Performance Analysis of NOMA Random Access

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***Abstract*—This letter applies non-orthogonal multiple access to uplink random access systems, where the transmit power from each user equipment (UE) is received at the base station as one of L target (received) powers. More precisely, if a backlogged UE's channel gain is greater than a threshold, it randomly aims at one of the target powers with channel inversion. Then, the BS decodes the received signals with successive interference cancellation. This letter investigates the performance of this system in terms of throughput, access delay, energy efficiency, and bistability.**

***Index Terms*—Non-orthogonal multiple access, bistability.**

I. Introduction

In DOWNLINK power-domain non-orthogonal multiple access (NOMA) [1], the base station (BS) transmits signals to multiple users at a single resource (time and frequency) by allocating a different power to each signal. Upon reception of it, each user equipment (UE) separates its own signal from the superimposed signal with successive interference cancellation (SIC) so that spectral efficiency (SE) can be improved. Due to such a high SE of NOMA, it has received explosive research interests as a promising candidate for the 5th generation (5G) mobile networks [2]. In downlink NOMA, the BS should estimate the channel gain of the UEs to allocate a proper transmit power to each signal, while the UEs need to estimate the BS's channel gain from the downlink reference signal in order to apply SIC to the received signal.

In expectation for enjoying high SE of downlink NOMA, it has been considered for uplink [3]–[7]. Outage probability was investigated for the system with two UEs in [3], and three UEs in [4], respectively, while the mean rate coverage probability was analyzed in [5] for the system with a large number of UEs distributed over the BS according to a Poisson cluster process. Note that the superimposed signal in downlink NOMA experiences the same fading and path-loss *collectively* until it reaches UEs so that the order of allocated powers are preserved. However, the packets transmitted over the uplink by UEs undergo different fading and path-loss, and arrive at the BS. Thus, without *channel inversion* [9], the order of the received powers of the signals is random, which is susceptible to outage. In [6] and [7], the channel inversion is thus utilized when UEs transmit their packet *based on slotted ALOHA* (S-ALOHA). Then, the packets can arrive at the BS with some predetermined distinct power levels, which facilitates the BS's decoding by SIC without an outage. Particularly the system with two levels of the target (received) powers is considered in [6], while a lower bound of throughput is examined for the system with multilevel target powers and multichannels in [7]. Compared to [6] and [7], this letter considers the system with multilevel target powers by taking into account that the channel of UEs experiences Rayleigh fading and pathloss. We obtain a significantly improved lower bound of throughput (packets/slot) of this NOMA random access compared to [7]. We further obtain the average access delay (slots) and consider the system stability, which are not considered in [7]. Additionally, we examine energy efficiency (EE) (packets/slot/joule) tigher than [7], and an optimal retransmission probability.

Manuscript received July 19, 2018; revised August 10, 2018; accepted August 17, 2018. Date of publication August 21, 2018; date of current version November 12, 2018. This work was supported in part by DST grant no. RP03514G, in part by IITP grant funded by the Korea government (MSIT) (No. 2015-0-00278, Research on Near-Zero Latency Network for 5G Immersive Service), and in part by the NRF grant funded by the Korea government (MSIT)(NRF-2017K1A3A1A19071179). The associate editor coordinating the review of this paper and approving it for publication was A. El Shafie.

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Digital Object Identifier 10.1109/LCOMM.2018.2866376

II. System Model

In NOMA random access systems of our interest, a BS is at the center of a circular coverage area with radius R (m) and a total of M UEs share the wireless uplink. Time is divided into slots of a constant size, and the length of each slot is equal to one packet transmission time, whereas time division duplex (TDD) is used to facilitate uplink channel estimation. We assume that at each slot the probability of a new packet arrival to a UE is σ and that each UE can hold only one packet. Once a UE has a packet to transmit, which is called *backlogged*, it will transmit the packet with probability r in the next slot based on Bernoulli trial.

Before transmitting the packet, each UE measures its channel gain, denoted by Y, through the downlink reference signal. With channel reciprocity of TDD, we assume that the uplink and downlink channel gains would not change rapidly, further remain identical for a short period of time in transmitting a packet. Using the channel inversion [9], the UE then takes its transmission power P_i/Y for $i \in \{1, 2, \ldots, L\}$ and $P_1 > P_2 > \cdots > P_L$ such that P_i can be the received power at the BS. Note that the UE selects P_i randomly, and we call P_i a target received power at the BS. The BS can successfully decode the received packets if the following three conditions are met: First, the packet with P_i can be successfully decoded if P_j's for $j < i$ are all successfully decoded. Secondly, it is the only packet that targets at P_i. If more than one packet with P_i are transmitted, all of them cannot be decoded, which is called *power collision*. Along with the first condition, upon this power collision the BS cannot decode all the packet with P_j for $j > i$. Thirdly, regarding the packets with P_j for $j > i$ (if they are transmitted) as interference, the BS decodes the packet with P_i if signal-to-interference noise ratio (SINR) satisfies

$$\frac{P_i}{\sum_{k=1}^{i-1}\epsilon_k+\sum_{j=i+1}^{L}P_j+N_0}\geq\Gamma, \tag{1}$$

where N_0 and Γ denote noise power and SINR threshold for the successful decoding, respectively. Particularly, ϵ_k indicates the residual power (or interference) of the packet with P_k for $k<i$ due to imperfect SIC, e.g., imperfect channel estimation of the BS. It is notable that such residual power largely depends on the number of levels L and the specific SIC technique adopted. While ϵ_k could be random in practice, we assume that there exists a small ϵ that satisfies $\Pr[\max(\epsilon_1,\cdots,\epsilon_L)\leq\epsilon]\approx 0$ which can be achieved by designing a good SIC technique. Additionally, if $N_0=1$ in (1), the minimum power level P_i of satisfying (1) for $i=\{1,2,\ldots,L\}$, i.e., replacing the inequality with the equality in (1), is obtained as $P_i=\epsilon\sum_{k=1}^{L+1-i}\Gamma^k(i+k-2)+\Gamma(1+\Gamma)^{L-i}$. It can be seen that P_i should increase to compensate for ϵ. As we focus on the effect of power collision and the design of SIC techniques is beyond the scope of this letter, we assume that such compensation is already taken into account for P_i. Additionally, we assume that UEs place the preamble in a packet in order for the BS to estimate channel gain, whereas the location of the preamble is different according to P_i. The BS notifies the outcome of a packet transmission, i.e., success, just before the beginning of the next slot. The UEs that do not receive the feedback shall retransmit their packet with probability r based on Bernoulli trial in each slot.

III. Analysis

Let us assume that the system has $M-m$ non-backlogged UEs at slot t. Since they shall have a packet to transmit with probability σ at each slot, the average input rate to the system $S_{in}(m)$ is expressed as

$$S_{in}(m)=\sum_{k=0}^{M-m}k\mathcal{B}_{M-m}^{k}(\sigma)=(M-m)\sigma, \tag{2}$$

where $\mathcal{B}_N^n(x)$ denotes a binomial distribution, i.e., $\mathcal{B}_N^n(x)=\binom{N}{n}x^n(1-x)^{N-n}$. Let us consider the mean number of UEs successfully transmitting their packet per slot, denoted by $S_{out}(m)$, if the system has m backlogged UEs at slot t. When the mean number of backlogged UEs is denoted by m^* *in the steady-state*, i.e., $t\to\infty$, m^* satisfies the following:

$$S_{in}(m^*)=S_{out}(m^*). \tag{3}$$

As the solution of (3), we take an integer m^* of minimizing $|S_{in}(m^*)-S_{out}(m^*)|$, since m^* is an argument of a binomial distribution, e.g., $\mathcal{B}_{m^*}^{k}(x)$. Let τ and $\overline{D}$ denote the system throughput and the average access delay, respectively. Once m^* is obtained, we get

$$\tau=S_{out}(m^*)\quad\text{and}\;\overline{D}=\frac{m^*}{S_{out}(m^*)}+1, \tag{4}$$

where Little's result is used to obtain $\overline{D}$ and one slot is added to $\overline{D}$ in order to explain one slot for a packet transmission. To apply Little's result, m^* and $S_{out}(m^*)$ can be interpreted as the number of users in a queuing system and the mean rate for each user to leave (or arrive) in the system.

Now we need to find $S_{out}(m)$. To do this, $p_{k,n}$ denotes the probability that k packets are successfully decoded when n packets are simultaneously transmitted. If each UE retransmits its packet with probability r, $S_{out}(m)$ is expressed as

$$S_{out}(m)=\sum_{n=1}^{m}\sum_{k=1}^{n}kp_{k,n}\mathcal{B}_m^n(r)=\sum_{n=1}^{m}\tau_n\mathcal{B}_m^n(r), \tag{5}$$

where τ_n is the conditional throughput, i.e., the mean number of packets decoded successfully when n UEs have transmitted. In [7], $p_{k,n}$ for $k=n$ is given as

$$p_{n,n}=\prod_{i=1}^{n-1}\left(1-\frac{i}{L}\right)\quad\text{for } n\leq L. \tag{6}$$

Notice that (6) is the probability that all the target received powers chosen by n UEs are different. In [7], by assuming that $p_{k,n}\approx 0$ for $k<n$ in (5), a lower bound of throughput is obtained. For $L=2$, the lower bound is exact. However, as we shall see in Section IV, as L increases, τ_n based only on (6) becomes incorrect. In order to improve it for the system with a large L, we need to consider the effect of $p_{k,n}$ which is nonzero for $k<n$ in general, i.e., the cases that a packet with a high target power is successfully decoded as long as (1) is satisfied, even if there are some packets with a lower target power. To this end, let us define $q_k(u)$ as the probability that among u possible power levels, k different power levels are selected by k UEs when there are a total of L power levels. For $1\leq k\leq u\leq L$, this can be expressed as $q_k(u)=\frac{u}{L}\cdot\frac{u-1}{L}\cdots\frac{u-k+1}{L}=\frac{u!}{L^k(u-k)!}$, where $\frac{u}{L}$ is the probability that a UE selects one of u possible power levels. For later notational convenience, we also define $q_0(0)=1$. For $1\leq k<n\leq L$, $p_{k,n}$ is then bounded as

$$p_{k,n}\geq\sum_{i=0}^{L-n}\left\{\binom{n}{k}q_k(k+i)\cdot\sum_{l=2}^{n-k}\left[\binom{n-k}{l}\left(\frac{1}{L}\right)^l\right.\right.$$
$$\left.\left.\times\, q_{n-k-l}(L-k-i-l)\right]\right\}, \tag{7}$$

which can be read in three steps. *First*, $\binom{n}{k}q_k(k+i)$ denotes the probability that k UEs select k different power levels out of the first $k+i$ power levels, i.e., $P_1,P_2,\ldots,P_{k+i}$. Hence, i power levels are not occupied. If the remaining $n-k$ UEs satisfy (1), the k UEs would make a successful transmission. *Second*, $\binom{n-k}{l}\left(\frac{1}{L}\right)^l$ is the probability that among the remaining $n-k$ UEs, exactly l UEs select the $(k+i+l)$-th power level. In this case, they have a power collision. Notice that as $l\cdot P_{k+i+l}\leq P_{k+i+1}+P_{k+i+2}+\cdots+P_{k+i+l}$, those l UEs still satisfy (1) for the successful transmissions of the first k UEs considered. *Finally*, since the l UEs have a power collision, the remaining $n-k-l$ UEs cannot succeed in transmissions. However, in order to ensure successful transmission of the first k UEs, the remaining $n-k-l$ UEs cannot choose the remaining $L-k-i-l$ power levels arbitrarily. For this, we consider $q_{n-k-l}(L-k-i-l)$, i.e., the probability that the remaining $n-k-l$ UEs select $n-k-l$ different power levels among the remaining $L-k-i-l$ power levels. Since the right hand side of (7) considers the partial cases when k out of n UEs succeed in transmissions for $n\leq L$, we have the inequality.

From τ_n, we can get an optimal retransmission probability given m backlogged UEs. For $L = 2, 3, 4$, the maximum of the conditional throughput τ_n is found at $n = 2$, while it is found at $n = 3$ for $L = 5$. Then, an optimal retransmission probability for backlogged UEs to take can be set as $r = \min(2/m, 1)$ for $L = 2, 3, 4$ and $r = \min(3/m, 1)$ for $L = 5$, where m is the number of backlogged UEs to be estimated every slot.

Let us consider EE. It is defined as a ratio of throughput to the average transmission power (or consumed energy) by all transmitting backlogged UEs. We assume that the channel gain Y consists of small-scale fading and pathloss, i.e., $Y = h{\cdot}x^{-\alpha}$, where h is an exponential random variable with unit mean, i.e., Rayleigh fading, and x is the distance between a UE and the BS with pathloss exponent α. Especially, we assume that UEs transmit a packet with P_i randomly targeted if $Y \geq \theta$, in which θ is the outage threshold. In (5), the retransmission probability r is expressed as $r = \mu \Pr[Y \geq \theta] = \mu(1 - F_Y(\theta))$, where μ is a control probability, and $F_Y(y) \triangleq \Pr\left[hr^{-\alpha} \leq y\right]$ is the cumulative distribution function (CDF) of Y. If $f_Y(y) = \frac{dF_Y(y)}{dy}$, the average transmission power for UEs to spend when their channel gain exceeds the threshold $P\overline{P}_L$ is obtained as $\overline{P}_L = \frac{1}{L}\sum_{i=1}^{L} P_i \int_\theta^\infty \frac{1}{y} f_Y(y)dy = \frac{(1+\Gamma)^L - 1}{L}\mathcal{Y}(\theta)$, where we have considered $\epsilon = 0$, and $\mathcal{Y}(\theta)$ is called a minimal transmit power for simplicity in this letter. Then, the EE is obtained as $\overline{E} = \tau/(\overline{P}_L \cdot \mu \cdot m^*)$, where since the probability that the channel gain is greater than the threshold is considered in calculating $\overline{P}_L$, we only need to multiply the average number of backlogged UEs m^* by the control probability. Thus, the denominator expresses the average transmission powers for backlogged UE to spend. Now to find $F_Y(y)$, from its definition, we can write

$$F_Y(y) = \int_0^R \left(1 - e^{-yx^\alpha}\right) f_R(x)dx, \tag{8}$$

where $f_R(x)$ is the pdf that a UE is *uniformly located in the BS of radius* R, that is, $f_R(x) = \frac{2x}{R^2}$ for $0 \leq x \leq R$. With some manipulations on (8), we obtain

$$F_Y(y) = \begin{cases} 1 + \dfrac{1}{yR^2}(e^{-yR^2} - 1), & \text{if } \alpha = 2, \\ 1 - \dfrac{2}{3(Ry^{\frac{1}{3}})^2}\gamma\left(\dfrac{2}{3}, yR^3\right), & \text{if } \alpha = 3, \\ 1 - \dfrac{1}{2R^2}\sqrt{\dfrac{\pi}{y}}\operatorname{erf}\left(\sqrt{y}R^2\right), & \text{if } \alpha = 4, \end{cases} \tag{9}$$

where $\gamma(s, z) = \int_0^z t^{a-1}e^{-t}dt$ denotes the incomplete lower Gamma function. From $F_Y(y)$, we can get $f_Y(y)$ as

$$f_Y(y) = \begin{cases} \dfrac{1}{(Ry)^2}\left(1 - (1 + yR^2)e^{-yR^2}\right), & \text{if } \alpha = 2, \\ \dfrac{2}{3R^2}\left(\dfrac{2}{3y^{\frac{5}{3}}}\gamma\left(\dfrac{2}{3}, yR^3\right) - \dfrac{R^2}{y}e^{-yR^3}\right), & \text{if } \alpha = 3, \\ \dfrac{1}{4R^2}\sqrt{\dfrac{\pi}{y^3}}\operatorname{erf}\left(\sqrt{y}R^2\right) - \dfrac{1}{2y}e^{-yR^4}, & \text{if } \alpha = 4. \end{cases}$$

Finally, let us consider the stability of this system. If there exists a unique $m^* \in [0, M]$ in (3), the system is said to be stable. Otherwise, it is said to be unstable. In particular, if there are multiple m^*'s, e.g., three, the system is called bistable, which is also regarded as an unstable system.

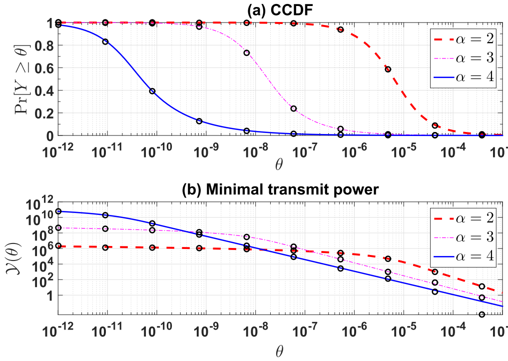

Fig. 1. CCDF of Y and the minimal transmit power with threshold θ.

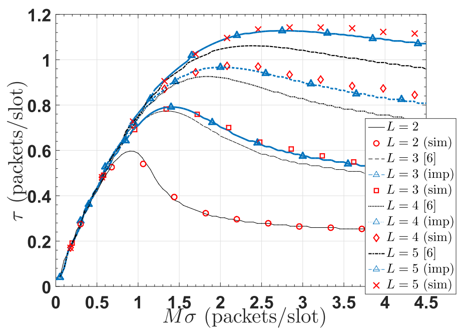

Fig. 2. Throughput with $M = 100$.

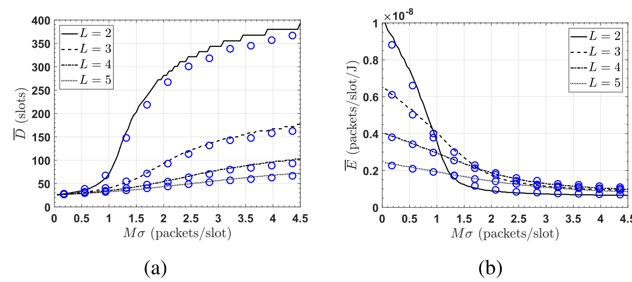

Fig. 3. (a) Average access delay; (b) energy efficiency with $M = 100$.

IV. Numerical Studies

This section compares our analysis with simulations and discusses the results. Fig. 1 shows the complementary CDF (CCDF) of Y and the minimal power $\mathcal{Y}(\theta)$ at various θ's, i.e., $\Pr[Y \geq \theta]$ to show the probability that UE's channel gain is good enough for a packet transmission for $R = 500$ (m). The lines denote the analysis, while the circles indicate the simulation results. As θ varies, it can be found that there exists a tradeoff between the probability for UEs to (re)transmit and the minimal transmit power and the dependency of $\Pr[Y \geq \theta]$ on α is substantial.

Figs. 2–3 illustrate τ, $\overline{D}$, and $\overline{E}$, respectively, for $R = 500$ (m), $\Gamma = 1$ with $N_0 = 1$, $M = 100$, and $\sigma \in [0.001, 0.045]$. Additionally, we set $\mu = 0.7979$, the outage threshold $\theta = 5 \times 10^{-9}$, and $\alpha = 4$, which gives $\Pr[Y \geq \theta] = 0.0501$; this gives $r = \mu \Pr[Y \geq \theta] = 0.04$. As an alternative, we can set $\theta = 7.85 \times 10^{-9}$ with $\mu = 1$, which also gives $r = 0.04$. This higher θ can improve EE. At the same threshold, if $\alpha = 3$, a high value of $\Pr[Y \geq \theta]$ is expected. We should adjust μ

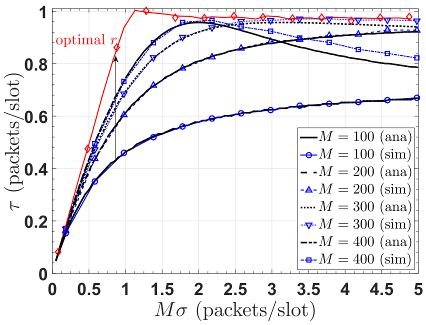

Fig. 4. Throughput with various population sizes.

in order to make the system stable. Thus, the role of μ is to neutralize the effect of θ with α to get a desired r, with which the system has no bistability or instability.

Fig. 2 depicts throughput τ for various L's. The lines without a mark indicate the lower bound based on (6) in [7], whereas the lines with symbol $\triangle$ show our improved lower bound, i.e., (6) together with (7). In comparison with simulations, our improved lower bound seems quite accurate for $L \leq 5$, while the lower bound in [7] becomes poor. If r increases for $L > 5$, our bound becomes inaccurate. However, the system setting of the power level that is more than five may occur rarely in practice. Note that τ and the mean access delay $\overline{D}$ are improved as L increases at the expense of transmit power. Since the analysis on τ is a lower bound, the average access delay in (4) is an upper bound, i.e., the analytical results of $\overline{D}$ is larger than simulations. As the improved lower bound on τ is tight, it is observed that the upper bound on $\overline{D}$ is also tight. Although the analysis on τ for $L = 2$ shows good agreement with the simulation, the graph of $\overline{D}$ seems staircase and shows a little discrepancy. This might be due to the fact that we select an integer m^* in (4). In Fig. 3(b) the best EE is found in the system with $L = 2$ when $M\sigma < 1$, whereas EE with $L = 3$ is reasonably good for $M\sigma > 1$.

Fig. 4 shows τ of the system with $L = 4$, $\Gamma = 1$, and $r = 0.01$, where $M\sigma$ remains a constant while M increases. Note that τ is quite insensitive to Γ. In other words, L strongly affects (the number of) the possible cases for a packet to be decoded successfully with respect to (1). It can be seen again that our improved lower bound accurately estimates τ as M increases. The line with symbol $\diamond$ indicates the simulation result for the system with $M = 100$, where the optimal retransmission probability $r = \min(2/m, 1)$ is used and the system knows m exactly every slot. We can see that τ is substantially enhanced and also find numerically that the maximum throughput achievable for $L = 2$, 3, 4, and 5 is 0.5967, 0.7826, 1.004, and 1.147, respectively.

Fig. 5 depicts the behavior of a bistable system with $M = 100$, $\sigma = 0.0086$, $\Gamma = 0.5$, $L = 3$, and $r = 0.0754$. In Fig. 5(a), the solid line $S_{in}(m)$ meets the (blue) dashed line $S_{out}(m)$ at three points, i.e., $m = 15$, 57, and 90. Fig. 5(c) depicts the probability density function (pdf) that the system has m backlogged UEs, which is bimodal; that is, two peaks are found around at $m = 15$ and 90. In accordance with it, it can be seen that the sample path of the number of backlogged UEs over time switches back and forth around from $m = 15$ and 90 in Fig. 5(d), which is not desirable.

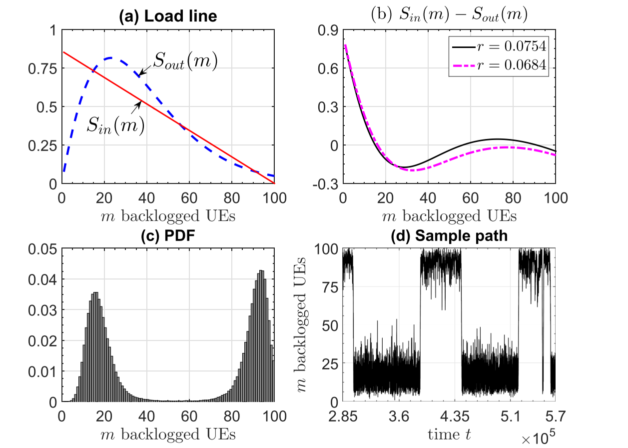

Fig. 5. Bistability of NOMA random access: $M = 100$, $\sigma = 0.0086$, $\Gamma = 0.5$, $L = 3$, $r = 0.0754$.

To eliminate it, increasing r does not help, since there will be still three points. On the other hand, reducing r gives a single $m = 17$ of satisfying (3) while getting rid of $m = 90$ as shown in Fig. 5(b). Since the difference between $S_{in}(m)$ and $S_{out}(m)$ is negative for $m > 17$, it implies that the average output is larger than the average input. Thus, it suggests that once the system detects such a bistability, it should ask UEs to use a lower r.

V. Conclusion

This letter has investigated throughput, mean access delay, EE, and stability of NOMA uplink random access systems, where Rayleigh fading and pathloss are taken into account. We have improved the lower bound on the throughput and the upper bound on the mean access delay. We have further showed an optimal retransmission probability to achieve the maximum throughput and the maximum achievable throughput for various levels of target powers.

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