

# Opportunistic Network Decoupling with Virtual Full-Duplex Operation in Multi-Source Interfering Relay Networks

Won-Yong Shin, *Senior Member, IEEE*, Vien V. Mai, Bang Chul Jung, *Senior Member, IEEE*, and Hyun Jong Yang, *Member, IEEE*

**Abstract**—We introduce a new achievability scheme, termed *opportunistic network decoupling (OND)*, operating in virtual full-duplex mode. In the scheme, a novel relay scheduling strategy is utilized in the  $K \times N \times K$  channel with interfering relays, consisting of  $K$  source-destination pairs and  $N$  half-duplex relays in-between them. A subset of relays using alternate relaying is opportunistically selected in terms of producing the minimum total interference level, thereby resulting in network decoupling. As our main result, it is shown that under a certain relay scaling condition, the OND protocol achieves  $K$  degrees of freedom even in the presence of interfering links among relays. Numerical evaluation is also shown to validate the performance of the proposed OND. Our protocol basically operates in a fully distributed fashion along with local channel state information, thereby resulting in relatively easy implementation.

**Index Terms**—Degrees of freedom (DoF), half-duplex, interference,  $K \times N \times K$  channel, opportunistic network decoupling (OND), relay, virtual full-duplex (FD)

## 1 INTRODUCTION

### 1.1 Previous Work

INTERFERENCE between wireless links has been taken into account as a critical problem in wireless communication systems. Recently, interference alignment (IA) was proposed for fundamentally solving the interference problem when there are two communication pairs [1]. It was shown in [2] that the IA scheme can achieve the optimal degrees of freedom (DoF), which is equal to  $K/2$ , in the  $K$ -user interference channel with time-varying channel coefficients. Since then, interference management schemes based on IA have been further developed and analyzed in various wireless network environments: multiple-input multiple-output (MIMO) interference networks [3], [4], X networks [5], and cellular networks [6], [7], [8], [9].

On the one hand, following up on these successes for single-hop networks, more recent and emerging work has studied multihop networks with multiple source-destination (S-D) pairs. For the two-user two-hop network with two

relays (referred to as the  $2 \times 2 \times 2$  interference channel), it was shown in [10] that interference neutralization combining with symbol extension achieves the optimal DoF. A more challenging network model is to consider  $K$ -user two-hop relay-aided interference channels, consisting of  $K$  source-destination (S-D) pairs and  $N$  helping relay nodes located in the path between S-D pairs, so-called the  $K \times N \times K$  channel. Several achievability schemes have been known for the network, but more detailed understanding is still in progress. By applying the result from [11] to the  $K \times N \times K$  channel, one can show that  $K/2$  DoF is achieved by using orthogonalize-and-forward relaying, which completely neutralizes interference at all destinations if  $N$  is greater than or equal to  $K(K-1)+1$ . Another achievable scheme, called aligned network diagonalization, was introduced in [12] and was shown to achieve the optimal DoF in the  $K \times N \times K$  channel while tightening the required number of relays. The scheme in [12] is based on the real interference alignment framework [7]. In [10], [12], however, the system model under consideration assumes that there is no interfering signal between relays and the relays are full-duplex. Moreover, in [13], the  $2 \times 2 \times 2$  interference channel with full-duplex relays interfering with each other was characterized and its DoF achievability was shown using aligned interference neutralization.<sup>1</sup>

On the other hand, there are lots of results on the usefulness of fading in the literature, where one can obtain the multiuser diversity gain in broadcast channels: opportunistic scheduling [15], opportunistic beamforming [16], and random beamforming [17]. Such opportunism can also be fully utilized in multi-cell uplink or downlink networks by using opportunistic interference alignment strategies [9],

1. The idea in [13] was later extended to the two-user three-hop network with four relays, i.e., the  $2 \times 2 \times 2 \times 2$  interference channel [14].

- W.-Y. Shin is with the Department of Computer Science and Engineering, Dankook University, Yongin 448-701, Republic of Korea. E-mail: wyshin@dankook.ac.kr.
- V.V. Mai is with the Department of Automatic Control, KTH Royal Institute of Technology, Stockholm SE-100 44, Sweden. E-mail: mai@kth.se.
- B.C. Jung is with the Department of Electronics Engineering, Chungnam National University, Daejeon 305-764, Republic of Korea. E-mail: bcjung@cnu.ac.kr.
- H.J. Yang is with the School of Electrical and Computer Engineering, UNIST, Ulsan 689-798, Republic of Korea. E-mail: hjyang@unist.ac.kr.

Manuscript received 11 Feb. 2016; revised 12 Sept. 2016; accepted 22 Sept. 2016. Date of publication 4 Oct. 2016; date of current version 27 June 2017. For information on obtaining reprints of this article, please send e-mail to: reprints@ieee.org, and reference the Digital Object Identifier below. Digital Object Identifier no. 10.1109/TMC.2016.2614979

[18], [19], [20]. Various scenarios exploiting the multiuser diversity gain have been studied not only in cooperative networks by applying an opportunistic two-hop relaying protocol [21], an opportunistic routing [22], and an opportunistic two-way relay selection [23], but also in cognitive radio networks with opportunistic scheduling [24], [25]. In addition, recent results [26], [27] have shown how to utilize the opportunistic gain when there are a large number of channel realizations. More specifically, to amplify signals and cancel interference, the idea of opportunistically pairing complementary channel instances has been studied in interference networks [26] and multi-hop relay networks [27]. In cognitive radio environments [28], opportunistic spectrum sharing was introduced by allowing secondary users to share the radio spectrum originally allocated to primary users via transmit adaptation in space, time, or frequency.

## 1.2 Main Contributions

In this paper, we study the  $K \times N \times K$  channel with interfering relays, which can be taken into account as one of practical multi-source interfering relay networks and be regarded as a fundamentally different channel model from the conventional  $K \times N \times K$  channel in [12]. Then, we introduce an *opportunistic network decoupling (OND)* protocol that achieves full DoF with comparatively easy implementation under the channel model. This work focuses on the  $K \times N \times K$  channel with one additional assumption that  $N$  half-duplex (HD) relays interfere with each other, which is a more feasible scenario. The scheme adopts the notion of the multiuser diversity gain for performing interference management over two hops. More precisely, in our scheme, a scheduling strategy is presented in time-division duplexing (TDD) two-hop environments with time-invariant channel coefficients, where a subset of relays is opportunistically selected in terms of producing the minimum total interference level. To improve the spectral efficiency, the *alternate relaying* protocol in [29], [30], [31] is employed with a modification, which eventually enables our system to operate in *virtual full-duplex* mode. As our main result, it turns out that in a high signal-to-noise ratio (SNR) regime, the OND protocol asymptotically achieves the min-cut upper bound of  $K$  DoF even in the presence of inter-relay interference and under the half-duplex assumption, provided that the number of relays,  $N$ , scales faster than  $\text{snr}^{3K-2}$ , which is the minimum number of relays required to guarantee our achievability result. Numerical evaluation also indicates that the OND protocol has higher sum-rates than those of other relaying methods under realistic network conditions (e.g., finite  $N$  and SNR) since the inter-relay interference is significantly reduced owing to the opportunistic gain. For comparison, the OND scheme without alternate relaying and the max-min SNR scheme are also shown as baseline schemes. Note that our protocol basically operates with local channel state information (CSI) at the transmitter and thus is suitable for distributed/decentralized networks.

Our main contributions are fourfold as follows:

- In the  $K \times N \times K$  channel with interfering relays, we introduce a new achievability scheme, termed OND with virtual full-duplex operation.
- Under the channel model, we completely analyze the optimal DoF, the required relay scaling condition, and the decaying rate of the interference level, where

TABLE 1  
Summary of Notations

Notation	Description
$\mathcal{S}_k$	$k$ th source
$\mathcal{D}_k$	$k$ th destination
$\mathcal{R}_k$	$k$ th relay
$h_{ik}^{(1)}$	channel coefficient from $\mathcal{S}_k$ to $\mathcal{R}_i$
$h_{ki}^{(2)}$	channel coefficient from $\mathcal{R}_i$ to $\mathcal{D}_k$
$h_{ik}^{(r)}$	channel coefficient between $\mathcal{R}_i$ and $\mathcal{R}_k$
$x_k^{(1)}(l)$	$l$ th transmitted symbol of $k$ th source
$\pi_s(k)$ ( $s = 1, 2$ )	indices of two relays helping $k$ th S-D pair
$x_k^{(1)}(l)$	$l$ th transmit symbol of $\mathcal{S}_k$
$x_{\pi_s(k)}^{(2)}(l)$	$l$ th transmit symbol of $\mathcal{R}_{\pi_s(k)}$
$\Pi_s$ ( $s = 1, 2$ )	two selected relay sets
$\tilde{L}_{i,k}$	scheduling metric in Step 1
$L_{i,k}^{\Pi_2}$	scheduling metric in Step 2
$\text{DoF}_{\text{total}}$	total number of DoF
$\text{sinr}_{\pi_s(k)}^{(1)}$	SINR at $\mathcal{R}_{\pi_s(k)}$
$\text{sinr}_{k,\pi_s(k)}^{(2)}$	SINR at $\mathcal{D}_k$ (from $\mathcal{R}_{\pi_s(k)}$ )

the OND scheme is shown to approach the min-cut upper bound on the DoF.

- Our achievability result (i.e., the derived DoF and relay scaling law) is validated via numerical evaluation.
- We perform extensive computer simulations with other baseline schemes.

## 1.3 Organization

The rest of this paper is organized as follows. In Section 2, we describe the system and channel models. In Section 3, the proposed OND scheme is specified and its lower bound on the DoF is analyzed. Section 4 shows an upper bound on the DoF. Numerical results of the proposed OND scheme are provided in Section 5. Finally, we summarize the paper with some concluding remarks in Section 6.

## 1.4 Notations

Throughout this paper,  $\mathbb{C}$ ,  $\mathbb{E}[\cdot]$ , and  $\lceil \cdot \rceil$  indicate the field of complex numbers, the statistical expectation, and the ceiling operation, respectively. Unless otherwise stated, all logarithms are assumed to be to the base 2.

Moreover, Table 1 summarizes the notations used throughout this paper. Some notations will be more precisely defined in the following sections when we introduce our channel model and achievability results.

## 2 SYSTEM AND CHANNEL MODELS

As one of two-hop cooperative scenarios, we consider the  $K \times N \times K$  channel model with interfering relays, which fits into the case where each S-D pair is geographically far apart and/or experiences strong shadowing (thus requiring the response to a huge challenge for achieving the target spectral efficiency). In the channel model, it is thus assumed that each source transmits its own message to the corresponding destination only through one of  $N$  relays, and thus there is no

direct path between an S-D pair. Note that unlike the conventional  $K \times N \times K$  channel, relay nodes are assumed to interfere with each other in our model. There are  $K$  S-D pairs, where each receiver is the destination of exactly one source node and is interested only in traffic demands of the source. As in the typical cooperative relaying setup,  $N$  relay nodes are located in the path between S-D pairs so as to help to reduce path-loss attenuations.

Suppose that each node is equipped with a single transmit antenna. Each relay node is assumed to operate in half-duplex mode and to fully decode, re-encode, and retransmit the source message, i.e., decode-and-forward protocol is taken into account. We assume that each node (either a source or a relay) has an average transmit power constraint  $P$ . Unlike the work in [10], [12],  $N$  relays are assumed to interfere with each other.<sup>2</sup> To improve the spectral efficiency, the alternate relaying protocol in [29], [30], [31] is employed with a modification. With alternate relaying, each selected relay node toggles between the transmit and listen modes for alternate time slots of message transmission of the sources. If  $N$  is sufficiently large, then it is possible to exploit the channel randomness for each hop and thus to obtain the opportunistic gain in multiuser environments. In this work, we do not assume the use of any sophisticated multiuser detection schemes at each receiver (either a relay or a destination node), thereby resulting in an easier implementation.

Now, let us turn to channel modeling. Let  $S_k$ ,  $D_k$ , and  $\mathcal{R}_i$  denote the  $k$ th source, the corresponding  $k$ th destination, and the  $i$ th relay node, respectively, where  $k \in \{1, \dots, K\}$  and  $i \in \{1, \dots, N\}$ . The terms  $h_{ik}^{(1)}$ ,  $h_{ki}^{(2)} \in \mathbb{C}$  denote the channel coefficients from  $S_k$  to  $\mathcal{R}_i$  and from  $\mathcal{R}_i$  to  $D_k$ , corresponding to the first and second hops, respectively. The term  $h_{in}^{(r)} \in \mathbb{C}$  indicates the channel coefficient between two relays  $\mathcal{R}_i$  and  $\mathcal{R}_n$ . All the channels are assumed to be Rayleigh, having zero-mean and unit variance, and to be independent across different  $i$ ,  $k$ ,  $n$ , and hop index  $r$ . We assume the block-fading model, i.e., the channels are constant during one block (e.g., frame), consisting of one scheduling time slot and  $L$  data transmission time slots, and changes to a new independent value for every block.

### 3 ACHIEVABILITY RESULTS

In this section, we describe the OND protocol, operating in virtual full-duplex mode, in the  $K \times N \times K$  channel with interfering relays. Then, its performance is analyzed in terms of achievable DoF along with a certain relay scaling condition. The decaying rate of the interference level is also analyzed. In addition, the OND protocol with no alternate relaying and its achievability result are shown for comparison.

#### 3.1 OND in the $K \times N \times K$ Channel with Interfering Relays

In this section, we introduce an OND protocol as the achievable scheme to guarantee the optimal DoF of the  $K \times N \times K$

channel with inter-relay interference, where  $2K$  relay nodes among  $N$  candidates are opportunistically selected for data forwarding in the sense of producing a sufficiently small amount of interference level. The proposed scheme is basically performed by utilizing the channel reciprocity of TDD systems.

Suppose that  $\pi_1(k)$  and  $\pi_2(k)$  denote the indices of two relays communicating with the  $k$ th S-D pair for  $k \in \{1, \dots, K\}$ . In this case, without loss of generality, assuming that the number of data transmission time slots,  $L$ , is an odd number, the specific steps of each node during one block are described as follows:

- Time slot 1: Sources  $S_1, \dots, S_K$  transmit their first encoded symbols  $x_1^{(1)}(1), \dots, x_K^{(1)}(1)$ , where  $x_k^{(1)}(l)$  represents the  $l$ th transmit symbol of the  $k$ th source node.<sup>3</sup> A set of  $K$  selected relay nodes,  $\Pi_1 = \{\pi_1(1), \dots, \pi_1(K)\}$ , operating in receive mode at each odd time slot, listens to  $x_1^{(1)}(1), \dots, x_K^{(1)}(1)$  (note that a relay selection strategy will be specified later). Other  $N - K$  relay nodes and destinations  $D_1, \dots, D_K$  remain idle.
- Time slot 2: The  $K$  sources transmit their encoded symbols  $x_1^{(1)}(2), \dots, x_K^{(1)}(2)$ . The  $K$  selected relays in the set  $\Pi_1$  forward their first re-encoded symbols  $x_{\pi_1(1)}^{(2)}(1), \dots, x_{\pi_1(K)}^{(2)}(1)$  to the corresponding  $K$  destinations. If the relays in  $\Pi_1$  successfully decode the corresponding symbols, then  $x_{\pi_1(k)}^{(2)}(1)$  is the same as  $x_k^{(1)}(1)$ . Another set of  $K$  selected relay nodes,  $\Pi_2 = \{\pi_2(1), \dots, \pi_2(K)\}$ , operating in receive mode at each even time slot, listens to and decodes  $x_1^{(1)}(2), \dots, x_K^{(1)}(2)$  while being interfered with by  $\mathcal{R}_{\pi_1(1)}, \dots, \mathcal{R}_{\pi_1(K)}$ . The  $K$  destinations receive and decode  $x_{\pi_1(1)}^{(2)}(1), \dots, x_{\pi_1(K)}^{(2)}(1)$  from  $\mathcal{R}_{\pi_1(1)}, \dots, \mathcal{R}_{\pi_1(K)}$ . The remaining  $N - 2K$  relays keep idle.
- Time slot 3: The  $K$  sources transmit their encoded symbols  $x_1^{(1)}(3), \dots, x_K^{(1)}(3)$ . The  $K$  relays  $\pi_2(1), \dots, \pi_2(K)$  forward their re-encoded symbols  $x_{\pi_2(1)}^{(2)}(2), \dots, x_{\pi_2(K)}^{(2)}(2)$  to the corresponding  $K$  destinations. Another  $K$  relays in  $\Pi_1$  receive and decode  $x_1^{(2)}(3), \dots, x_K^{(2)}(3)$  while being interfered with by  $\mathcal{R}_{\pi_2(1)}, \dots, \mathcal{R}_{\pi_2(K)}$ . The  $K$  destinations receive and decode  $x_{\pi_2(1)}^{(2)}(2), \dots, x_{\pi_2(K)}^{(2)}(2)$  from  $\mathcal{R}_{\pi_2(1)}, \dots, \mathcal{R}_{\pi_2(K)}$ . The remaining  $N - 2K$  relays keep idle.
- The processes in time slots 2 and 3 are repeated to the  $(L - 1)$ th time slot.
- Time slot  $L$ : The  $K$  relays in  $\Pi_2$  forward their re-encoded symbols  $x_{\pi_2(1)}^{(2)}(L - 1), \dots, x_{\pi_2(K)}^{(2)}(L - 1)$  to the corresponding  $K$  destinations. The  $K$  sources and the other  $N - K$  relays remain idle.

At each odd time slot  $l$  (i.e.,  $l = 1, 3, \dots, L$ ), let us consider the received signal at each selected relay for the first hop

2. If we can cancel the interfering signals among multiple relays, then the existing achievable scheme of the  $K \times N \times K$  channel can also be applied here.

3. For notational convenience, we use scalar notation instead of vector notation for each coding block from source nodes, but the size of each symbol is assumed to be sufficiently long to achieve the Shannon-theoretic channel capacity.

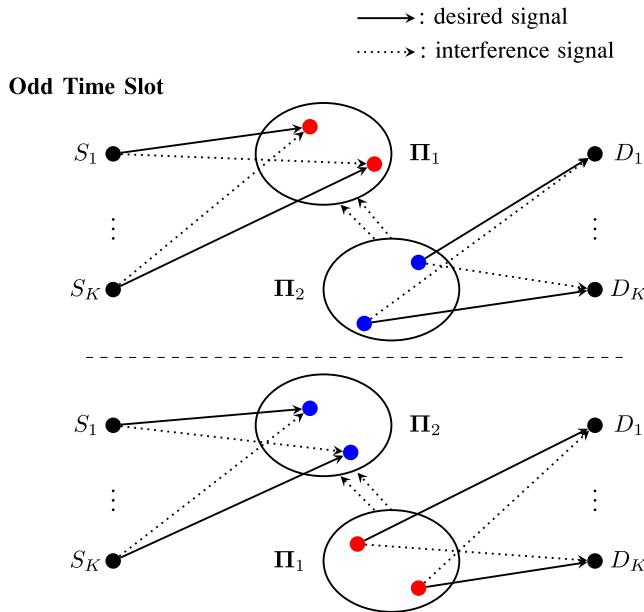


Fig. 1. The overall procedure of our OND scheme in the  $K \times N \times K$  channel with interfering relays.

and the received signal at each destination for the second hop, respectively.

For the first hop (Phase 1), the received signal  $y_{\pi_1(i)}^{(1)}(l) \in \mathbb{C}$  at  $\mathcal{R}_{\pi_1(i)}$  is given by

$$y_{\pi_1(i)}^{(1)}(l) = \sum_{k=1}^K h_{\pi_1(i)k}^{(1)} x_k^{(1)}(l) + \sum_{n=1}^K h_{\pi_1(i)\pi_2(n)}^{(r)} x_{\pi_2(n)}^{(2)}(l-1) + z_{\pi_1(i)}^{(1)}(l), \quad (1)$$

where  $x_k^{(1)}(l)$  and  $x_{\pi_2(n)}^{(2)}(l-1)$  are the  $l$ th transmit symbol of  $\mathcal{S}_k$  and the  $(l-1)$ th transmit symbol of  $\mathcal{R}_{\pi_2(n)}$ , respectively. As addressed earlier, if relay  $\mathcal{R}_{\pi_2(k)}$  successfully decodes the received symbol, then it follows that  $x_{\pi_2(k)}^{(2)}(l-1) = x_k^{(1)}(l-1)$ . The received signal  $y_{\pi_1(i)}^{(1)}(l)$  at  $\mathcal{R}_{\pi_1(i)}$  is corrupted by the independent and identically distributed (i.i.d.) and circularly symmetric complex additive white Gaussian noise (AWGN)  $z_{\pi_1(i)}^{(1)}(l)$  having zero-mean and variance  $N_0$ . Note that the second term in the right-hand side (RHS) of (1) indicates the inter-relay interference, which occurs when the  $K$  relays in the set  $\Pi_1$ , operating in receive mode, listen to the sources, the relays are interfered with by the other set  $\Pi_2$ , operating in transmit mode. Note that when  $l = 1$ , relays have no symbols to transmit, and the second term in the RHS of (1) becomes zero. Similarly when  $l = L$ , sources do not transmit symbols, and the first term in the RHS of (1) becomes zero.

For the second hop (Phase 2), assuming that the  $K$  selected relay nodes transmit their data packets simultaneously, the received signal  $y_k^{(2)}(l) \in \mathbb{C}$  at  $\mathcal{D}_k$  is given by

$$y_k^{(2)}(l) = \sum_{n=1}^K h_{k\pi_2(n)}^{(2)} x_{\pi_2(n)}^{(2)}(l-1) + z_k^{(2)}(l), \quad (2)$$

where  $z_k^{(2)}(l)$  is the i.i.d. AWGN having zero-mean and variance  $N_0$ . We also note that when  $l = 1$ , there are no signals from relays.

Likewise, at each even time slot (i.e.,  $l = 2, 4, \dots, L-1$ ), the received signals at  $\mathcal{R}_{\pi_2(i)}$  and  $\mathcal{D}_k$  (i.e., the first and second hops) are given by

$$y_{\pi_2(i)}^{(1)}(l) = \sum_{k=1}^K h_{\pi_2(i)k}^{(1)} x_k^{(1)}(l) + \sum_{n=1}^K h_{\pi_2(i)\pi_1(n)}^{(r)} x_{\pi_1(n)}^{(2)}(l-1) + z_{\pi_2(i)}^{(1)}(l)$$

and

$$y_k^{(2)}(l) = \sum_{n=1}^K h_{k\pi_1(n)}^{(2)} x_{\pi_1(n)}^{(2)}(l-1) + z_k^{(2)}(l),$$

respectively. The illustration of the aforementioned OND protocol is geographically shown in Fig. 1 (two terms  $\tilde{L}_{\pi_1(k),k}^{\Pi_2}$  and  $L_{\pi_2(k),k}^{\Pi_1}$  are specified later in the following relay selection steps).

Now, let us describe how to choose two types of relay sets,  $\Pi_1$  and  $\Pi_2$ , among  $N$  relay nodes, where  $N$  is sufficiently large (the minimum  $N$  required to guarantee the DoF optimality will be analyzed in Section 3.2).

### 3.1.1 Step 1 (The First Relay Set Selection)

Let us first focus on selecting the set  $\Pi_1 = \{\mathcal{R}_{\pi_1(1)}, \dots, \mathcal{R}_{\pi_1(K)}\}$ , operating in receive and transmit modes in odd and even time slots, respectively. For every scheduling period, it is possible for relay  $\mathcal{R}_i$  to obtain all the channel coefficients  $h_{ik}^{(1)}$  and  $h_{ki}^{(2)}$  by using a pilot signaling sent from all of the source and destination nodes due to the channel reciprocity before data transmission, where  $i \in \{1, \dots, N\}$  and  $k \in \{1, \dots, K\}$  (note that this is our local CSI assumption). When  $\mathcal{R}_i$  is assumed to serve the  $k$ th S-D pair  $(\mathcal{S}_k, \mathcal{D}_k)$ , it then examines both i) how much interference is received from the other sources and ii) how much interference is generated by itself to the other destinations, by computing the following scheduling metric  $\tilde{L}_{i,k}$ :

$$\tilde{L}_{i,k} = \sum_{\substack{m=1 \\ m \neq k}}^K \left( |h_{im}^{(1)}|^2 + |h_{mi}^{(2)}|^2 \right), \quad (3)$$

where  $i \in \{1, \dots, N\}$  and  $k \in \{1, \dots, K\}$ . We remark that the first term  $\sum_{m=1, m \neq k}^K |h_{im}^{(1)}|^2$  in (3) denotes the sum of interference power received at  $\mathcal{R}_i$  for the first hop (i.e., Phase 1). On the other hand, the second term  $\sum_{m=1, m \neq k}^K |h_{mi}^{(2)}|^2$  indicates the sum of interference power generating at  $\mathcal{R}_i$ , which can be interpreted as the *leakage of interference* to the  $K-1$  receivers expect for the corresponding destination, for the second hop (i.e., Phase 2) under the same assumption.

Suppose that a short duration Clear to Send (CTS) message is transmitted by the destination who finds its desired relay node (or the master destination). Then according to the computed metrics  $\tilde{L}_{i,k}$  in (3), a timer-based method can be used for the relay selection similarly

as in [32].<sup>4</sup> Note that the method based on the timer is considerably suitable in distributed systems in the sense that information exchange among all the relay nodes can be minimized. At the beginning of every scheduling period, the relay  $\mathcal{R}_i$  computes the set of  $K$  scheduling metrics,  $\{\tilde{L}_{i,1}, \dots, \tilde{L}_{i,K}\}$ , and then starts its own timer with  $K$  initial values, which can be set to be proportional to the  $K$  metrics.<sup>5</sup> Thus, there exist  $NK$  metrics over the whole relay nodes, and we need to compare them so as to determine who will be selected. The timer of the relay  $\mathcal{R}_{\pi_1(\hat{k})}$  with the least one  $\tilde{L}_{\pi_1(\hat{k}),\hat{k}}$  among  $NK$  metrics will expire first, where  $\pi_1(\hat{k}) \in \{1, \dots, N\}$  and  $\hat{k} \in \{1, \dots, K\}$ . The relay then transmits a short duration RTS message, signaling its presence, to the other  $N - 1$  relays, where each RTS message is composed of  $\lceil \log_2 K \rceil$  bits to indicate which S-D pair the relay wants to serve. Thereafter, the relay  $\mathcal{R}_{\pi_1(\hat{k})}$  is first selected to forward the  $\hat{k}$ th S-D pair's packet. All the other relays are in listen mode while waiting for their timer to be set to zero (i.e., to expire). At the stage of deciding who will send the second RTS message, it is assumed that the other relays are not allowed to communicate with the  $\hat{k}$ th S-D pair, and thus the associated metrics  $\{\tilde{L}_{1,\hat{k}}, \dots, \tilde{L}_{\pi_1(\hat{k})-1,\hat{k}}, \tilde{L}_{\pi_1(\hat{k})+1,\hat{k}}, \dots, \tilde{L}_{N,\hat{k}}\}$  are discarded with respect to timer operation. If another relay has an opportunity to send the second RTS message of  $\lceil \log_2(K - 1) \rceil$  bits in order to declare its presence, then it is selected to communicate with the corresponding S-D pair. When such  $K$  RTS messages, consisting of at most  $K \lceil \log_2 K \rceil$  bits, are sent out in consecutive order, i.e., the set of  $K$  relays,  $\Pi_1 = \{\mathcal{R}_{\pi_1(1)}, \dots, \mathcal{R}_{\pi_1(K)}\}$ , is chosen, the timer-based algorithm for the first relay set selection terminates, yielding no RTS collision with high probability. We remark that when  $K = 1$  (i.e., the single S-D pair case),  $K$  relay nodes are *arbitrarily* chosen as the first relay set  $\Pi_1$  since there is no interference in this step.

### 3.1.2 Step 2 (The Second Relay Set Selection)

Now let us turn to choosing the set of  $K$  relay nodes (among  $N - K$  candidates),  $\Pi_2 = \{\pi_2(1), \dots, \pi_2(K)\}$ , operating in receive and transmit modes in even and odd time slots, respectively. Using  $K$  RTS messages broadcasted from the  $K$  relay nodes in the set  $\Pi_1$ , it is possible for relay node  $\mathcal{R}_i \in \{1, \dots, N\} \setminus \Pi_1$  to compute the sum of inter-relay interference power generated from the relays in  $\Pi_1$ , denoted by  $\sum_{k=1}^K \left| h_{i\pi_1(k)}^{(r)} \right|^2$ . When  $\mathcal{R}_i$  is again assumed to serve the  $k$ th S-

4. The reception of a CTS message, which is transmitted from a certain destination, triggers the initial timing process at each relay. Therefore, no explicit timing synchronization protocol is required among the relays [32], [33]. Moreover, it is worth noting that the overhead of relay selection is a small fraction of one transmission block with small collision probability [32]. Since our relay selection procedure is performed sequentially over all of the S-D pairs and the already selected relays for a certain S-D pair are not allowed to take part in the selection process for another S-D pair, the collision probability is thus at most  $2K$  times that of the single S-D pair case [32].

5. To avoid a situation such that a malicious relay deliberately sets its timer to a smaller value so as to win the chance, prior to the relay selection process, a secret key may be shared among legitimate nodes including relays. If a malicious relay who did not share the key wants to participate in communication, then one can neglect his/her message (e.g., Request to Send (RTS) message).

D pair  $(\mathcal{S}_k, \mathcal{D}_k)$ , it examines both i) how much interference is received from the undesired sources and the selected relays in the set  $\Pi_1$  for the first hop and ii) how much interference is generated by itself to the other destinations by computing the following metric  $L_{i,k}^{\Pi_2}$ , termed *total interference level (TIL)*:

$$\begin{aligned} L_{i,k}^{\Pi_2} &= \tilde{L}_{i,k} + \sum_{k=1}^K \left| h_{i\pi_1(k)}^{(r)} \right|^2 \\ &= \sum_{\substack{m=1 \\ m \neq k}}^K \left( \left| h_{im}^{(1)} \right|^2 + \left| h_{mi}^{(2)} \right|^2 \right) + \sum_{k=1}^K \left| h_{i\pi_1(k)}^{(r)} \right|^2, \end{aligned} \quad (4)$$

where  $i \in \{1, \dots, N\}$  and  $k \in \{1, \dots, K\}$ . We note that Steps 1 and 2 cannot be exchangeable due to the fact that the inter-relay interference term  $\sum_{k=1}^K \left| h_{i\pi_1(k)}^{(r)} \right|^2$  is measured after determining the first relay set  $\Pi_1$ . If the relay set selection order is switched, then the metric TIL in (4) will not be available.

According to the computed TIL  $L_{i,k}^{\Pi_2}$ , we also apply the timer-based method used in Step 1 for the second relay set selection. The relay  $\mathcal{R}_i \in \{1, \dots, N\} \setminus \Pi_1$  computes the set of  $K$  TILs,  $\{L_{i,1}^{\Pi_2}, \dots, L_{i,K}^{\Pi_2}\}$ , and then starts its timer with  $K$  initial values, proportional to the  $K$  TILs. Thus, we need to compare  $(N - K)K$  TIL metrics over the relay nodes in the set  $\{1, \dots, N\} \setminus \Pi_1$  in order to determine who will be selected as the second relay set. The rest of the relay set selection protocol (i.e., RTS message exchange among relay nodes) almost follows the same line as that of Step 1. The timer-based algorithm for the second relay set selection terminates when  $K$  RTS messages are sent out in consecutive order. Then,  $K$  relay nodes having a sufficiently small amount of TIL  $L_{i,k}^{\Pi_2}$  are selected as the second relay set  $\Pi_2$ .

**Remark 1.** Owing to the channel reciprocity of TDD systems, the sum of inter-relay interference power received at any relay  $R_i \in \Pi_1$ ,  $\sum_{k=1}^K \left| h_{i\pi_2(k)}^{(r)} \right|^2$ , also turns out to be sufficiently small when  $N$  is large. That is, it is also guaranteed that  $K$  selected relays in the set  $\Pi_1$  have a sufficiently small amount of TIL.

**Remark 2.** The overhead of each scheduling time slot (i.e., the total number of bits required for exchanging RTS messages among the relay nodes) can be made arbitrarily small, compared to one transmission block. From the fact that  $K$  RTS messages, consisting of at most  $K \lceil \log_2 K \rceil$  bits, are sent out in each relay set selection step, only  $2K \lceil \log_2 K \rceil$  bit transmission could suffice.

### 3.1.3 Step 3 (Data Transmission)

The  $2K$  selected relays request data transmission to their desired source nodes. Each source  $(\mathcal{S}_k)$  then starts to transmit data to the corresponding destination  $(\mathcal{D}_k)$  via one of its two relay nodes alternately  $(\mathcal{R}_{\pi_1(k)} \text{ or } \mathcal{R}_{\pi_2(k)})$ , which was specified earlier. If the TILs of the selected relays are arbitrarily small, then i) the associated undesired source-relay and relay-destination channel links and ii) the inter-relay channel links are all in deep fade. In Section 3.2, we will show that it is possible to choose such relays with the help of the multiuser diversity gain.

At the receiver side, each relay or destination detects the signal sent from its desired transmitter, while simply

treating interference as Gaussian noise. Thus, no multiuser detection is performed at each receiver, thereby resulting in an easier implementation.

### 3.2 Analysis of a Lower Bound on the DoF

In this section, using the scaling argument bridging between the number of relays,  $N$ , and the received SNR (refer to [9], [18], [19], [20] for the details), we shall show 1) the lower bound on the DoF of the  $K \times N \times K$  channel with interfering relays as  $N$  increases and 2) the minimum  $N$  required to guarantee the achievability result. The total number of DoF, denoted by  $\text{DoF}_{\text{total}}$ , is defined as [2]

$$\text{DoF}_{\text{total}} = \sum_{k=1}^K \left( \lim_{\text{snr} \rightarrow \infty} \frac{T_k(\text{snr})}{\log \text{snr}} \right),$$

where  $T_k(\text{snr})$  denotes the transmission rate of source  $S_k$ . Using the OND framework in the  $K \times N \times K$  channel with interfering relays where  $L$  transmission slots per block are used, the achievable  $\text{DoF}_{\text{total}}$  is lower-bounded by

$$\text{DoF}_{\text{total}} \geq \frac{L-1}{L} \sum_{k=1}^K \sum_{s=1}^2 \left( \lim_{\text{snr} \rightarrow \infty} \frac{\frac{1}{2} \log \left( 1 + \min \left\{ \text{snr}_{\pi_s(k)}^{(1)}, \text{snr}_{k,\pi_s(k)}^{(2)} \right\} \right)}{\log \text{snr}} \right), \quad (5)$$

where  $\text{snr}_{\pi_s(k)}^{(1)}$  denotes the received signal-to-interference-and-noise ratio (SINR) at the relay  $\mathcal{R}_{\pi_s(k)}$  and  $\text{snr}_{k,\pi_s(k)}^{(2)}$  denotes the received SINR at the destination  $\mathcal{D}_k$  when the relay  $\mathcal{R}_{\pi_s(k)}$  transmits the desired signal ( $s=1,2$  and  $k \in \{1, \dots, K\}$ ). More specifically, the above SINRs can be formally expressed as<sup>6</sup>

$$\text{snr}_{\pi_s(i)}^{(1)} = \frac{P |h_{\pi_s(i)i}^{(1)}|^2}{N_0 + P \sum_{\substack{k=1 \\ k \neq i}}^K |h_{\pi_s(i)k}^{(1)}|^2 + P \sum_{k=1}^K |h_{\pi_s(i),\pi_s(k)}^{(r)}|^2}$$

$$\text{snr}_{i,\pi_s(i)}^{(2)} = \frac{P |h_{i\pi_s(i)}^{(2)}|^2}{1 + P \sum_{\substack{k=1 \\ k \neq i}}^K |h_{i\pi_s(k)}^{(2)}|^2},$$

where the second term in the denominator of  $\text{snr}_{\pi_s(i)}^{(1)}$  indicates the interference power at relay  $\pi_s(i)$  received from the sources while the third term indicates the inter-relay interference, and the second term in the denominator of  $\text{snr}_{i,\pi_s(i)}^{(2)}$  indicates the interference power at the destination  $\mathcal{D}_i$  received from the active relays. Here,  $\tilde{s} = 3 - s$ , i.e.,  $\tilde{s} = 2$  if  $s = 1$ , and vice versa.

We focus on the first relay set  $\Pi_1$ 's perspective to examine the received SINR values according to each time slot. Let us first denote  $L_{\pi_1(i),i}^{\Pi_1} \triangleq \tilde{L}_{\pi_1(i),i} + \sum_{k=1}^K |h_{\pi_1(i),\pi_2(k)}^{(r)}|^2$  for  $i \in \{1, \dots, K\}$ . For the first hop, at time slot  $2t-1$  (i.e., each

odd time slot),  $t \in \{1, 2, \dots, \frac{L-1}{2}\}$ , the received  $\text{snr}_{\pi_1(i)}^{(1)}$  at  $\mathcal{R}_{\pi_1(i)}$  is lower-bounded by

$$\begin{aligned} \text{snr}_{\pi_1(i)}^{(1)} &\geq \frac{\text{snr} |h_{\pi_1(i)i}^{(1)}|^2}{1 + \text{snr} \sum_{\substack{k=1 \\ k \neq i}}^K \left( |h_{\pi_1(i)k}^{(1)}|^2 + |h_{k\pi_1(i)}^{(2)}|^2 \right) + \text{snr} \sum_{k=1}^K |h_{\pi_1(i),\pi_2(k)}^{(r)}|^2} \\ &= \frac{\text{snr} |h_{\pi_1(i)i}^{(1)}|^2}{1 + \text{snr} \left( \tilde{L}_{\pi_1(i),i} + \sum_{k=1}^K |h_{\pi_1(i),\pi_2(k)}^{(r)}|^2 \right)} \\ &= \frac{\text{snr} |h_{\pi_1(i)i}^{(1)}|^2}{1 + \text{snr} L_{\pi_1(i),i}^{\Pi_1}} \\ &\geq \frac{\text{snr} |h_{\pi_1(i)i}^{(1)}|^2}{1 + \text{snr} \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1}}, \end{aligned} \quad (6)$$

where  $\tilde{L}_{\pi_1(i),i}$  indicates the scheduling metric in (3) when  $\mathcal{R}_{\pi_1(i)}$  is assumed to serve the  $i$ th S-D pair ( $S_i, \mathcal{D}_i$ ). For the second hop, at time slot  $2t$  (i.e., each even time slot),  $t \in \{1, 2, \dots, \frac{L-1}{2}\}$ , the received  $\text{snr}_{i,\pi_1(i)}^{(2)}$  at  $\mathcal{D}_i$  is lower-bounded by

$$\begin{aligned} \text{snr}_{i,\pi_1(i)}^{(2)} &\geq \frac{\text{snr} |h_{i\pi_1(i)}^{(2)}|^2}{1 + \text{snr} \sum_{i=1}^K \sum_{\substack{k=1 \\ k \neq i}}^K |h_{i\pi_1(k)}^{(2)}|^2} \\ &\geq \frac{\text{snr} |h_{i\pi_1(i)}^{(2)}|^2}{1 + \text{snr} \sum_{i=1}^K \sum_{\substack{k=1 \\ k \neq i}}^K \left( |h_{\pi_1(i)k}^{(1)}|^2 + |h_{k\pi_1(i)}^{(2)}|^2 \right)} \\ &= \frac{\text{snr} |h_{i\pi_1(i)}^{(2)}|^2}{1 + \text{snr} \sum_{i=1}^K \tilde{L}_{\pi_1(i),i}} \\ &\geq \frac{\text{snr} |h_{i\pi_1(i)}^{(2)}|^2}{1 + \text{snr} \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1}}, \end{aligned} \quad (7)$$

where the second inequality holds due to the channel reciprocity. The term  $\sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1}$  in the denominator of (6) and (7) needs to scale as  $\text{snr}^{-1}$ , i.e.,  $\sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1} = O(\text{snr}^{-1})$ , so that both  $\text{snr}_{\pi_1(k)}^{(1)}$  and  $\text{snr}_{k,\pi_1(k)}^{(2)}$  scale as  $\Omega(\text{snr})$  with increasing SNR, which eventually enables to achieve the DoF of  $\frac{L-1}{2}$  per S-D pair from (5).<sup>7</sup> Even if such a bounding technique in (6) and (7) leads to a loose lower bound on the SINR, it is sufficient to prove our achievability result in terms of DoF and relay scaling law.

Now, let us turn to the second relay set  $\Pi_2$ . Similarly as in (6), for the first hop, at time slot  $2t$ ,  $t \in \{1, 2, \dots, \frac{L-1}{2}\}$ , the received  $\text{snr}_{\pi_2(i)}^{(1)}$  at  $\mathcal{R}_{\pi_2(i)}$  is lower-bounded by

6. Note that at the first time slot for the relays  $\{\pi_1(i)\}_{i=1}^K$ , the third term in the denominator of  $\text{snr}_{\pi_1(i)}^{(1)}$  (i.e., the inter-relay interference term) becomes zero.

7. We use the following notation: i)  $f(x) = O(g(x))$  means that there exist constant  $C$  and  $c$  such that  $f(x) \leq Cg(x)$  for all  $x > c$ . ii)  $f(x) = \Omega(g(x))$  if  $g(x) = O(f(x))$ . iii)  $f(x) = \omega(g(x))$  means that  $\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = 0$  [34].

$$\text{snr}_{\pi_2(i)}^{(1)} \geq \frac{\text{snr} |h_{\pi_2(i)i}^{(1)}|^2}{1 + \text{snr} \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2}}, \quad (8)$$

where  $L_{\pi_2(i),i}^{\Pi_2}$  indicates the TIL in (4) when  $\mathcal{R}_{\pi_2(i)}$  is assumed to serve the  $i$ th S-D pair  $(S_i, \mathcal{D}_i)$ . For the second hop, at time slot  $2t + 1$ ,  $t \in \{1, 2, \dots, \frac{L-1}{2}\}$ , the received  $\text{snr}_{i,\pi_2(i)}^{(2)}$  at  $\mathcal{D}_i$  can also be lower-bounded by

$$\text{snr}_{i,\pi_2(i)}^{(2)} \geq \frac{\text{snr} |h_{i,\pi_2(i)}^{(2)}|^2}{1 + \text{snr} \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2}}. \quad (9)$$

The next step is thus to characterize the three metrics  $\tilde{L}_{i,k}$ ,  $L_{i,k}^{\Pi_1}$ , and  $L_{i,k}^{\Pi_2}$  ( $i \in \{1, \dots, N\}$  and  $k \in \{1, \dots, K\}$ ) and their cumulative density functions (CDFs) in the  $K \times N \times K$  channel with interfering relays, which is used to analyze the lower bound on the DoF and the required relay scaling law in the model under consideration. Since it is obvious to show that the CDF of  $L_{i,k}^{\Pi_1}$  is identical to that of  $L_{i,k}^{\Pi_2}$ , we focus only on the characterization of  $L_{i,k}^{\Pi_2}$ . The scheduling metric  $\tilde{L}_{i,k}$  follows the chi-square distribution with  $2(2K - 2)$  degrees of freedom since it represents the sum of i.i.d.  $2K - 2$  chi-square random variables with 2 degrees of freedom. Similarly, the TIL  $L_{i,k}^{\Pi_2}$  follows the chi-square distribution with  $2(3K - 2)$  degrees of freedom. The CDFs of the two metrics  $\tilde{L}_{i,k}$  and  $L_{i,k}^{\Pi_2}$  are given by

$$\mathcal{F}_{\tilde{L}}(\ell) = \frac{\gamma(2K - 2, \ell/2)}{\Gamma(2K - 2)} \quad (10)$$

$$\mathcal{F}_L(\ell) = \frac{\gamma(3K - 2, \ell/2)}{\Gamma(3K - 2)}, \quad (11)$$

respectively, where  $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$  is the Gamma function and  $\gamma(z, x) = \int_0^x t^{z-1} e^{-t} dt$  is the lower incomplete Gamma function [35, eqn. (8.310.1)]. We start from the following lemma.

**Lemma 1.** For any  $0 < \ell \leq 2$ , the CDFs of the random variables  $\tilde{L}_{i,k}$  and  $L_{i,k}^{\Pi_2}$  in (10) and (11) are lower-bounded by  $\mathcal{F}_{\tilde{L}}(\ell) \geq C_1 \ell^{2K-2}$  and  $\mathcal{F}_L(\ell) \geq C_2 \ell^{3K-2}$ , respectively, where

$$C_1 = \frac{e^{-1} 2^{-(2K-2)}}{\Gamma(2K-1)} \quad (12)$$

$$C_2 = \frac{e^{-1} 2^{-(3K-2)}}{\Gamma(3K-1)}, \quad (13)$$

and  $\Gamma(z)$  is the Gamma function.

**Proof.** The detailed proof of this argument is omitted here since it essentially follows the similar line to the proof of [9, Lemma 1] with a slight modification.  $\square$

In the following theorem, we establish our first main result by deriving the lower bound on the total DoF in the  $K \times N \times K$  channel with interfering relays.

**Theorem 1.** Suppose that the OND scheme with alternate relaying is used for the  $K \times N \times K$  channel with interfering relays. Then, for  $L$  data transmission time slots,

$$\text{DoF}_{\text{total}} \geq \frac{(L-1)K}{L}$$

is achievable if  $N = \omega(\text{snr}^{3K-2})$ .

**Proof.** From (5),(6), (7), (8), and (9), the OND scheme achieves  $\text{DoF}_{\text{total}} \geq \frac{L-1}{L} K$  provided that the two values  $\text{snr} \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1}$  and  $\text{snr} \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2}$  are less than or equal to some constant  $\epsilon_0 > 0$ , independent of SNR, for all S-D pairs. Then, a lower bound on the achievable  $\text{DoF}_{\text{total}}$  is given by

$$\text{DoF}_{\text{total}} \geq \mathcal{P}_{\text{OND}} \frac{(L-1)K}{L},$$

which indicates that  $\frac{L-1}{L} K$  DoF is achievable for a fraction  $\mathcal{P}_{\text{OND}}$  of the time for actual transmission, where

$$\begin{aligned} \mathcal{P}_{\text{OND}} &= \lim_{\text{snr} \rightarrow \infty} \Pr \left\{ \text{snr} \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1} \leq \epsilon_0 \text{ and } \text{snr} \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2} \leq \epsilon_0 \right\}. \end{aligned} \quad (14)$$

We now examine the relay scaling condition such that  $\mathcal{P}_{\text{OND}}$  converges to one with high probability. For the simplicity of the proof, suppose that the first and the second relay sets  $\Pi_1$  and  $\Pi_2$  are selected out of two mutually exclusive relaying candidate sets  $\mathcal{N}_1$  and  $\mathcal{N}_2$ , respectively, i.e.,  $\mathcal{N}_1, \mathcal{N}_2 \subset \{1, \dots, N\}$ ,  $\mathcal{N}_1 \cap \mathcal{N}_2 = \emptyset$ ,  $\mathcal{N}_1 \cup \mathcal{N}_2 = \{1, \dots, N\}$ ,  $\Pi_1 \subset \mathcal{N}_1$ , and  $\Pi_2 \subset \mathcal{N}_2$ . Then, we are interested in how  $|\mathcal{N}_1|$  and  $|\mathcal{N}_2|$  scale with SNR in order to guarantee that  $\mathcal{P}_{\text{OND}}$  tends to one, where  $|\mathcal{N}_s|$  denotes the cardinality of  $\mathcal{N}_s$  for  $s = 1, 2$ . From (14), we further have

$$\begin{aligned} \mathcal{P}_{\text{OND}} &= \lim_{\text{snr} \rightarrow \infty} \left( \Pr \left\{ \text{snr} \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1} \leq \epsilon_0 \right\} \right. \\ &\quad \left. \cdot \Pr \left\{ \text{snr} \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2} \leq \epsilon_0 \right\} \right). \end{aligned} \quad (15)$$

Let  $\mathcal{B}_m \triangleq \mathcal{N}_2 \setminus \{\pi_2(\ell)\}_{\ell=1}^{m-1}$  with  $\{\pi_2(\ell)\}_{\ell=1}^0 = \emptyset$  and  $|\mathcal{B}_m|$  be the candidate set associated with the second relay set and the  $m$ th S-D pair and its cardinality, respectively. For a constant  $\epsilon_0 > 0$ , we can bound the second term in (15) as follows:

$$\begin{aligned} &\Pr \left\{ \sum_{i=1}^K L_{\pi_2(i),i}^{\Pi_2} \leq \frac{\epsilon_0}{\text{snr}} \right\} \\ &\geq \Pr \left\{ \max_{1 \leq i \leq K} L_{\pi_2(i),i}^{\Pi_2} \leq \frac{\epsilon_0}{K \text{snr}} \right\} \\ &\stackrel{(a)}{\geq} 1 - \Pr \left\{ \exists i : L_{\pi_2(i),i}^{\Pi_2} \geq \frac{\epsilon_0}{K \text{snr}} \right\} \\ &\stackrel{(b)}{\geq} 1 - \sum_{i=1}^K \Pr \left\{ L_{\pi_2(i),i}^{\Pi_2} \geq \frac{\epsilon_0}{K \text{snr}} \right\} \\ &\stackrel{(c)}{\geq} 1 - K \Pr \left\{ \min_{j \in \mathcal{N}_2} L_{j,i}^{\Pi_2} \geq \frac{\epsilon_0}{K \text{snr}} \right\} \\ &\stackrel{(d)}{\geq} 1 - K \left( 1 - \mathcal{F}_L \left( \frac{\epsilon_0}{K \text{snr}} \right) \right)^{|\mathcal{B}_i|} \\ &\stackrel{(e)}{\geq} 1 - K \left( 1 - C_2 \left( \frac{\epsilon_0}{K \text{snr}} \right)^{3K-2} \right)^{|\mathcal{N}_2| - K + 1}, \end{aligned} \quad (16)$$

where the inequality (a) holds from the De Morgan's law; (b) follows from the union bound; (c) follows since  $L_{\pi_2(i),i}^{\Pi_2} = \min_{j \in \mathcal{N}_2} L_{j,i}^{\Pi_2}$ ; (d) follows since  $L_{j,i}^{\Pi_2}$  are the i.i.d. random variables  $\forall j \in \mathcal{N}_2$  for a given  $i$ , owing to the fact that the channels are i.i.d. variables; and (e) follows from Lemma 1 with  $C_2 = \frac{e^{-1} 2^{-(3K-2)}}{\Gamma(3K-1)}$  since  $0 < \frac{\epsilon_0}{\text{snr}} \leq 2$  as  $\text{snr} \rightarrow \infty$  and from the fact that  $|\mathcal{B}_i| \geq |\mathcal{N}_2| - K + 1$ .

We now pay our attention to the first term in (15), which can be bounded by

$$\begin{aligned} \Pr \left\{ \sum_{i=1}^K L_{\pi_1(i),i}^{\Pi_1} \leq \frac{\epsilon_0}{\text{snr}} \right\} &\geq \Pr \left\{ \max_{1 \leq i \leq K} L_{\pi_1(i),i}^{\Pi_1} \leq \frac{\epsilon_0}{K \text{snr}} \right\} \\ &= \left( \Pr \left\{ L_{\pi_1(i),i}^{\Pi_1} \leq \frac{\epsilon_0}{K \text{snr}} \right\} \right)^K, \end{aligned} \quad (17)$$

where the equality follows from the fact that  $L_{\pi_1(i),i}^{\Pi_1}$  and  $L_{\pi_1(j),j}^{\Pi_1}$  for  $i \neq j$  are the functions of different random variables and these are independent of each other. By letting  $K_i = \sum_{k=1}^K \left| h_{\pi_1(i),\pi_2(k)}^{(r)} \right|^2$ , by the definition of  $L_{\pi_1(i),i}^{\Pi_1}$ , we have

$$\begin{aligned} \Pr \left\{ L_{\pi_1(i),i}^{\Pi_1} \leq \frac{\epsilon_0}{K \text{snr}} \right\} &= 1 - \Pr \left\{ \tilde{L}_{\pi_1(i),i} + K_i \geq \frac{\epsilon_0}{K \text{snr}} \right\} \\ &\geq 1 - \Pr \left\{ \tilde{L}_{\pi_1(i),i} \geq \frac{\epsilon_0}{2K \text{snr}} \right\} - \Pr \left\{ K_i \geq \frac{\epsilon_0}{2K \text{snr}} \right\}, \end{aligned} \quad (18)$$

where the inequality follows from the fact that for any random variables  $X$  and  $Y$ ,  $\Pr\{X + Y \geq \epsilon\} \leq \Pr\{X \geq \frac{\epsilon}{2}\} + \Pr\{Y \geq \frac{\epsilon}{2}\}$  [38]. In the same manner, let  $\mathcal{A}_m \triangleq \mathcal{N}_1 \setminus \{\pi_1(\ell)\}_{\ell=1}^{m-1}$  with  $\{\pi_1(\ell)\}_{\ell=1}^{m-1} = \emptyset$  and  $|\mathcal{A}_m|$  be the candidate set associated with the first relay set and the  $m$ th S-D pair and its cardinality, respectively. Then, we can bound the first two terms in the RHS of (18) as follows:

$$\begin{aligned} 1 - \Pr \left\{ \tilde{L}_{\pi_1(i),i} \geq \frac{\epsilon_0}{2K \text{snr}} \right\} &= 1 - \Pr \left\{ \min_{j \in \mathcal{N}_1} \tilde{L}_{j,i} \geq \frac{\epsilon_0}{2K \text{snr}} \right\} \\ &= 1 - \left( 1 - \mathcal{F}_{\tilde{L}} \left( \frac{\epsilon_0}{2K \text{snr}} \right) \right)^{|\mathcal{A}_i|} \\ &\geq 1 - \left( 1 - C_1 \left( \frac{\epsilon_0}{2K \text{snr}} \right)^{2K-2} \right)^{|\mathcal{N}_1| - K + 1}, \end{aligned} \quad (19)$$

where the last inequality follows from Lemma 1 with  $C_1 = \frac{e^{-1} 2^{-(2K-2)}}{\Gamma(2K-1)}$ . Finally, from (16), it follows that  $\Pr\{K_i \geq \frac{\epsilon_0}{2K \text{snr}}\}$  tends to zero as  $|\mathcal{N}_2|$  grows large by noting that  $L_{\pi_2(i),i}^{\Pi_2} = \tilde{L}_{\pi_2(i),i} + K_i$  due to the reciprocal property of TDD systems. From (16), (18), and (19), it is obvious that if  $|\mathcal{N}_1|$  and  $|\mathcal{N}_2|$  scale faster than  $\text{snr}^{2K-2}$  and  $\text{snr}^{3K-2}$ , respectively, then

$$\lim_{\text{snr} \rightarrow \infty} \left( 1 - C_1 \left( \frac{\epsilon_0}{2K \text{snr}} \right)^{2K-2} \right)^{|\mathcal{N}_1| - K + 1} = 0 \quad (20)$$

$$\lim_{\text{snr} \rightarrow \infty} \left( 1 - C_2 \left( \frac{\epsilon_0}{K \text{snr}} \right)^{3K-2} \right)^{|\mathcal{N}_2| - K + 1} = 0. \quad (21)$$

Therefore,  $\mathcal{P}_{\text{OND}}$  asymptotically approaches one, which means that the DoF of  $\frac{(L-1)K}{L}$  is achievable with high probability if  $N = |\mathcal{N}_1| + |\mathcal{N}_2| = \omega(\text{snr}^{3K-2})$ . This completes the proof of the theorem.  $\square$

Note that the lower bound on the DoF asymptotically approaches  $K$  for large  $L$ , which implies that our system operates in virtual full-duplex mode. The parameter  $N$  required to obtain full DoF (i.e.,  $K$  DoF) needs to increase exponentially with the number of S-D pairs,  $K$ , in order to make the sum of  $3K - 2$  interference terms in the TIL metric (4) non-increase with increasing SNR at each relay.<sup>8</sup> Here, from the perspective of each relay in  $\Pi_2$ , the SNR exponent  $3K - 2$  indicates the total number of interference links and stems from the following three factors: the sum of interference power received from other sources, the sum of interference power generated to other destinations, and the sum of inter-relay interference power generated from the relays in  $\Pi_1$ . From Theorem 1, let us provide the following interesting discussions regarding the DoF achievability.

**Remark 3.**  $K$  DoF can be achieved by using the proposed OND scheme in the  $K \times N \times K$  channel with interfering links among relay nodes, if the number of relay nodes,  $N$ , scales faster than  $\text{snr}^{3K-2}$  and the number of transmission slots in one block,  $L$ , is sufficiently large. In this case, all the interference signals are almost nulled out at each selected relay by exploiting the multiuser diversity gain. In other words, by applying the OND scheme to the interference-limited  $K \times N \times K$  channel such that the channel links are inherently coupled with each other, the links among each S-D path via one relay can be completely decoupled, thus enabling us to achieve the same DoF as in the interference-free channel case.

**Remark 4.** It is not difficult to show that the centralized relay selection method that maximizes the received SINR (at either the relay or the destination) using global CSI at the transmitter, which is a combinatorial problem with exponential complexity, gives the same relay scaling result  $N = \omega(\text{snr}^{3K-2})$  along with full DoF. However, even with our OND scheme using a decentralized relay selection based only on local CSI, the same achievability result is obtained, thus resulting in a much easier implementation.

### 3.3 The TIL Decaying Rate

In this section, we analyze the TIL decaying rate under the OND scheme with alternate relaying, which is meaningful since the desired relay scaling law is closely related to the TIL decaying rate with respect to  $N$  for given SNR.

Let  $L_{K^{\text{th}}-\text{min}}$  denote the  $K$ th smallest TIL among the ones that  $N$  selected relay nodes compute. Since the  $K$  relays yielding the TIL values up to the  $K$ th smallest one are selected, the  $K$ th smallest TIL is the largest among the TILs

<sup>8</sup> Since  $\text{snr}^{2K-2}$  scales slower than  $\text{snr}^{3K-2}$ , it does not affect the performance in terms of DoF and relay scaling laws.



that the selected relays compute. Similarly as in [36], by Markov's inequality, a lower bound on the average decaying rate of  $L_{K^{\text{th}}-\text{min}}$  with respect to  $N$ ,  $\mathbb{E}\left[\frac{1}{L_{K^{\text{th}}-\text{min}}}\right]$ , is given by

$$\mathbb{E}\left[\frac{1}{L_{K^{\text{th}}-\text{min}}}\right] \geq \frac{1}{\epsilon} \Pr(L_{K^{\text{th}}-\text{min}} \leq \epsilon), \quad (22)$$

where the inequality always holds for  $\epsilon > 0$ . We denote  $\mathcal{P}_K(\epsilon)$  as the probability that there are only  $K$  relays satisfying  $\text{TIL} \leq \epsilon$ , which is expressed as

$$\mathcal{P}_K(\epsilon) = \binom{N}{K} \mathcal{F}_L(\epsilon)^K (1 - \mathcal{F}_L(\epsilon))^{N-K}, \quad (23)$$

where  $\mathcal{F}_L(\epsilon)$  is the CDF of the TIL. Since  $\Pr(L_{K^{\text{th}}-\text{min}} \leq \epsilon)$  is lower-bounded by  $\Pr(L_{K^{\text{th}}-\text{min}} \leq \epsilon) \geq \mathcal{P}_K(\epsilon)$ , a lower bound on the average TIL decaying rate is given by

$$\mathbb{E}\left[\frac{1}{L_{K^{\text{th}}-\text{min}}}\right] \geq \frac{1}{\epsilon} \mathcal{P}_K(\epsilon). \quad (24)$$

The next step is to find the parameter  $\hat{\epsilon}$  that maximizes  $\mathcal{P}_K(\epsilon)$  in terms of  $\epsilon$  in order to provide the tightest lower bound.

**Lemma 2.** *When a constant  $\hat{\epsilon}$  satisfies the condition  $\mathcal{F}_L(\hat{\epsilon}) = K/N$ ,  $\mathcal{P}_K(\hat{\epsilon})$  in (23) is maximized for a given  $N$ .*

**Proof.** To find the parameter  $\hat{\epsilon}$  that maximizes  $\mathcal{P}_K(\epsilon)$ , we take the first derivative with respect to  $\epsilon$ , resulting in

$$\begin{aligned} & \frac{\partial \mathcal{P}_K(\epsilon)}{\partial \epsilon} \\ &= \frac{\partial \mathcal{F}_T(\epsilon)}{\partial \epsilon} \binom{N}{K} \mathcal{F}_T(\epsilon)^{K-1} (1 - \mathcal{F}_T(\epsilon))^{N-K-1} (K - N \mathcal{F}_T(\epsilon)), \end{aligned}$$

which is zero when

$$\hat{\epsilon} = \mathcal{F}_T^{-1}\left(\frac{K}{N}\right). \quad (25)$$

The parameter  $\hat{\epsilon}$  is the unique value that maximizes  $\mathcal{P}_K(\epsilon)$  since

$$\begin{aligned} \left. \frac{\partial \mathcal{P}_K(\epsilon)}{\partial \epsilon} \right|_{\epsilon} &> 0 \text{ if } 0 < \epsilon < \hat{\epsilon} \\ \left. \frac{\partial \mathcal{P}_K(\epsilon)}{\partial \epsilon} \right|_{\epsilon} &> 0 \text{ if } \epsilon \geq \hat{\epsilon}, \end{aligned}$$

which completes the proof of the lemma.  $\square$

Now, we establish our second main theorem, which shows a lower bound on the TIL decaying rate with respect to  $N$ .

**Theorem 2.** *Suppose that the OND scheme with alternate relaying is used for the  $K \times N \times K$  channel with interfering relays. Then, the decaying rate of TIL is lower-bounded by*

$$\mathbb{E}\left[\frac{1}{L_{K^{\text{th}}-\text{min}}}\right] \geq \Theta\left(N^{\frac{1}{3K-2}}\right). \quad (26)$$

**Proof.** As shown in (24), the TIL decaying rate is lower-bounded by the maximum of  $\frac{1}{\epsilon} \mathcal{P}_K(\epsilon)$  over  $\epsilon$ . By Lemma 2,  $\mathcal{P}(\hat{\epsilon})$  is maximized when  $\hat{\epsilon} = \mathcal{F}_L^{-1}\left(\frac{K}{N}\right)$ . Thus, we have

$$\begin{aligned} \mathbb{E}\left[\frac{1}{L_{K^{\text{th}}-\text{min}}}\right] &\geq \frac{1}{\mathcal{F}_L^{-1}(K/N)} \binom{N}{K} \left(\frac{K}{N}\right)^K \left(1 - \frac{K}{N}\right)^{N-K} \\ &\geq \frac{1}{\mathcal{F}_L^{-1}(K/N)} \left(\frac{N-K+1}{N}\right)^K \left(1 - \frac{K}{N}\right)^{N-K} \\ &\geq \frac{1}{\mathcal{F}_L^{-1}(K/N)} \left(\frac{1}{K}\right)^K e^{-K} \\ &\geq \Theta\left(N^{\frac{1}{3K-2}}\right), \end{aligned}$$

where the second and third inequalities hold since  $\binom{N}{K} \geq \left(\frac{N-K+1}{K}\right)^K$  and  $\left(1 - \frac{K}{N}\right)^{N-K} \geq \left(1 - \frac{K}{N}\right)^N \geq e^{-K}$ , respectively. By Lemma 1, it follows that  $\frac{1}{\mathcal{F}_L^{-1}(K/N)} \geq \left(\frac{C_1 N}{K}\right)^{\frac{1}{3K-2}}$ , where  $C_1$  is given by (12). Hence, the last inequality also holds, which completes the proof of the theorem.  $\square$

From Theorem 2, the following valuable insight is provided: the smaller SNR exponent of the relay scaling law, the faster TIL decaying rate with respect to  $N$ . This asymptotic result will be verified in a finite  $N$  regime via numerical evaluation in Section 5.

### 3.4 OND Without Alternate Relaying

For comparison, the OND scheme without alternate relaying is also explained in this section.

It is worth noting that there exists a trade-off between the lower bound on the DoF and the minimum number of relays required to guarantee our achievability result by additionally introducing the OND protocol without alternate relaying. In the scheme, the first relay set  $\Pi_1$  only participates in data forwarding. That is, the second relay set  $\Pi_2$  does not need to be selected for the OND protocol without alternate relaying. Specifically, the steps of each node during one block are then described as follows:

- Time slot 1: Sources  $\mathcal{S}_1, \dots, \mathcal{S}_K$  transmit their first encoded symbols  $x_1^{(1)}(1), \dots, x_K^{(1)}(1)$ , where  $x_k^{(1)}(l)$  represents the  $l$ th transmitted symbol of the  $k$ th source node. A set of  $K$  selected relay nodes,  $\Pi_1 = \{\pi_1(1), \dots, \pi_1(K)\}$ , operating in receive mode at each odd time slot, listens to  $x_1^{(1)}(1), \dots, x_K^{(1)}(1)$ . Other  $N - K$  relay nodes and destinations  $\mathcal{D}_1, \dots, \mathcal{D}_K$  remain idle.
- Time slot 2: The  $K$  relays in the set  $\Pi_1$  forward their first re-encoded symbols  $x_{\pi_1(1)}^{(2)}(1), \dots, x_{\pi_1(K)}^{(2)}(1)$  to the corresponding  $K$  destinations. The  $K$  destinations receive from  $\mathcal{R}_{\pi_1(1)}, \dots, \mathcal{R}_{\pi_1(K)}$  and decode  $x_{\pi_1(1)}^{(2)}(1), \dots, x_{\pi_1(K)}^{(2)}(1)$ . The remaining  $N - K$  relays keep idle.
- The processes in time slots 1 and 2 are repeated to the  $(L - 1)$ th time slot.
- Time slot  $L$ : The  $K$  relays in  $\Pi_1$  forward their re-encoded symbols  $x_{\pi_1(1)}^{(2)}(L - 1), \dots, x_{\pi_1(K)}^{(2)}(L - 1)$  to

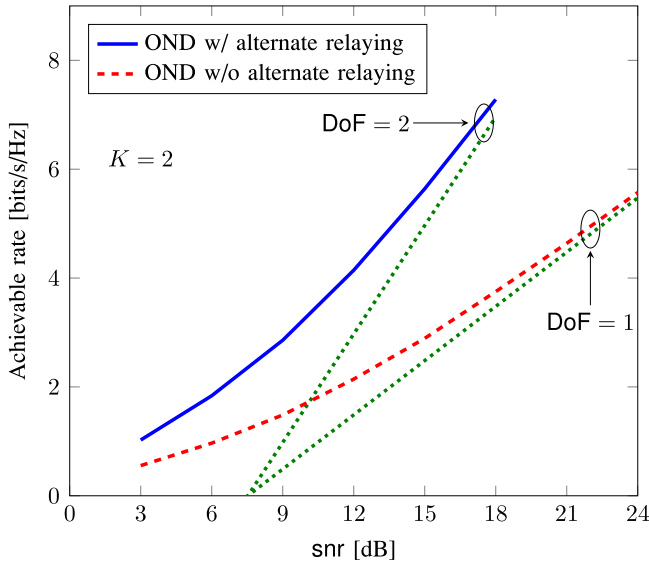


Fig. 2. The achievable sum-rates as a function of snr when  $K = 2$  and  $N = \text{snr}^4$  in the  $K \times N \times K$  channel. It is assumed that  $N = \text{snr}^4$  and  $N = \text{snr}^2$  are used for the OND schemes with and without alternate relaying, respectively.

the corresponding  $K$  destinations. The  $K$  sources and the other  $N - K$  relays remain idle.

When  $\mathcal{R}_i$  is assumed to serve the  $k$ th S-D pair  $(S_k, \mathcal{D}_k)$  ( $i \in \{1, \dots, N\}$  and  $k \in \{1, \dots, K\}$ ), it computes the scheduling metric  $\tilde{L}_{i,k}$  in (3). According to the computed  $\tilde{L}_{i,k}$ , a timer based method is used for relay selection as in Section 3.1.1. Because there is no inter-relay interference for the OND scheme without alternate relaying, it is expected that the minimum required  $N$  to achieve the optimal DoF is significantly reduced. Our third main theorem is established as follows.

**Theorem 3.** *Suppose that the OND scheme without alternate relaying is used for the  $K \times N \times K$  channel. Then, for  $L$  data transmission time slots,*

$$\text{DoF}_{\text{total}} \geq \frac{K}{2}$$

*is achievable if  $N = \omega(\text{snr}^{2K-2})$ .*

**Proof.** The detailed proof of this argument is omitted here since it basically follows the same line as the proof of Theorem 1.  $\square$

In Section 5, it will be also seen that in a finite  $N$  regime, there exists the case even where the OND without alternate relaying outperforms that of the OND with alternate relaying in terms of achievable sum-rates via computer simulations.

## 4 UPPER BOUND FOR DOF

In this section, to show the optimality of the proposed OND scheme in the  $K \times N \times K$  channel with interfering relays, which consists of  $K$  S-D pairs and  $N$  relay nodes, we derive an upper bound on the DoF using the cut-set bound [37] as a counterpart of the lower bound on the total DoF in Section 3.2. Suppose that  $\tilde{N}$  relay nodes are active, i.e., receive packets and retransmit their re-encoded ones, simultaneously,

where  $\tilde{N} \in \{1, \dots, N\}$ . This is a generalized version of our transmission since it is not characterized how many relays need to be activated simultaneously to obtain the optimal DoF. We consider the two cuts  $L_1$  and  $L_2$  dividing our network into two parts in a different manner. Let  $\mathcal{S}_{L_i}$  and  $\mathcal{D}_{L_i}$  denote the sets of sources and destinations, respectively, for the cut  $L_i$  in the network ( $i = 1, 2$ ). For the  $K \times N \times K$  channel model with interfering relays, we now use the fact that there is no direct path between an S-D pair. Then, it follows that under  $L_1$ ,  $K$  transmit nodes in  $\mathcal{S}_{L_1}$  are on the left of the network, while  $\tilde{N}$  active relay nodes and  $K$  (final) destination nodes in  $\mathcal{D}_{L_2}$  are on the right and act as receivers. In this case, we can create the  $K \times (\tilde{N} + K)$  multiple-input multiple-output channel between the two sets of nodes separated by the cut. Similarly, the  $(\tilde{N} + K) \times K$  MIMO channel are obtained under the cut  $L_2$ . It is obvious to show that DoF for the two MIMO channels is upper-bounded by  $K$ . Hence, it turns out that even with the half-duplex assumption, our lower bound on the DoF based on the OND with alternate relaying asymptotically approaches this upper bound on the DoF for large  $L$ .

Note that this upper bound is generally derived regardless of whether the number of relays,  $N$ , tends to infinity or not, whereas the scaling condition  $N = \omega(\text{snr}^{3K-2})$  is included in the achievability proof.

## 5 NUMERICAL EVALUATION

In this section, we perform computer simulations to validate the achievability result of the proposed OND scheme in Section 3 for finite parameters  $N$  and SNR in the  $K \times N \times K$  channel model with interfering relays. In our simulation, the channel coefficients in (1) and (2) are generated  $1 \times 10^5$  times for each system parameter.

Fig. 2 shows the achievable sum-rates of the  $K \times N \times K$  channel for the OND schemes with and without alternate relaying according to snr in dB scale when  $K = 2$ . Note that  $N$  is set to a different scalable value according to snr, i.e.,  $N = \text{snr}^{3K-2}$  for the OND with alternate relaying and  $N = \text{snr}^{2K-2}$  for the OND without alternate relaying, respectively, to see whether the slope of each curve follows the DoF in Theorems 1 and 2. In the figure, the dotted green lines are also plotted to indicate the first order approximation of the achievable rates with a proper bias, where the slopes are given by  $K$  and  $K/2$  for the OND schemes with and without alternate relaying, respectively.

In Fig. 3, the log-log plot of the average TIL of the OND with alternate relaying versus  $N$  is shown for the  $K \times N \times K$  channel when  $K = 2, 3$ .<sup>9</sup> It can be seen that the TIL tends to decrease linearly with  $N$ . It is further seen how many relays are required with the OND scheme with alternate relaying to guarantee that the TIL is less than a small constant for a given parameter  $K$ . In this figure, the dashed lines are also plotted from theoretical results in Theorem 2 with a proper bias to check the slope of the TIL. We can see that the TIL decaying rates are consistent with the relay scaling law condition in Theorem 1. More specifically, the TIL is

9. Even if it seems unrealistic to have a great number of relays in cooperative relay networks, the range for parameter  $N$  is taken into account to precisely see some trends of curves varying with  $N$ .

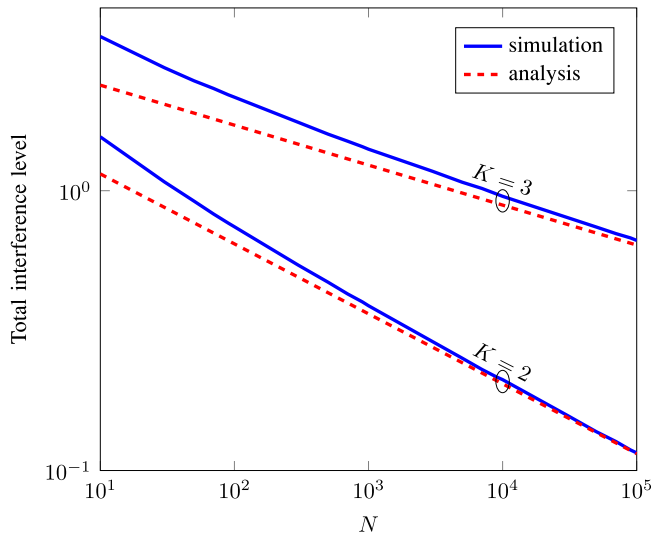


Fig. 3. The average TIL versus  $N$  when  $K = 2, 3$  in the  $K \times N \times K$  channel.

reduced as  $N$  increases with the slope of 0.25 for  $K = 2$  and 0.143 for  $K = 3$ , respectively.

Fig. 4 illustrates the achievable sum-rates of the  $K \times N \times K$  channel for the OND schemes with and without alternate relaying versus snr (in dB scale) when  $K = 2$  and  $N = 50, 100, 200$ . We can see that in a finite  $N$  regime, there exists the case where the OND without alternate relaying outperforms that of the OND with alternate relaying. This is because for finite  $N$ , the achievable sum-rates for the alternate relaying case tend to approach a floor with increasing SNR faster than no alternate relaying case due to more residual interference in each dimension. We can also see that the crossing points slightly move to the right as  $N$  increases; this is due to the fact that our OND scheme with alternate relaying always benefits from having more relays for selection, thus resulting in more multiuser diversity gain. This highly motivates us to operate our system in a switched fashion when the relay selection scheme is chosen

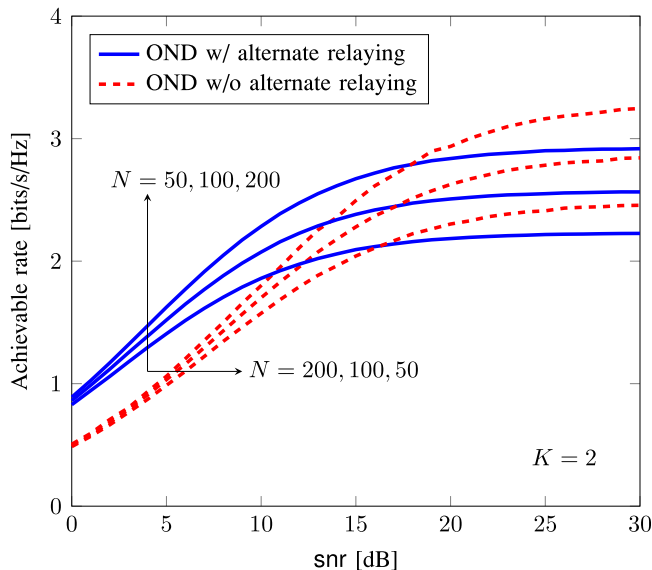


Fig. 4. The achievable sum-rates versus snr when  $K = 2$  and  $N = 100, 200$  in the  $K \times N \times K$  channel. Both OND schemes with and without alternate relaying are compared.

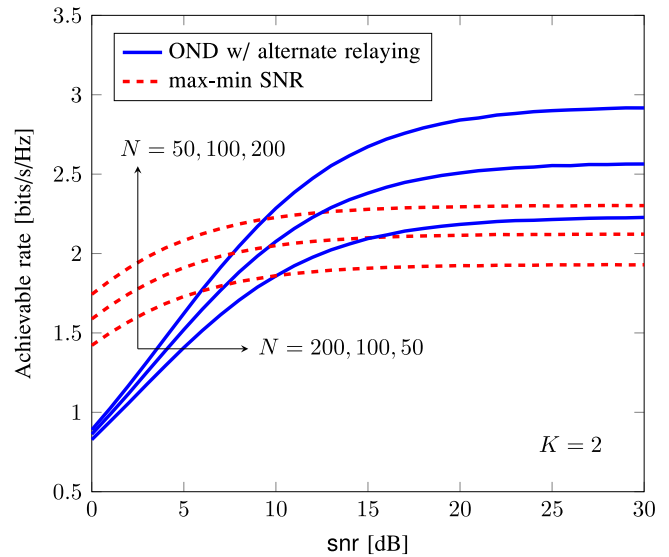


Fig. 5. The achievable sum-rates versus snr when  $K = 2$  and  $N = 100, 200$  in the  $K \times N \times K$  channel. The OND scheme with alternate relaying and the max-min SNR scheme are compared.

between the OND schemes with and without alternate relaying depending on the operating regime of our system.

To further ascertain the efficacy of our scheme, a performance comparison is performed with a baseline scheduling. Specifically, in the *max-min SNR* scheme, each S-D pair selects one relay  $\mathcal{R}_i$  ( $i \in \{1, \dots, N\}$ ) such that the minimum out of the desired channel gains of two communication links (either from  $\mathcal{S}_k$  to  $\mathcal{R}_i$  or from  $\mathcal{R}_i$  to  $\mathcal{D}_k$ ) becomes the maximum among the associated minimum channel gains over all the unselected relays. This max-min SNR scheme is well-suited for relay-aided systems if interfering links are absent. The achievable sum-rates are illustrated in Fig. 5 according to snr (in dB scale) when  $K = 2$  and  $N = 100, 200$ . We can see that our OND scheme with alternate relaying outperforms this baseline scheme beyond a certain low SNR point. We also see that the rate gaps increase when  $N$  increases in the high SNR regime. On the other hand, for fixed  $N$ , the sum-rates of the max-min scheme are slightly changed with respect to snr due to more residual interference in each dimension.

## 6 CONCLUDING REMARKS

An efficient distributed OND protocol operating in virtual full-duplex mode was proposed for the  $K \times N \times K$  channel with interfering relays, referred to as one of multi-source interfering relay networks. A novel relay scheduling strategy with alternate half-duplex relaying was presented in two-hop environments, where a subset of relays is opportunistically selected in terms of producing the minimum total interference level, thereby resulting in network decoupling. It was shown that the OND protocol asymptotically achieves full DoF even in the presence of inter-relay interference and half-duplex assumption, provided that the number of relays,  $N$ , scales faster than  $\text{snr}^{3K-2}$ . Numerical evaluation was also shown to verify that our scheme outperforms the other relay selection methods under realistic network conditions (e.g., finite  $N$  and SNR) with respect to sum-rates.

Suggestions for future research in this area include the extension to the MIMO  $K \times N \times K$  channel and the optimal design of joint beamforming and scheduling under the MIMO model.

## ACKNOWLEDGMENTS

This work was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2014R1A1A2054577) and by the Ministry of Science, ICT & Future Planning (MSIP) (2015R1A2A1A15054248). The material in this paper was presented in part at the IEEE International Conference on Communications, Sydney, Australia, June 2014 [39]. Hyun Jong Yang is the corresponding author. This research was conducted while V.V. Mai was with Dankook University, Yongin 448-701, Republic of Korea.

## REFERENCES

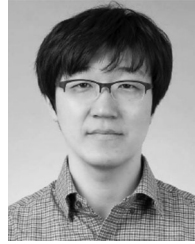
- [1] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, "Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3457–3470, Aug. 2008.
- [2] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the  $K$ -user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.
- [3] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3309–3322, Jun. 2011.
- [4] T. Gou and S. A. Jafar, "Degrees of freedom of the  $K$ -user  $M \times N$  MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 12, pp. 6040–6057, Dec. 2010.
- [5] S. A. Jafar and S. Shamai(Shitz), "Degrees of freedom region of the MIMO X channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 151–170, Jan. 2008.
- [6] C. Suh and D. Tse, "Interference alignment for cellular networks," in *Proc. 46th Annu. Allerton Conf. Commun. Control Comput.*, Sep. 2008, pp. 1037–1044.
- [7] A. S. Motahari, O. Gharan, M.-A. Maddah-Ali, and A. K. Khandani, "Real interference alignment: Exploiting the potential of single antenna systems," *IEEE Trans. Inf. Theory*, vol. 60, no. 8, pp. 4799–4810, Aug. 2014.
- [8] B. C. Jung and W.-Y. Shin, "Opportunistic interference alignment for interference-limited cellular TDD uplink," *IEEE Commun. Lett.*, vol. 15, no. 2, pp. 148–150, Feb. 2011.
- [9] B. C. Jung, D. Park, and W.-Y. Shin, "Opportunistic interference mitigation achieves optimal degrees-of-freedom in wireless multicell uplink networks," *IEEE Trans. Commun.*, vol. 60, no. 7, pp. 1935–1944, Jul. 2012.
- [10] T. Gou, S. A. Jafar, C. Wang, S.-W. Jeon, and S.-Y. Chung, "Aligned interference neutralization and the degrees of freedom of the  $2 \times 2 \times 2$  interference channel," *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4381–4395, Jul. 2012.
- [11] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 379–389, Feb. 2007.
- [12] I. Shomorony and A. S. Avestimehr, "Degrees of freedom of two-hop wireless networks: 'Everyone gets the entire cake'," in *Proc. 50th Annu. Allerton Conf. Commun. Control Comput.*, Oct. 2012, pp. 1935–1942.
- [13] T. Gou, C. Wang, and S. A. Jafar, "Aligned interference neutralization and the degrees of freedom of the  $2 \times 2 \times 2$  interference channel with interfering relays," in *Proc. 49th Annu. Allerton Conf. Commun. Control Comput.*, Sep. 2011, pp. 1041–1047.
- [14] T. Gou, C. Wang, and S. A. Jafar, "Degrees of freedom of a class of non-layered two unicast wireless networks," in *Proc. 45th Asilomar Conf. Signals Syst. Comput.*, Nov. 2011, pp. 1707–1711.
- [15] R. Knopp and P. Humblet, "Information capacity and power control in single cell multiuser communications," in *Proc. IEEE Int. Conf. Commun.*, Jun. 1995, pp. 331–335.
- [16] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, Aug. 2002.
- [17] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial side information," *IEEE Trans. Inf. Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.
- [18] H. J. Yang, W.-Y. Shin, B. C. Jung, and A. Paulraj, "Opportunistic interference alignment for MIMO interfering multiple access channels," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2180–2192, May 2013.
- [19] H. J. Yang, B. C. Jung, W.-Y. Shin, and A. Paulraj, "Codebook-based opportunistic interference alignment," *IEEE Trans. Signal Process.*, vol. 62, no. 11, pp. 2922–2937, Jun. 2014.
- [20] H. J. Yang, W.-Y. Shin, B. C. Jung, C. Suh, and A. Paulraj, "Opportunistic downlink interference alignment," in *Proc. IEEE Int. Symp. Inf. Theory*, Jun./Jul. 2014, pp. 1588–1592.
- [21] S. Cui, A. M. Haimovich, O. Somekh, and H. V. Poor, "Opportunistic relaying in wireless networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 11, pp. 5121–5137, Nov. 2009.
- [22] W.-Y. Shin, S.-Y. Chung, and Y. H. Lee, "Parallel opportunistic routing in wireless networks," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6290–6300, Oct. 2013.
- [23] H. J. Yang, W.-Y. Shin, and B. C. Jung, "On the degrees of freedom of the large-scale interfering two-way relay network," *IEEE Trans. Veh. Technol.*, vol. 65, no. 11, pp. 9442–9450, Nov. 2016.
- [24] C. Shen and M. P. Fitz, "Opportunistic spatial orthogonalization and its application to fading cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 5, no. 1, pp. 182–189, Feb. 2011.
- [25] T. W. Ban, W. Choi, B. C. Jung, and D. K. Sung, "Multi-user diversity in a spectrum sharing system," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 102–106, Jan. 2009.
- [26] B. Nazer, M. Gastpar, S. A. Jafar, and P. Viswanath, "Ergodic interference alignment," in *Proc. IEEE Int. Symp. Inf. Theory*, Jun./Jul. 2009, pp. 1769–1773.
- [27] S.-W. Jeon and S.-Y. Chung, "Capacity of a class of linear binary field multisource relay networks," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6405–6420, Oct. 2013.
- [28] R. Zhang and Y. C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 88–102, Feb. 2008.
- [29] Y. Fan, C. Wang, J. Thompson, and H. V. Poor, "Recovering multiplexing loss through successive relaying using repetition coding," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp. 4484–4493, Dec. 2007.
- [30] F. Xue and S. Sandhu, "Cooperation in a half-duplex Gaussian diamond relay channel," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3806–3814, Oct. 2007.
- [31] R. Zhang, "Characterizing achievable rates for two-path digital relaying," in *Proc. IEEE Int. Conf. Commun.*, May 2008, pp. 1113–1117.
- [32] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.
- [33] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3450–3460, Sep. 2007.
- [34] D. E. Knuth, "Big Omicron and big Omega and big Theta," *ACM SIGACT News*, vol. 8, pp. 18–24, Apr.–Jun. 1976.
- [35] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. San Diego, CA, USA: Academic, 2000.
- [36] J. Jose, S. Subramanian, X. Wu, and J. Li, "Opportunistic interference alignment in cellular downlink," in *Proc. 50th Annu. Allerton Conf. Commun. Control Comput.*, Oct. 2012, pp. 1529–1534.
- [37] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, New York, NY, USA: Wiley, 1991.
- [38] Z. Lin and Z. Bai, *Probability Inequalities*, New York, NY, USA: Springer, 2011.
- [39] W.-Y. Shin, H. J. Yang, and B. C. Jung, "Opportunistic network decoupling in multi-source interfering relay networks," in *Proc. IEEE Conf. Commun.*, Jun. 2014, pp. 2671–2676.



**Won-Yong Shin** (S'02-M'08-SM'16) received the BS degree in electrical engineering from Yonsei University, Seoul, Korea, in 2002, and the MS and PhD degrees in electrical engineering and computer science from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 2004 and 2008, respectively. From February 2008 to April 2008, he was a visiting scholar in the School of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts. From September 2008 to April 2009, he was in the Brain Korea Institute and CHiPS, KAIST, as a postdoctoral fellow. From August 2008 to April 2009, he was with Lumatic Inc., Daejeon, Korea, as a visiting researcher. In May 2009, he joined Harvard University as a postdoctoral fellow and was promoted to a research associate in October 2011. Since March 2012, he has been in the Department of Computer Science and Engineering, Dankook University, Yongin, Korea, where he is currently an assistant professor (*early tenure*). His research interests include the areas of information theory, communications, signal processing, mobile computing, big data analytics, and online social networks analysis. He has served as an associate editor of the *IEICE Transactions on Fundamentals of Electronics, Communications, Computer Sciences*, the *IEIE Transactions on Smart Processing & Computing*, and the *Journal of Korea Information and Communications Society*. He also served as an organizing committee member of the 2015 IEEE Information Theory Workshop. He received the Bronze Prize of Samsung Humantech Paper Contest (2008) and the KICS Haedong Young Scholar Award (2016). He is a senior member of the IEEE.



**Vien V. Mai** received the BE degree in electronic and telecommunication from the Posts & Telecommunications Institute of Technology, Ho Chi Minh, Viet Nam, in 2013, and the MS degree in electronics and radio engineering from Kyung Hee University, Yongin, Korea, in 2015. He is currently working toward the PhD degree at the KTH Royal Institute of Technology, Stockholm, Sweden. From September 2015 to July 2016, he was in the Communications and Networking Laboratory, Dankook University, Yongin, Korea, as a researcher. Since August 2016, he has been with the KTH Royal Institute of Technology, Stockholm, Sweden. His research interests include wireless communication, information theory, massive MIMO, distributed detection and estimation, and energy harvesting.



**Bang Chul Jung** (S'02-M'08-SM'14) received the BS degree in electronics engineering from Ajou University, Suwon, Korea, in 2002, and the MS and PhD degrees in electrical & computer engineering from KAIST, Daejeon, Korea, in 2004 and 2008, respectively. He was a senior researcher/research professor with the KAIST Institute for Information Technology Convergence, Daejeon, Korea, from January 2009 to February 2010. From March 2010 to August 2015, he was a faculty of Gyeongsang National University, Tongyeong, Korea. He is currently an associate professor in the Department of Electronics Engineering, Chungnam National University, Daejeon, Korea. His research interests include 5G mobile communication systems, statistical signal processing, opportunistic communications, compressed sensing, interference management, interference alignment, random access, relaying techniques, device-to-device networks, in-network computation, and network coding. He received the 5th IEEE Communication Society Asia-Pacific Outstanding Young Researcher Award in 2011. He also received the Bronze Prize of Intel Student Paper Contest in 2005, the First Prize of KAIST's Invention Idea Contest in 2008, the Bronze Prize of Samsung Humantech Paper Contest in 2009, the Outstanding Research Award of Institute of Marine Industry in Gyeongsang National University in 2013, the Gaechuck Award for Excellence in Teaching of Gyeongsang National University in 2014, and the Outstanding Paper Award in Spring Conference of the Korea Institute of Information and Communication Engineering in 2015. He is a senior member of the IEEE.



**Hyun Jong Yang** (M'12) received the BS degree in electrical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 2004, and the MS and PhD degrees in electrical engineering from KAIST, in 2006 and 2010, respectively. From Aug. 2010 to Aug. 2011, he was a research fellow with the Korea Institute Ocean Science Technology, Daejeon, Korea. From Oct. 2011 to Oct. 2012, he worked as a post-doctoral researcher in the Electrical Engineering Department, Stanford University, Stanford, California. From Oct. 2012 to Aug. 2013, he was a Staff II systems design engineer with Broadcom Corporation, Sunnyvale, California, where he developed physical-layer algorithms for LTE-A MIMO receivers. In addition, he was a delegate of Broadcom in 3GPP standard meetings for RAN1 Rel-12 technologies. Since Aug. 2013, he has been an assistant professor in the School of Electrical and Computer Engineering, UNIST, Ulsan, Korea. His fields of interests include algorithms and theory for wireless communication and their applications and implementation. He is a member of the IEEE.

▷ For more information on this or any other computing topic, please visit our Digital Library at [www.computer.org/publications/dlib](http://www.computer.org/publications/dlib).