems is investigated by using a non-cooperative game model. It can be easily shown that the CR systems use as many bands as possible if they are not being charged for using bands. However, excess inter-system interference may deteriorate the overall system capacity. Thus, we also analyse the optimal number of bands that are used by each CR system, which maximises the overall capacity of the CR systems. To the best of our knowledge, the proposed technique is the first distributed pricing-based spectrum access technique in the GD-based CR networks.

2 System model

We consider the multi-band CR environment. The system model is described in the following sections.

2.1 CR system model

We assume that there are M vacant bands whose bandwidth is W. We also assume that $\mathcal{N}$ is the set of CR systems where $|\mathcal{N}| = N$. These CR systems are distributed randomly and uniformly over an area A. The transmission power of each CR system is limited to $P_{\rm T}$. Therefore, when the CR system uses multiple bands simultaneously, we assume that the transmission power is divided equally and allocated to each band, e.g. if the CR system uses m bands, the transmission power allocated to each band becomes $(P_{\rm T}/m)$.

We assume that the channel gain of the lth CR system due to path loss is G_l. We also assume that the fast fading of the ith channel of the lth CR system is $h_{l,i}$, which is modelled as circularly symmetric complex Gaussian (CSCG) [11]. Moreover, we consider additive white Gaussian noise whose power is N_0W. The CR systems concerned are non-cooperative and try to maximise their own utilities only, i.e. they are not interested in maximising the overall capacity of whole CR systems. When multiple CR systems use the same band, they interfere with each other and this inter-system interference will greatly affect the channel capacity of CR systems.

2.2 Operation of A geolocation database

A GD has a complete set of information on vacant bands. It can also enable CR systems to use vacant bands by providing information on vacant bands. To use the vacant bands, each CR system needs to send a request to the GD. In the request, individual CR systems have to indicate the number of bands that they want to use. We assume that the number of bands requested by the lth CR system is k_l. After receiving those requests, the database assigns a set of bands, $\mathscr{K}_l$, whose cardinality is k_l, i.e. $|\mathscr{K}_l| = k_l$, to the lth CR system. $\mathscr{K}_l$ is randomly selected by the database from the whole set of vacant bands (The selection of bands in $\mathscr{K}_l$ can be optimised for further improvement of the overall capacity. However, to do so, the complete channel information of all CR systems including interference channels has to be known to the GD and a complex integer programming has to be solved. Therefore, in practice, it will be hard for the database to allocate specific set of bands optimally to the CR system]. If $k_l > M$, all the available bands are assigned to the CR system. Note that bands can be shared by multiple CR systems such that it is possible to have $\mathscr{K}_{l_1} \cap \mathscr{K}_{l_2} \neq \varnothing$ for $l_1 \neq l_2$.

CR systems have to pay for using vacant bands. We consider a simple linear pricing scheme in which the amount of payment by a CR system is proportional to the number of bands utilised by the CR system. Let the unit price of one band be c. Then, the amount of payment by the lth CR system for using k_l bands becomes ck_l. Given that the CR system has to pay for utilising bands, it carefully chooses how many bands to use, which is k_l, based on the expected capacity that can be achieved in the bands. Therefore, the CR system needs to estimate the capacity that can be achieved by using bands and to do so, the amount of interference in the bands has to be known to the CR system. To facilitate the calculation of the capacity, we assume that the database broadcasts the average value of inverse of interference and noise to CR systems (Herein, the average value of inverse of interference and noise is used, because using the average value of interference and noise might result in wrong estimation of capacity due to the cases in which interference and noise are significant. For example, we can consider the case in which bandwidth and signal power of a CR system are 1, and the sum of noise and interference of the CR system is 1 with probability 0.99 and it is 10,000 with probability 0.01. In this case, the expected capacity is 0.99 bits/s. Moreover, the average value of interference and noise is 100.99 while the average value of inverse of interference and noise is 0.99. If the average value of interference and noise is used, the estimated capacity becomes 0.01 bits/s while if the average value of inverse of interference and noise is used, the estimated capacity becomes 0.99 bits/s, which is identical with the actual expected capacity. Therefore, the use of inverse of interference and noise can be justified from this example.), which we denote as I_{E}^{-1}, and the CR system calculates the expected capacity by using this information.

3 Game formulation and channel capacity of CR systems

In this section, we model the behaviour of CR systems in determining the number of bands to be used, by using a game theory. We also derive the overall channel capacity of CR systems and find the number of bands that each CR system has to use for the maximisation of the overall capacity of whole CR systems.

3.1 Game formulation

We formulate the behaviour of CR systems as a non-cooperative game because each CR system is rational and only concerns its utility. The game model is formulated as follows:

- *Players*: CR systems.
- *Strategy*: The strategy of lth CR system is adjusting k_l, which is the number of bands to be used. In the analysis, we assume that k_l is a non-negative real number, i.e. $k_l \in \mathbb{R}^+ \cup \{0\}$; however, in practice, k_l has to be a non-negative integer value, i.e. $k_l \in \mathbb{Z}^+ \cup \{0\}$. To solve this problem, an integer value which is the closest to k_l can be used. Moreover, we use $\boldsymbol{k}_{-1}$ to denote the vector of number of bands used by all CR systems except the lth CR system.
- *Payoff*: The payoff of lth CR system, which we denote as V_l, is the expected capacity minus cost (Our payoff formulation is similar to that used in [12] in which the cost is the function of transmit power.) to use k_l bands, where the cost is ck_l. The expected capacity of lth CR system is written as

$$\mathbb{E}\left[\sum_{i \in \mathscr{K}_l} W\log_2\left(1 + \frac{G_l|h_{l,i}|^2 P_{\mathrm{T}}}{(N_0 W + I_{l,i}(\boldsymbol{k}_{-1}))k_l}\right)\right], \tag{1}$$

  where $I_{l,i}(\boldsymbol{k}_{-1})$ is the amount of interference caused by the other CR systems in band i. As we have stated in our system model, each CR system will use I_{E}^{-1} which is informed by a GD, instead of $(1/N_0 W + I_{l,i}(\boldsymbol{k}_{-1}))$ in calculating expected capacity. Moreover, $h_{l,i}$ is CSCG and independent and identically distributed. Then, V_l is shown as (see (2)) where $E_1(x)$ is the exponential integral, which is expressed as $E_1(x) = \int_x^\infty (\mathrm{e}^{-u}/u)\mathrm{d}u$ [6].

Then, the Nash equilibrium of the considered game, which we denote as $\hat{k}_l$, is found from the derivative of (2) and it can be found by solving

$$W\left[\mathrm{e}^{(\hat{k}_l/G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1})} E_1\left(\frac{\hat{k}_l}{G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1}}\right)\left(1 + \frac{\hat{k}_l}{G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1}}\right) - 1\right] = c \log 2. \tag{3}$$

By letting $c = 0$, we can examine the behaviour of CR systems when bands are used for free. Let U_l be the payoff lth CR system when $c = 0$, which is represented as

$$U_l = \frac{Wk_l}{\log 2}\mathrm{e}^{(k_l/G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1})} E_1\left(\frac{k_l}{G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1}}\right). \tag{4}$$

From the following proposition, it can be shown that U_l is an increasing function of k_l.

Proposition 1: U_l is an increasing function of k_l for non-negative k_l, i.e. $(\partial U_l/\partial k_l) > 0$.

Proof: $(\partial U_l/\partial k_l)$ is calculated as (see (5)) We can find that $\mathrm{e}^z E_1(z)(1 + z) > 1$ for all $z \geq 0$ and accordingly $(\partial U_l/\partial k_l) > 0$ for all $k_l \geq 0$. Therefore, U_l is an increasing function for all non-negative k_l □.

Remark 1: Proposition 1 shows that each CR system uses as many bands as possible when it is not being charged for utilising bands, i.e. $c = 0$. Therefore, we can expect that without pricing of band,

$$\begin{aligned} V_l &= Wk_l\left(\int_0^\infty \log_2\left(1 + \frac{G_l x P_{\mathrm{T}} I_{\mathrm{E}}^{-1}}{k_l}\right)\mathrm{e}^{-x}\mathrm{d}x\right) - ck_l \\ &= \frac{Wk_l}{\log 2}\left(-\left[\mathrm{e}^{-x}\log\left(1 + \frac{G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1}}{k_l}x\right)\right]_0^\infty + \int_0^\infty \mathrm{e}^{-x}\frac{(G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1}/k_l)}{1 + (G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1}/k_l)x}\mathrm{d}x\right) - ck_l \\ &= k_l\left(\frac{W}{\log 2}\mathrm{e}^{(k_l/G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1})} E_1\left(\frac{k_l}{G_l P_{\mathrm{T}} I_{\mathrm{E}}^{-1}}\right) - c\right), \end{aligned} \tag{2}$$

inter-system interference can become severe and the capacity of CR systems can seriously be deteriorated due to this excess inter-system interference.

3.2 Overall channel capacity of CR systems

Now, we derive the overall channel capacity of CR systems and find the optimal number of bands to be allocated to each CR system for the maximisation of the overall capacity. Here, we assume that the amount of interference in each band is a function of the average number of bands used by individual CR systems (It is also possible to take into account more accurate interference formulation, e.g. $I_{l,i}(\boldsymbol{k}_{-l})$. However, in this case, the problem becomes too complicated to analyse and it will be hard to get any meaningful insights from it.). Let $f_{\mathrm{I}^{-1}}(\boldsymbol{k})$ be the inverse of interference and noise in each band where $\boldsymbol{k}$ is the average value of k_l, i.e. $\boldsymbol{k} = (1/N)\sum_{l \in \mathcal{N}} k_l$. Then, by using the derivation in (2), it is easy to show that the expected capacity of whole CR systems becomes

$$\sum_{l \in \mathcal{N}} \frac{Wk_l}{\log 2}\left(\mathrm{e}^{(k_l/G_lP_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\boldsymbol{k}))}E_1\left(\frac{k_l}{G_lP_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\boldsymbol{k})}\right)\right). \tag{6}$$

Let $\hat{k}_l^{SO}$ be the optimal number of bands of lth CR system to maximise the overall capacity. From the derivative of (6), it can be shown that $\hat{k}_l^{SO}$ has to satisfy the following equality: (see (7)) where $\hat{\boldsymbol{k}}^{SO} = (1/N)\sum_{j \in \mathcal{N}} \hat{k}_j^{SO}$. It is worth pointing out that $(\partial f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})/\partial k_j)$ is the same for all $j \in \mathcal{N}$.

However, it is challenging to find $\hat{k}_l^{SO}$ from (7), because (7) has to be satisfied for all $l \in \mathcal{N}$ and consequently, N equations have to be solved jointly. To simplify the calculation of $\hat{k}_l^{SO}$, let $\hat{k}_l^{SO}$ be proportional to the relative channel gain of lth CR system such that

$$\hat{k}_l^{SO} = \hat{\boldsymbol{k}}^{SO}\frac{G_l}{\boldsymbol{G}_{\mathrm{avg}}}, \tag{8}$$

where $\boldsymbol{G}_{\mathrm{avg}} = (1/N)\sum_{j \in \mathcal{N}} G_j$, i.e. the average channel gain of CR systems. Then, (7) can be rewritten as (see (9)) By using (9), $\hat{\boldsymbol{k}}^{SO}$ can be found by solving single equation instead of solving N equations jointly. Therefore, the calculation of $\hat{k}_l^{SO}$ becomes much easier.

Remark 2: It should be noted that the overall capacity of CR systems is maximised when the number of bands allocated to each CR system is proportional to the relative channel gain of the CR system.

We note that $f_{\mathrm{I}^{-1}}(\boldsymbol{k})$ plays a significant role in determining $\hat{\boldsymbol{k}}^{SO}$. However, it is challenging to derive the exact form of $f_{\mathrm{I}^{-1}}(\boldsymbol{k})$. Therefore, we use an approximation to derive $f_{\mathrm{I}^{-1}}(\boldsymbol{k})$ such that only the effect of a dominant interference source, i.e. the nearest CR system that uses the same band is taken into account in the formulation of $f_{\mathrm{I}^{-1}}(\boldsymbol{k})$. Moreover, we use the simplified path loss model in which the channel gain between two wireless entities separated by d is $\beta d^{-\alpha}$, where β is the path loss constant and α is the path loss exponent [11, 13]. Then, $f_{\mathrm{I}^{-1}}(\boldsymbol{k})$ can be found through the following proposition.

Proposition 2: $f_{\mathrm{I}^{-1}}(\boldsymbol{k})$ can be approximated as

$$f_{\mathrm{I}^{-1}}(\boldsymbol{k}) = \eta(\alpha)\,\boldsymbol{k}^{-((\alpha/2)-1)}, \tag{10}$$

where $\eta(\alpha) = \beta^{-1}P_{\mathrm{T}}^{-1}((\pi N/MA))^{-(\alpha/2)}\Gamma(((2+\alpha)/2))$. Moreover, $\Gamma(\cdot)$ is a Gamma function, i.e. $\Gamma(z) = \int_0^{\infty} t^{z-1}\mathrm{e}^{-t}\mathrm{d}t$ [11].

Proof: Let the distance between one CR system and its nearest CR system that uses the same band be $d^{\min}$. Since the average number of bands used by each CR system is $\boldsymbol{k}$, the density of CR systems which use the same band, which we denote as λ_{CR}, becomes $\lambda_{\mathrm{CR}} = (N\boldsymbol{k}/MA)$. In our system model, CR systems are distributed randomly and uniformly. Therefore, the distribution of CR systems can be approximated as a homogeneous Poisson point process [14, 15]. Using this approximation, the probability distribution function (PDF) of $d^{\min}$, $f_{d^{\min}}(r)$, is described as [14, 15]

$$f_{d^{\min}}(r) = 2\lambda_{\mathrm{CR}}\pi r\mathrm{e}^{-\lambda_{\mathrm{CR}}\pi r^2}, \quad r \in \mathbb{R}^{+}. \tag{11}$$

$$\begin{aligned}\frac{\partial U_l}{\partial k_l} &= \frac{W}{\log 2}\left(\mathrm{e}^{(k_l/G_lP_{\mathrm{T}}I_{\mathrm{E}}^{-1})}E_1\left(\frac{k_l}{G_lP_{\mathrm{T}}I_{\mathrm{E}}^{-1}}\right)\right) + \frac{Wk_l}{\log 2}\frac{1}{G_lP_{\mathrm{T}}I_{\mathrm{E}}^{-1}}\left(\mathrm{e}^{(k_l/G_lP_{\mathrm{T}}I_{\mathrm{E}}^{-1})}E_1\left(\frac{k_l}{G_lP_{\mathrm{T}}I_{\mathrm{E}}^{-1}}\right)\right) - \frac{W}{\log 2}\\ &= \frac{W}{\log 2}\left(\mathrm{e}^{(k_l/G_lP_{\mathrm{T}}I_{\mathrm{E}}^{-1})}E_1\left(\frac{k_l}{G_lP_{\mathrm{T}}I_{\mathrm{E}}^{-1}}\right)\left(1 + \frac{k_l}{G_lP_{\mathrm{T}}I_{\mathrm{E}}^{-1}}\right) - 1\right).\end{aligned} \tag{5}$$

$$\begin{aligned}&\mathrm{e}^{(\hat{k}_l^{SO}/G_lP_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO}))}E_1\left(\frac{\hat{k}_l^{SO}}{G_lP_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}\right)\left(1 + \frac{\hat{k}_l^{SO}}{G_lP_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}\right) - 1\\ &= \sum_{j=1}^{N}\frac{(\hat{k}_j^{SO})^2}{G_jP_{\mathrm{T}}(f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO}))^2}\left(\mathrm{e}^{(\hat{k}_j^{SO}/G_jP_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO}))}E_1\left(\frac{\hat{k}_j^{SO}}{G_jP_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}\right) - \frac{G_jP_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}{\hat{k}_j^{SO}}\right)\frac{\partial f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}{\partial k_j},\end{aligned} \tag{7}$$

$$\begin{aligned}&\mathrm{e}^{(\hat{\boldsymbol{k}}^{SO}/\boldsymbol{G}_{\mathrm{avg}}P_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO}))}E_1\left(\frac{\hat{\boldsymbol{k}}^{SO}}{\boldsymbol{G}_{\mathrm{avg}}P_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}\right)\left(1 + \frac{\hat{\boldsymbol{k}}^{SO}}{\boldsymbol{G}_{\mathrm{avg}}P_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}\right) - 1\\ &= \left(\mathrm{e}^{(\hat{\boldsymbol{k}}^{SO}/\boldsymbol{G}_{\mathrm{avg}}P_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO}))}E_1\left(\frac{\hat{\boldsymbol{k}}^{SO}}{\boldsymbol{G}_{\mathrm{avg}}P_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}\right) - \frac{\boldsymbol{G}_{\mathrm{avg}}P_{\mathrm{T}}f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}{\hat{\boldsymbol{k}}^{SO}}\right)\frac{(\hat{\boldsymbol{k}}^{SO})^2}{\boldsymbol{G}_{\mathrm{avg}}P_{\mathrm{T}}(f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO}))^2}\frac{\partial f_{\mathrm{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}{\partial\hat{\boldsymbol{k}}^{SO}}.\end{aligned} \tag{9}$$

```
Algorithm 1 Procedure of Proposed Pricing Scheme
Initialize $c$ and $I_E^{-1}$ to their initial values.
while $\hat{k}_l^{SO} \neq \hat{k}_l$ for any $l \in \mathcal{N}$ do
    A geolocation database broadcasts $c$ and $I_E^{-1}$ to CR systems.
    $l$-th CR system measures $G_l$ and sends this information to the database along with a request of $\hat{k}_l$ bands where $\hat{k}_l$ is calculated from (3).
    The geolocation database allocates the set of bands to each CR system according to its request and also updates $\mathbf{G}_{avg}$.
    CR systems transmit data by using assigned bands and report actual interference measured in the bands to the geolocation database.
    The geolocation database updates $I_E^{-1}$ and $\eta(\alpha)$ by using the interference information and calculates $\hat{k}_l^{SO}$ from (8) and (14).
    The geolocation database updates $c$.
end while
```

Fig. 1 *Procedure of proposed pricing scheme*

Let G_{INT}^{-1} be the expected value of inverse channel gain between the CR system and its nearest interfering CR system. From (11) and the simplified path loss model, G_{INT}^{-1} is derived as

$$\begin{aligned} G_{\text{INT}}^{-1} &= \int_0^\infty \beta^{-1} r^\alpha 2\lambda_{\text{CR}}\pi r \mathrm{e}^{-\lambda_{\text{CR}}\pi r^2} \mathrm{d}r \\ &= \beta^{-1} 2\lambda_{\text{CR}}\pi \int_0^\infty r^{\alpha+1} \mathrm{e}^{-\lambda_{\text{CR}}\pi r^2} \mathrm{d}r \\ &= \beta^{-1}(\lambda_{\text{CR}}\pi)^{-(\alpha/2)}\Gamma\left(\frac{2+\alpha}{2}\right) = \kappa \boldsymbol{k}^{-(\alpha/2)}, \end{aligned} \tag{12}$$

where $\kappa = \beta^{-1}((\pi N/MA))^{-(\alpha/2)}\Gamma(((2+\alpha)/2))$ and $\Gamma(\cdot)$ is a Gamma function [11].

To finalise the derivation of $f_{\text{I}^{-1}}(\boldsymbol{k})$, the transmission power of the interfering CR system has to be taken into account. Given that the average number of bands used by individual CR systems is $\boldsymbol{k}$, the average transmission power on each band becomes $(P_{\text{T}}/\boldsymbol{k})$. Therefore, $f_{\text{I}^{-1}}(\boldsymbol{k})$ is written as

$$\begin{aligned} f_{\text{I}^{-1}}(\boldsymbol{k}) &= \frac{G_{\text{INT}}^{-1}}{(P_{\text{T}}/\boldsymbol{k})} \\ &= \eta(\alpha)\boldsymbol{k}^{-((\alpha/2)-1)}, \end{aligned} \tag{13}$$

where $\eta(\alpha) = \beta^{-1}P_{\text{T}}^{-1}((\pi N/MA))^{-(\alpha/2)}\Gamma(((2+\alpha)/2))$ □.

Remark 3: From (10), we observe that as $\boldsymbol{k}$ increases, $f_{\text{I}^{-1}}(\boldsymbol{k})$ decreases, in other words, the amount of interference in each band increases as the number of bands used by individual CR systems increases. It justifies the necessity for controlling the number of bands used by individual CR systems. Moreover, in practice, the path loss exponent, which is α, is in the range $2 \le \alpha \le 4$ [13] and consequently, $(1/f_{\text{I}^{-1}}(\boldsymbol{k}))$ which corresponds to the amount of interference in each band, is in $\mathrm{o}(\boldsymbol{k})$, i.e. $(1/f_{\text{I}^{-1}}(\boldsymbol{k})) \in \mathrm{o}(\boldsymbol{k})$ where $\mathrm{o}(\cdot)$ is the little-o notation ($h(\boldsymbol{k}) \in \mathrm{o}(g(\boldsymbol{k}))$ is equivalent to $\lim_{\boldsymbol{k} \to \infty}(h(\boldsymbol{k})/g(\boldsymbol{k})) = 0$ [16]). Therefore, the rate of growth of interference by increasing $\boldsymbol{k}$ is slower than linear. Moreover, we note that as α increases, the rate of growth also increases, i.e. the effect of $\boldsymbol{k}$ on interference increases. For example, when $\alpha = 2$, $(1/f_{\text{I}^{-1}}(\boldsymbol{k})) = (1/\eta(2))$ and the interference is constant with respect to $\boldsymbol{k}$; however, when $\alpha = 4$, $(1/f_{\text{I}^{-1}}(\boldsymbol{k})) = (\boldsymbol{k}/\eta(4))$ and the interference increases linearly with $\boldsymbol{k}$.

Based on (10), we can summarise (9) as

$$\frac{\mathrm{e}^y E_1(y)}{y\mathrm{e}^y E_1(y) - 1} = -\frac{\alpha}{2}, \tag{14}$$

where $y = ((\hat{\boldsymbol{k}}^{SO})^{(\alpha/2)}/\boldsymbol{G}_{\text{avg}}P_{\text{T}}\eta(\alpha))$. By using (14), we can easily calculate $\hat{\boldsymbol{k}}^{SO}$ and accordingly $\hat{k}_l^{SO}$ can be readily derived from (8).

4 Optimal price of vacant bands

Now, we are ready to derive the optimal price of band, which we denote as $\hat{c}$, that achieves the maximum overall capacity of CR systems. Let $I_{\text{E}}^{-1} = f_{I^{-1}}(\hat{\boldsymbol{k}}^{SO})$, in other words, the information of interference notified by a GD coincides with $f_{\text{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})$. Then, we can set $\hat{c}$ as follows:

$$\hat{c} = \frac{W}{\log 2}\sum_{j \in \mathcal{N}} \frac{(\hat{k}_j^{SO})^2}{G_j P_{\text{T}}(I_{\text{E}}^{-1})^2}\left(\mathrm{e}^{(\hat{k}_j^{SO}/G_j P_{\text{T}} I_{\text{E}}^{-1})} E_1\left(\frac{\hat{k}_j^{SO}}{G_j P_{\text{T}} I_{\text{E}}^{-1}}\right) - \frac{G_j P_{\text{T}} I_{\text{E}}^{-1}}{\hat{k}_j^{SO}}\right)\frac{\partial f_{\text{I}^{-1}}(\hat{\boldsymbol{k}}^{SO})}{\partial k_j}, \tag{15}$$

We can check that the maximum overall capacity is achieved by using $\hat{c}$ in (15), because the Nash equilibrium in (3) becomes the same with (7) which corresponds to the optimal point for maximising the overall capacity of CR systems.

In the following, we describe the operation of the proposed pricing scheme. Initially, a GD sets price of band, c, and inverse of interference and noise, I_{E}^{-1}, to their initial values and announces these values to CR systems. Then, lth CR system measures its channel gain, which is G_l, and $\hat{k}_l$ is calculated from (3) by using c and I_{E}^{-1} which were notified by the database. After calculating $\hat{k}_l$, the CR system sends a request for $\hat{k}_l$ bands to the database along with the information of G_l. The GD assigns bands to individual CR systems according to their requests and it also updates $\boldsymbol{G}_{\text{avg}}$ by using the information of G_l. Then, the CR systems transmit data through assigned bands and report actual interference experienced in the bands to the database. Based on the collected information of interference from CR systems, the database updates I_{E}^{-1} and also updates the value of $\eta(\alpha)$ by using $\eta(\alpha) = f_{\text{I}^{-1}}(\hat{\boldsymbol{k}})\hat{\boldsymbol{k}}^{((\alpha/2)-1)}$, where $\hat{\boldsymbol{k}}$ is the average number of bands used by individual CR systems, to adjust channel parameter. Then, the database calculates $\hat{k}_l^{SO}$ from (8) and (14). Finally, the database updates c from (3) based on new $\hat{k}_l^{SO}$ and broadcasts updated c and I_{E}^{-1} to CR systems. The update iterates until $\hat{k}_l^{SO}$ and $\hat{k}_l$ are the same for all $l \in \mathcal{N}$. The details of proposed algorithm are in Fig. 1.

5 Performance evaluation

We evaluate the performance of the proposed pricing scheme in various environments. In performance evaluation, we randomly

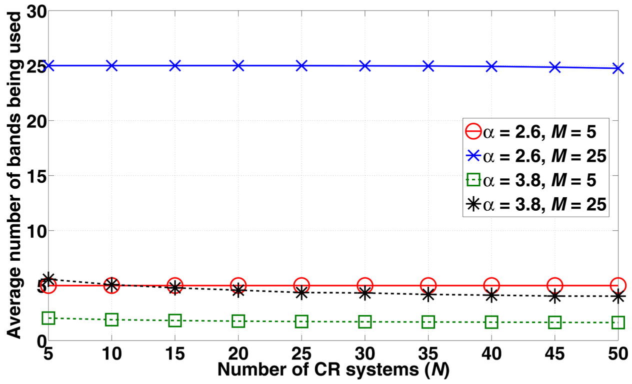

Fig. 2 *Average number of bands used by each CR system vs. number of CR systems*

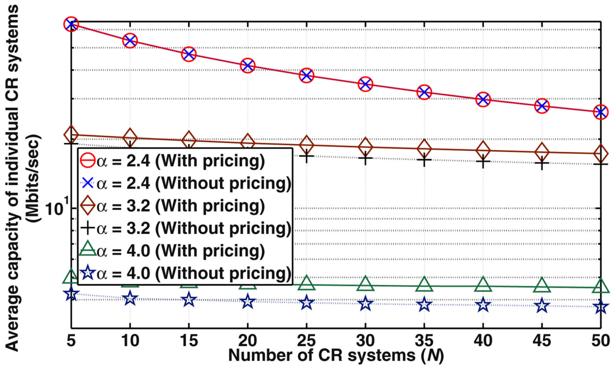

Fig. 3 *Average capacity of individual CR systems vs. number of CR systems for different α when M = 15*

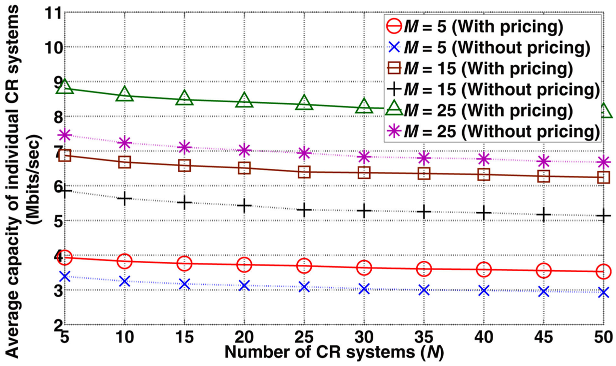

Fig. 4 *Average capacity of individual CR systems vs. number of CR systems for different M when α = 3.8*

deploy CR systems which are comprised of a base station and a mobile station, in an area of 1000 m × 1000 m, and let radius of coverage of each CR system be 100 m. We also assume the path loss constant β to be $10^{3.453}$ [17]. We further assume $P_{\mathrm{T}} = 1\,\mathrm{mW}$, $N_0 = -174\,\mathrm{dBm/Hz}$ [17] and that the size of each band is 6 MHz [2].

In Fig. 2, we show the average number of bands used by individual CR systems, $\hat{k}$, in the proposed pricing scheme. We assume that α can be either 2.6 or 3.8, the number of vacant bands, M, can be either 5 or 25, and the number of CR systems, N, varies from 5 to 50. It is worth pointing out that without pricing of bands, CR systems can use bands for free and $\hat{k}$ becomes the same with M, as we have shown in Proposition 1. We observe from Fig. 2 that $\hat{k}$ decreases as N increases, because the bands become more crowded by CR systems, i.e. the amount of inter-system interference in bands increases. To alleviate the inter-system interference, the number of bands used by individual CR systems has to be reduced. For the same reason, $\hat{k}$ decreases as M decreases.

It should be noted that when $\alpha = 2.6$, $\hat{k}$ is almost the same with M, which implies that each CR system uses the maximum possible number of bands. This is due to the fact that the effect of increasing $\hat{k}$ diminishes when α is close to 2, as we have noted in Remark 3. To be more specific, when α is close to 2, the amount of interference in bands, i.e. $(1/f_{\Gamma^{-1}}(\hat{k}))$, increases very slowly with increasing $\hat{k}$. Therefore, the interference in bands is nearly constant and using the maximum possible number of bands for each CR system maximises the overall system capacity (This can be easily shown by using the proof of Proposition 1). Moreover, we can also conjecture that the improvement of the capacity by using the proposed pricing scheme will be limited when α is close to 2, because the amount of interference in bands is nearly constant regardless of varying the number of bands used by CR systems.

In Figs. 3 and 4, we show the average capacity of individual CR systems, which is obtained from the overall capacity of CR systems by dividing N. We assume that $M = 15$ in Fig. 3 and $\alpha = 3.8$ in Fig. 4 [17]. In the simulations, we assume that a GD (i) charges and (ii) does not charge CR systems for using bands. Note that log-scale is used for y-axis in Fig. 3 while linear scale is used for y-axis in Fig. 4. We note from the simulation results that the average capacity of each CR system decreases as N increases because the amount of inter-system interference increases. We can also see that the capacity of the CR system can be improved when the proposed pricing scheme is used, e.g. in Fig. 3, the improvement of capacity is 12% and 22% when $\alpha = 3.2$ and $\alpha = 4.0$, respectively. It is due to the fact that the price of band is determined such that the number of bands used by individual CR systems is adjusted to achieve the maximum capacity. When no charging strategy is used, a CR system uses as many bands as possible regardless of the network condition, e.g. the amount of inter-system interference, and accordingly the system capacity deteriorates severely.

Moreover, we find that the improvement of capacity afforded by the proposed pricing scheme is large when the amount of inter-system interference is large, i.e. M is small or N is large, because the management of band usage of each CR system becomes more effective. Furthermore, it is worth pointing out that the capacity improvement due to the proposed pricing scheme becomes negligible when α is close to 2, because the amount of interference does not change significantly by adjusting the number of bands used by each CR system, as we have conjectured in the simulation result of Fig. 2. Therefore, our proposed scheme is more appropriate for the environment in which α is high, e.g. indoor environments.

6 Conclusions

In this paper, we proposed the pricing-based distributed spectrum access technique for CR networks with GD, in which the cost of vacant bands is determined to be proportional to the number of bands used by the CR system. As a main result, we analysed the optimal price for the vacant bands to maximise the overall system capacity by modelling the behaviour of CR systems as a non-cooperative game. It was observed that all CR systems certainly try to use vacant bands as many as possible if the price approaches to 0. Extensive simulation results showed that the capacity of CR systems is significantly improved with the proposed pricing-based spectrum management technique.

7 Acknowledgment

This research was supported in part by the National GNSS Research Center programme of Defense Acquisition Program Administration and Agency for Defense Developmentin and by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2015R1D1A1A01057529).

8 References

[1] Mitola, J., Maguire, G.Q.: 'Cognitive radio: making software radios more personal', *IEEE Pers. Commun.*, 1999, **6**, (4), pp. 13–18

[2] Lee, W., Cho, D.-H.: ‘Enhanced spectrum sensing scheme in cognitive radio systems with MIMO antennae’, *IEEE Trans. Veh. Technol.*, 2011, **60**, (3), pp. 1072–1085

[3] Nekovee, M., Irnich, T., Karlsson, J.: ‘Worldwide trends in regulation of secondary access to white spaces using cognitive radio’, *IEEE Wirel. Commun.*, 2012, **19**, (4), pp. 32–40

[4] Wang, J., Ghosh, M., Challapali, K.: ‘Emerging cognitive radio applications: a survey’, *IEEE Commun. Mag.*, 2011, **49**, (3), pp. 74–81

[5] Hwang, Y., Kim, S.-l., Sung, K.W., *et al.*: ‘Scenario making for assessment of secondary spectrum access’, *IEEE Wirel. Commun.*, 2012, **19**, (4), pp. 25–31

[6] Lee, W., Cho, D.-H.: ‘Comparison of channel state acquisition schemes in cognitive radio systems’, *IEEE Trans. Wirel. Commun.*, 2014, **13**, (4), pp. 2295–2307

[7] Peh, E., Liang, Y.-C., Guan, Y.L., *et al.*: ‘Optimization of cooperative sensing in cognitive radio networks: a sensing-throughput tradeoff view’, *IEEE Trans. Veh. Technol.*, 2009, **58**, (9), pp. 5294–5299

[8] Qu, Q., Milstein, L., Vaman, D.: ‘Cognitive radio based multi-user resource allocation in mobile *Ad Hoc* networks using multi-carrier CDMA modulation’, *IEEE J. Sel. Areas Commun.* 2008, **26**, (1), pp. 70–82

[9] Stotas, S., Nallanathan, A.: ‘Optimal sensing time and power allocation in multiband cognitive radio networks’, *IEEE Trans. Commun.*, 2011, **59**, (1), pp. 226–235

[10] Wang, F., Krunz, M., Cui, S.: ‘Price-based spectrum management in cognitive radio networks’, *IEEE J. Sel. Areas Commun.*, 2008, **2**, (1), pp. 74–87

[11] Lee, W., Cho, D.-H.: ‘Simultaneous RTS and sequential CTS considering multiple cooperative relays’, *IEEE Trans. Veh. Technol.*, 2013, **62**, (5), pp. 2369–2374

[12] Fu, S., Wu, B., Wen, H., *et al.*: ‘Transmission scheduling and game theoretical power allocation for interference coordination in CoMP’, *IEEE Trans. Wirel. Commun.*, 2014, **13**, (1), pp. 112–123

[13] Goldsmith, A.: ‘*Wireless communications*’ (Cambridge University Press, 2005)

[14] Srinivasa, S., Haenggi, M.: ‘Distance distributions in finite uniformly random networks: theory and applications’, *IEEE Trans. Veh. Technol.*, 2010, **59**, (2), pp. 940–949

[15] Lee, W., Cho, D.-H.: ‘Channel selection and spectrum availability check scheme for cognitive radio systems considering user mobility’, *IEEE Commun. Lett.*, 2013, **17**, (3), pp. 463–466

[16] Kaliski, B.: ‘*O-notation*’ (Springer Encyclopedia of Cryptography and Security, 2005), pp. 447–448

[17] IEEE 802.20: ‘IEEE 802.20 channel models document’, 2007 [Online]. Available: http://www.ieee802.org/20/Documents.htm